CHAPTER IV

EMISSIONS AT HALF THE EQUATORIAL GYROFREQUENCY AND THE GROUP RESONANCE OF A WAVE PACKET

4.1 INTRODUCTION

So far we have studied only the effects of the resonant wave particle interactions on particle distribution functions in plasmas with special reference to the magnetospheric conditions and have discussed their possible relevance to the stimulation of VLF emissions. Looking back to the observed properties of VLF emissions, already mentioned in chapter I, we must remember that any mechanism proposed for the generation of these emissions should have the inherent capability to account for their properties.

One of the very cutstanding features of VLF emissions is that they tend to occur at a frequency close to half the equatcrial gyrofrequency (written as $(1/2)\,\Omega_{eq}$ hereafter) along the field line of propagation. It has also been observed that the artificial stimulation of these emissions is highly favourable if the frequency of the triggering pulse is close to $(1/2)\ \Omega_{eq}$.

This suggests that the wave particle interaction for waves at frequency $(1/2)\ \Omega_{eq}$ has some special features associated with it that favourably contribute to the triggering of emissions near that frequency. Therefore, it is desirable that the resonance condition and the propagation characteristics for the whistler mode waves be examined carefully to understand their behaviour at the frequency $\omega = \frac{1}{2}\ \Omega_{eq}$. Once this behaviour is understood and its difference from that at other frequencies is brought about, detailed investigation can be carried out to see whether it can really contribute to the favourable emissions at $\omega = (1/2)\ \Omega_{eq}$.

4.2 WHISTLER CHARACTERISTICS AT $\omega \simeq \frac{1}{2}\Omega_{eq}$:

The speeds of the Landau resonant and the gyroresonant particles are same if the wave frequency $\omega = \frac{1}{2}\,\Omega$.

This can be very easily seen from the general resonance condition:

$$\omega - k_{\parallel} v_{\parallel} = n\Omega$$

For the whistler mode waves, it gives

$$v_{\parallel L} = \frac{\omega}{k_{\parallel}} \quad \text{for } n = 0$$

and
$$v_{\parallel G} = \frac{\omega - \Omega}{k_{\parallel}} \quad \text{for } n = 1$$

Where $v_{\parallel L}$ and $v_{\parallel G}$ represent the Landau and the gyro resonant velocities respectively. Since $\omega < \Omega$ for whistlers, $v_{\parallel G}$ is always negative. However, the speeds $|v_{\parallel L}|$ and $|v_{\parallel G}|$ would be equal if $\omega = \Omega - \omega$ or $\omega = \frac{1}{2}\Omega$. This was pointed out by Dungey as an important feature that might contribute to preferential triggering at

$\omega = \frac{1}{2}\Omega_{eq}$. However, we have obtained the following two additional properties which can play a rather important role in triggering these emissions at this particular frequency.

i) The group velocity of a pulse would be roughly equal to the phase velocity at its central frequency provided the pulse is centred at half the electron gyrofrequency in a uniform magnetic field.

This can be easily inferred from the following expressions for the phase and the group velocities of a whistler mode pulse:

$$v_p = \frac{c}{\omega_p}\,\omega^{1/2}(\Omega\cos\theta - \omega)^{1/2}$$

and $$v_g = \frac{c}{\omega_p}\,\omega^{1/2}(\Omega\cos\theta - \omega)^{1/2}\left\{\frac{\Omega\cos\theta - \omega}{(\Omega\cos\theta)/2}\right\}$$

It is evident that $v_p = v_g$ at $\omega = \frac{1}{2}\Omega\cos\theta$. But θ has to be a small angle for the whistler mode dispersion relation to hold and therefore $\cos\theta$ may be taken as unity. Alternatively, we can say that

$$v_p \approx v_g \quad \text{at} \quad \omega = \frac{1}{2}\Omega \quad .$$

ii) The phase velocity v_p of a whistler mode wave has a maximum at $\omega \approx \frac{1}{2}\Omega$:

From the expression for v_p for a whistler mode wave quoted above, it is easy to see that it will have a maximum at $\omega = \frac{1}{2}\Omega \cos\theta$, the maximum value of v_p being $c\Omega\cos\theta/(2\omega_p)$. Since θ is small for whistler mode propagation, it turns out that the maximum in the phase velocity occurs for a value of ω close to $\frac{1}{2}\Omega$. The maximum of the Landau resonance velocity $\frac{\omega}{k_{\parallel}}$ would be $\frac{c\Omega}{2\omega_p}$ which is independent of the propagation angle θ. It may also be noted that particles with $v_{\parallel} = \frac{c\Omega}{2\omega_p}$ would gyro resonate with a pulse having frequency $\omega = \frac{1}{2}\Omega$ and wave vector anti parallel to $\vec{B}_o$.

Figure (4.1) shows the variation of the phase velocity of a pulse with its frequency ω . The maximum occuring at $\omega = \frac{1}{2}\Omega \cos\theta$ may be noted.

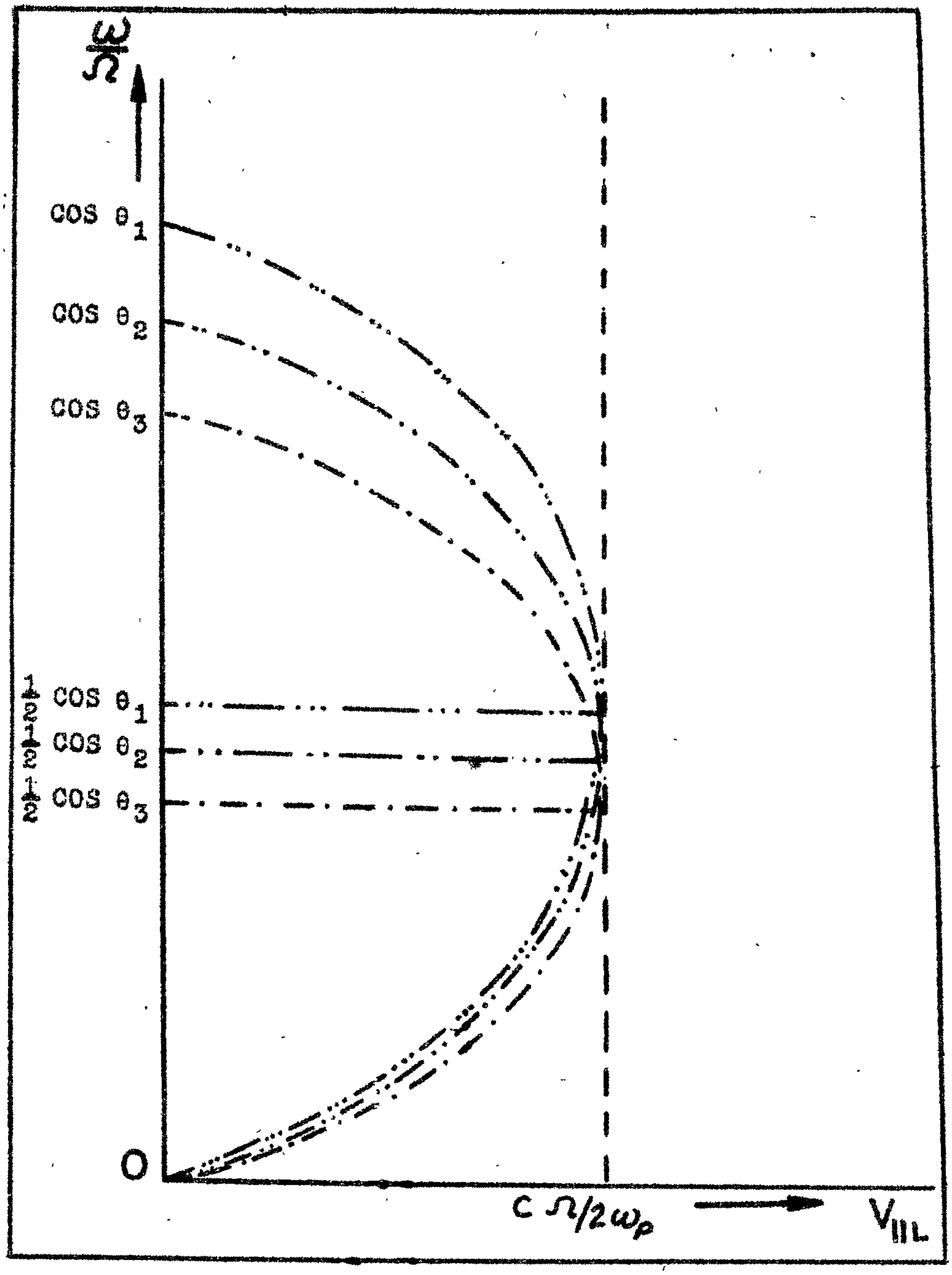

FIG 4.1

VARIATION OF $V_{\parallel L}$ THE LANDAU RESONANT VELOCITY, WITH NORMALISED FREQUENCY ω/Ω, FOR THREE DIFFERENT PROPAGATION ANGLES θ_1, θ_2 AND θ_3.

We would like to see if one or more of the above mentioned characteristics for whistlers at $\omega \simeq \frac{1}{2}\Omega$ can contribute to the mechanism for the observed favourable emissions at $\omega \simeq \frac{1}{2}\Omega_{eq}$.

4.3 A SHORT DESCRIPTION OF ABDELLA'S MODEL:

It was first pointed out by Dungey that the speeds of the Landau resonant and the gyroresonant particles are same at $\omega = \frac{1}{2}\Omega$ for whistler mode waves. Ashour Abdalla (1972), on this basis, qualitatively argued that her model for VLF emissions which was primarily based on gyroresonant effects would be able to explain the favourable emissions at $\omega = \frac{1}{2}\Omega_{eq}$ if the Landau resonant effects are also included in the thecry.

She considered the gyroresonant interaction of a succession of whistler mode pulses with particles mirroring in earth's radiation belts. The process leads to a diffusion of particles in velocity space.

The mirroring particles have their $v_{\parallel}$ maximum at the equator and the $v_{\parallel}$ that satisfies the gyro-resonant condition for a wave has a minimum there. The behaviour of $v_{\parallel}$ and $v_{\parallel G}$ has been shown in fig.4.2 which gives a plot of $v_{\parallel}$ and $|v_{\parallel G}|$ against distance S measured from the equator along a field line. Three curves have been shown for the variation of each of $v_{\parallel}$ and $|v_{\parallel G}|$ to clearly illustrate how they vary with S. The waves with different frequencies will have different curves for the corresponding $|v_{\parallel G}|$ as shown by the lables I, II and III. Similarly the changes in the $v_{\parallel}$ of the particles with S for different values of $v_{\parallel eq}$ will be given by different curves as shown by (i), (ii) and (iii). Here the $v_{\perp eq}$ of all the three particles has been assumed to be the same. If for a given ω, $|v_{\parallel eq}| < |v_{\parallel G eq}|$, no gyroresonance can take place (The subscript 'eq ' refers to the equatorial value). Therefore, the diffusion co-efficients for $|v_{\parallel eq}| < |v_{\parallel G eq}|$ would be negligibly small.

If $v_{\parallel eq} = v_{\parallel G eq}$, the resonance takes

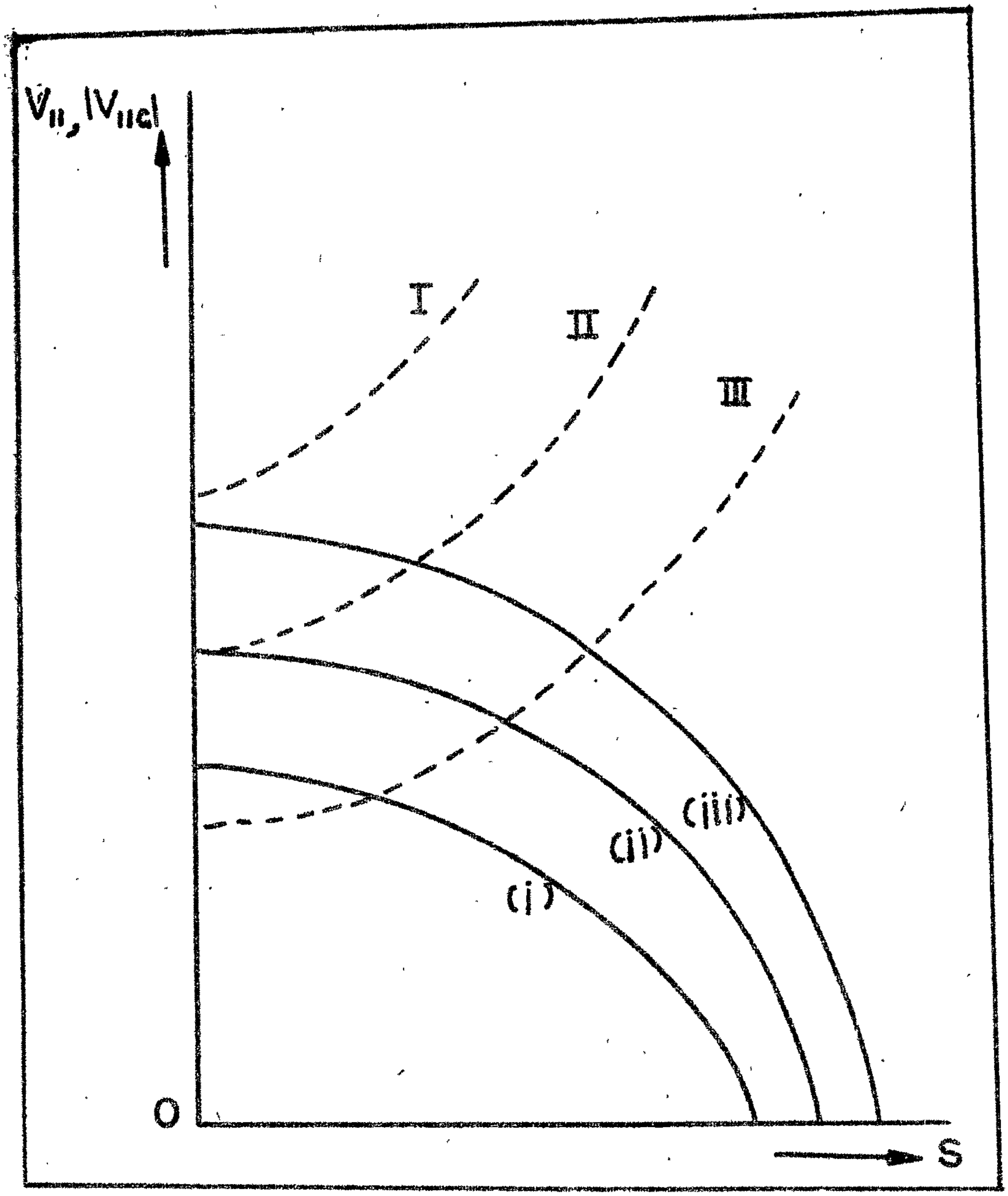

FIG 4.2

VARIATION OF $V_{\parallel}$ (CONTINUOUS CURVES) AND $|V_{\parallel G}|$ (DOTTED CURVES) WITH S, THE DISTANCE ALONG A FIELD LINE MEASURED FROM THE EQUATOR.

place right at the equator and is quite strong and therefore the diffusion co-efficient will suddenly rise at $v_{\parallel eq} = |v_{\parallel G eq}|$. As $v_{\parallel eq}$ exceeds $|v_{\parallel G eq}|$, the resonance point shifts more and more away from the equator where the nonuniformity of the field does not allow the resonance to be quite strong. Therefore the diffusion co-efficient would slowly go on decreasing as $v_{\parallel eq}$ exceeds more and more beyond $|v_{\parallel G eq}|$. Fig. 4.3 shows a typical plot of diffusion co-efficients against $|v_{\parallel eq}|$.

It is found that the changes in $v_{\parallel}$ and $v_{\perp}$ are not independent of each other. In the equatorial region the two changes are related by the following expression:

$$\frac{dv_{\perp eq}}{dv_{\parallel eq}} = - \frac{v_{\parallel eq}}{v_{\perp eq}\,(1 - \omega/\Omega_{eq})} \quad \text{---(4.1)}$$

which, on integration gives diffusion curves given by

$$\frac{v_{\perp eq}^{2}}{1 - \omega/\Omega_{eq}} + v_{\parallel eq}^{2} = \text{constt} \quad \text{---(4.2)}$$

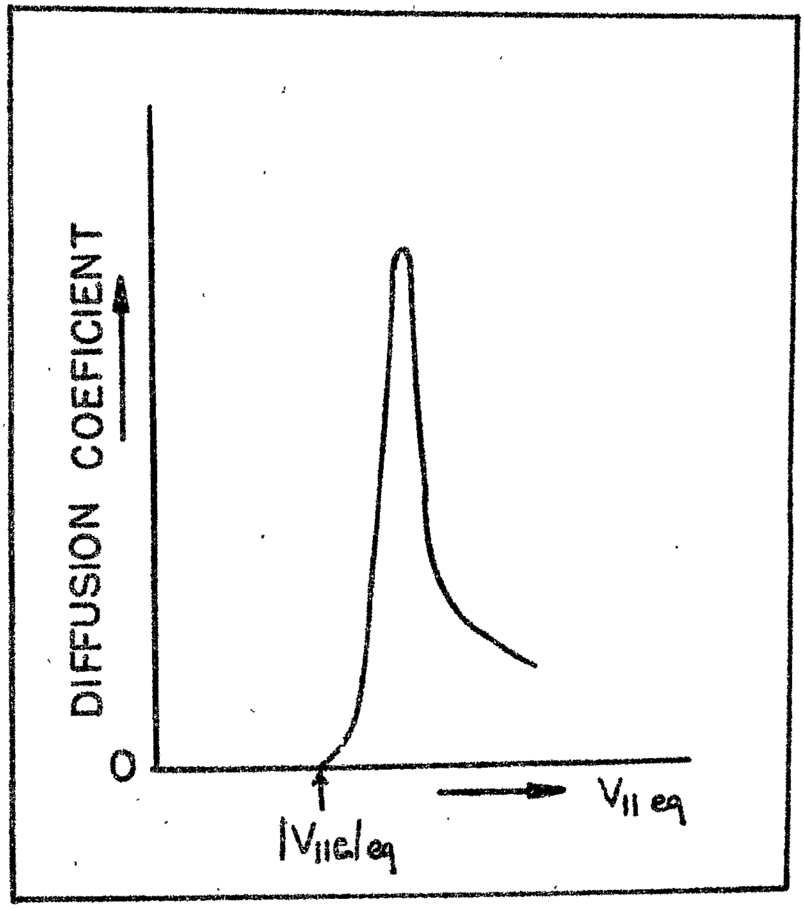

FIG 4.3

NATURE OF VARIATION OF DIFFUSION COEFFICIENT WITH $|V_{\parallel}|_{eq}$

(After Ashour Abdalla, 1972)

Which represents a family of ellipses (see fig.4.4). Since a loss cone α_0 exists in the magnetospheric particle pitch angle distribution, the particles lying close to the loss cone boundary would slowly diffuse into the loss cone along the diffusion curves and get lost. Since the diffusion co-efficients are quite large at $v_{\parallel eq} = |v_{\parallel G\, eq}|$ in the velocity space, the most efficient loss of particles would occur in a region which is close to the point of intersection of the lines $\alpha = \alpha_0$ and $v_{\parallel eq} = |v_{\parallel G\, eq}|$. The fast diffusion occuring there will lead to the formation of a slot in the velocity distribution of the particles. The large negative value of the gradient of f with $|v_{\parallel}|$ near the slot leads to large growth rates for resonant waves at corresponding frequencies (fig.4.5).

She considers the effects of resonances to be similar to those of collisions as far as the changes in the energy W and the magnetic moment μ of the particles are concerned. The analogy is further strengthened by the assumption that the distribution of phase angle between the velocity vector and the wave

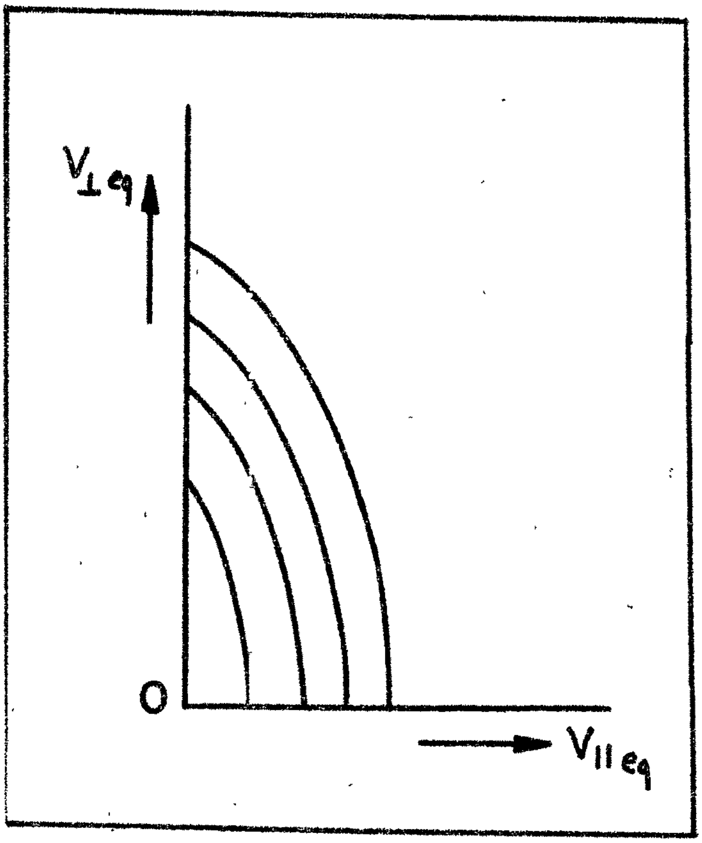

FIG 4.4

NATURE OF THE EQUATORIAL GYRORESONANCE DIFFUSION CURVES IN $V_{\parallel} - V_{\perp}$ SPACE

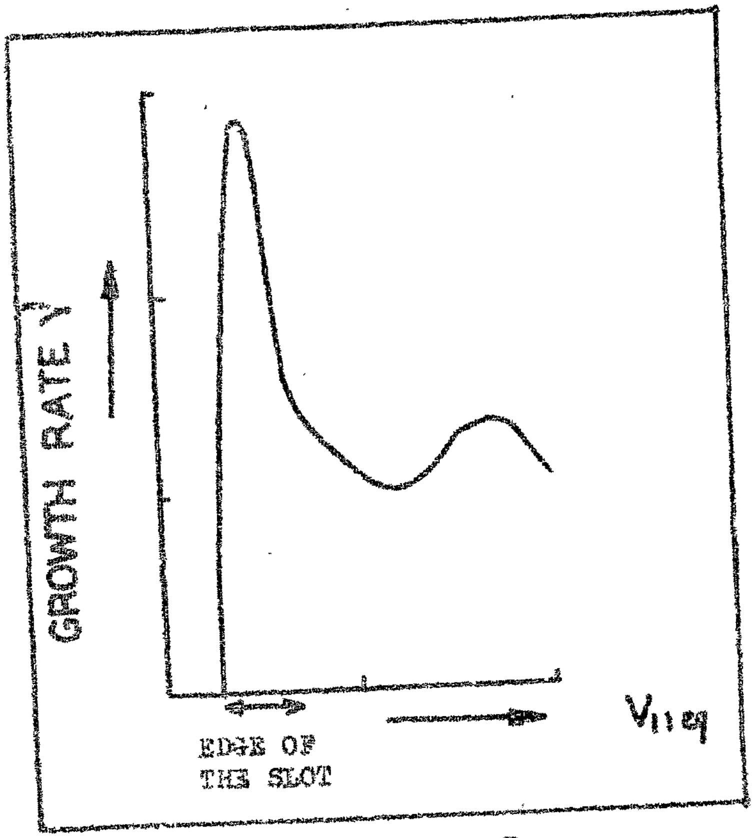

FIG 4.5

TYPICAL BEHAVIOUR OF GROWTH RATE γ WITH VARIATION IN $V_{\parallel eq}$

(Based on the calculations of Ashour Abdalla, 1972) field vector is random and therefore changes in W and μ are also random. Thus the analysis of the process can be handled using the one dimensional Fokker Planck equation.

4.4. A PARALLEL MODEL FOR LANDAU RESONANT INTERACTION:

The Landau resonant velocity $v_{\parallel L} = \frac{\omega}{k_{\parallel}}$ increases as we move away from the equator along a field line. On the other hand, particle $v_{\parallel}$ will decrease as it moves away from the equator along the same field line. The behaviour of $v_{\parallel L}$ here, is similar to that of $|v_{\parallel G}|$ illustrated in figure (4.2). It is evident that a particle must have a $v_{\parallel eq}$ greater than $v_{\parallel L eq}$ in order to resonate at some point or other on the field line with a given wave. This would mean that for values of $v_{\parallel eq} < v_{\parallel L eq}$, the diffusion co-efficients in velocity space, caused by the Landau resonant interaction would be practically zero. For $v_{\parallel eq} \geq v_{\parallel L eq}$ the value of the diffusion co-efficient will depend upon the position of the point on the field line at which the resonance takes place. The resonance for $v_{\parallel eq} \simeq v_{\parallel L eq}$ will take place near the equator and would therefore be strong and will last longer giving large diffusion co-efficients. The resonance for $v_{\parallel eq} >> v_{\parallel L eq}$ would take place quite away from the equator and its effective duration would be small because of the dominance of the nonuniform magnetic field. This would lead to comparatively smaller diffusion co-efficients. Thus the nature of the diffusion co-efficient curve plotted against the distance from equator remains same as in the gyroresonance case (fig.4.3).

It is found that the changes in $v_{\perp}$ during Landau resonance are negligible and $\frac{dv_{\perp eq}}{dv_{\parallel eq}}$ may be considered nearly equal to zero. Thus the resonant diffusion of particles will occur along the straight lines given by $v_{\perp eq}$ = constant. The region of strongest diffusion will lie in the close proximity of the point of intersection of the lines given by $v_{\parallel eq} = v_{\parallel L eq}$ and $\alpha = \alpha_0$ which defines the loss cone. Through a mechanism similar to the one discussed by Abdalla for gyro resonant case, a slot will be developed in the distribution function in velocity space for the Landau resonance case also.

The negative gradients of f with $v_{\parallel}$ developed at the lower $v_{\parallel}$ boundary of the Landau resonant slot will give growth to the perturbations at corresponding frequency.

The two slots are, in general, formed at two different places in the velocity space, one near $v_{\parallel eq} = |v_{\parallel G eq}|$ and the other near $v_{\parallel eq} = v_{\parallel L eq}$. If $v_{\parallel L eq}$ and $|v_{\parallel G eq}|$ are equal, the two slots will overlap and the effect of the slots on the growth rate shall be enhanced. This might cause the favourable emissions at $\omega = \frac{1}{2}\Omega_{eq}$.

4.5 DUNGEY'S SUGGESTION AND ITS CRITICAL APPRECIATION:

Dungey had suggested earlier that for pulses with their central frequency $\omega = \frac{1}{2}\Omega_{eq}$, the effect of the Landau resonant diffusion would reinforce the effect of the gyroresonant diffusion in distorting the distribution function and thus enhancing the growth of the perturbations. This is because the Landau resonant and the gyroresonant particle speeds are same. However, the Landau resonance effects are usually weak compared to the gyroresonant effects mainly because the propagation angle θ for whistler mode waves is small and for small values of θ, $E_{\parallel}$, the parallel component of the wave electric field $\vec{E}$ is rather small compared to $E_{\perp}$, the perpendicular component of $\vec{E}$.

Under such circumstances, contribution of Landau resonant effect to the gyroresonant effect at $\omega = \frac{1}{2}\Omega_{eq}$ may not be able to account for the observed favourable emissions at this frequency unless there is some other process which strengthens the Landau resonance effects at $\omega = \frac{1}{2}\Omega_{eq}$. The simple combinations of Landau and the gyro resonances may not be able to explain the preferential triggering of VLF emissions at $\omega = \frac{1}{2}\Omega_{eq}$.

4.6 THE GROUP RESONANCE OF A WAVEPACKET AT $\omega = \frac{1}{2}\Omega$.

The effect of a resonance would be strengthened if the waves at different frequencies inside a wavepacket simultaneously resonate with the same particle and thus reinforce the effect of each other. We would like here to investigate the condition governing the simultaneous resonance of a group of whistler mode waves with the plasma particles.

A wave with frequency ω and wave vector $\vec{k}$ inside a wave packet will experience 'Landau resonance' (hereafter in this section mentioned only as 'resonance') with a particle if

$$\omega - k v_{\parallel} \cos\theta = 0 \qquad ---(4.3)$$

where $v_{\parallel}$ is the particle velocity component along $\vec{B}o$ and θ the angle between $\vec{k}$ and $\vec{B}o$.

Another wave in the same wave packet with frequency $\omega + \Delta\omega$ and wave vector $\vec{k} + \Delta\vec{k}$ will have resonance with the same particle if

$$(\omega + \Delta\omega) - (k + \Delta k)\, v_{\parallel} \cos\theta = 0 \qquad \text{---}(4.4)$$

Assuming $\Delta \vec{k}$ to be in the same direction as $\vec{k}$ and solving equations (4.3) and (4.4) together, we get

$$\Delta\omega = v_{\parallel}\, \Delta k \cos\theta$$

or, $$\frac{\Delta\omega}{\Delta k} = v_{\parallel} \cos\theta \qquad \text{---}(4.5)$$

From equation (4.3),

$$v_{\parallel} \cos\theta = \frac{\omega}{k}$$

Substituting this in eq. (4.5) we get

$$\frac{\Delta\omega}{\Delta k} = \frac{\omega}{k}$$

which in the limit $\Delta\omega \rightarrow 0$ and $\Delta k \rightarrow 0$ passes over to

$$\frac{d\omega}{dk} = \frac{\omega}{k} \qquad \text{---}(4.6)$$

Equation (4.6) can be interpreted as the following:

'All the waves within a narrow band wavepacket will resonate with the same particle at the same time if the condition

$$v_{\parallel} \cos\theta = v_{po} = v_g \qquad \text{--- (4.7)}$$

is satisfied where v_g is the group velocity of the wavepacket and v_{po}, the phase velocity of the central wave of the wavepacket'.

Effectively speaking, the whole group of waves inside the pulse comes in resonance with the same set of particles thereby making the interaction stronger and more effective.

The condition $v_{po} = v_g$ may not always be satisfied for all types of plasma waves. Nevertheless, for whistlers, a frequency exists at which the condition is fulfilled. The cold plasma whistler mode dispersion relation

$$\frac{c^2k^2}{\omega^2} = \frac{\omega_p^2}{\omega(\Omega\cos\theta - \omega)}$$

leads to the following expressions for v_{po} and v_g :

$$v_{po} = \frac{c}{\omega_p}\left\{\omega(\Omega\cos\theta - \omega)\right\}^{1/2} \quad (4.8)$$

and

$$v_g = v_{po}\,\frac{\Omega\cos\theta - \omega}{\frac{1}{2}\Omega\cos\theta} \quad (4.9)$$

From (4.8) and (4.9), it is clear that $v_{po} = v_g$ at $\omega = \frac{1}{2}\Omega\cos\theta$. Since θ is small, a whistler mode pulse centred around $\omega \simeq \frac{1}{2}\Omega_{eq}$ would undergo most efficient Landau resonance with electrons having appropriate $v_{\parallel}$. We will refer to this phenomenon of the resonance of the whole group of waves inside a narrow band pulse as the 'group resonance' or 'pulse resonance' here after.

4.7 CONCLUSIONS:

Thus we have seen how the group resonance of a wave packet comes into picture when the condition $v_{po} = v_g$ is satisfied. The group resonance essentially means the reinforcement of the effects ~~of~~ of different waves by one another and this feature is absent in the ordinary resonance of a wave with particles.

Here the important point is that the range of particle $v_{\parallel}$ affected by the waves is quite narrow. The whole spectrum of waves exerts its effect on particles in this narrow range thereby making the resonant effects quite strong.

Thus, although the Landau resonance of whistlers is, in general, weak, it becomes quite strong at $\omega = \frac{1}{2} \Omega$. In the light of this fact we must reconsider Dungey's suggestion that the joint effect of the Landau and the gyro resonances might account for the favourable triggering at $\omega = \frac{1}{2} \Omega_{eq}$. But before studying the effects of the two together let us consider qualitatively how the group Landau resonance can give rise to emissions.

The Landau resonance will cause diffusion of particles in velocity space along diffusion curves given by $v_\perp$ = constt. where different values of the constant refer to different members of the family (see fig.4.6). The maximum diffusion will take place near the group resonant $v_\parallel$. The diffusion at all other places can be neglected in comparison to this. The direction of diffusion will be from the contours of higher f values to those of the lower f values and therefore it will point towards the loss cone boundary (see fig.4.6). The region close to the point of intersection of group resonant $v_\parallel$ and the loss cone boundary would be affected to the maximum limit. Particles from this region will migrate into the loss cone and consequently a slot will be developed at that point which will give growth to perturbation at the corresponding frequency.

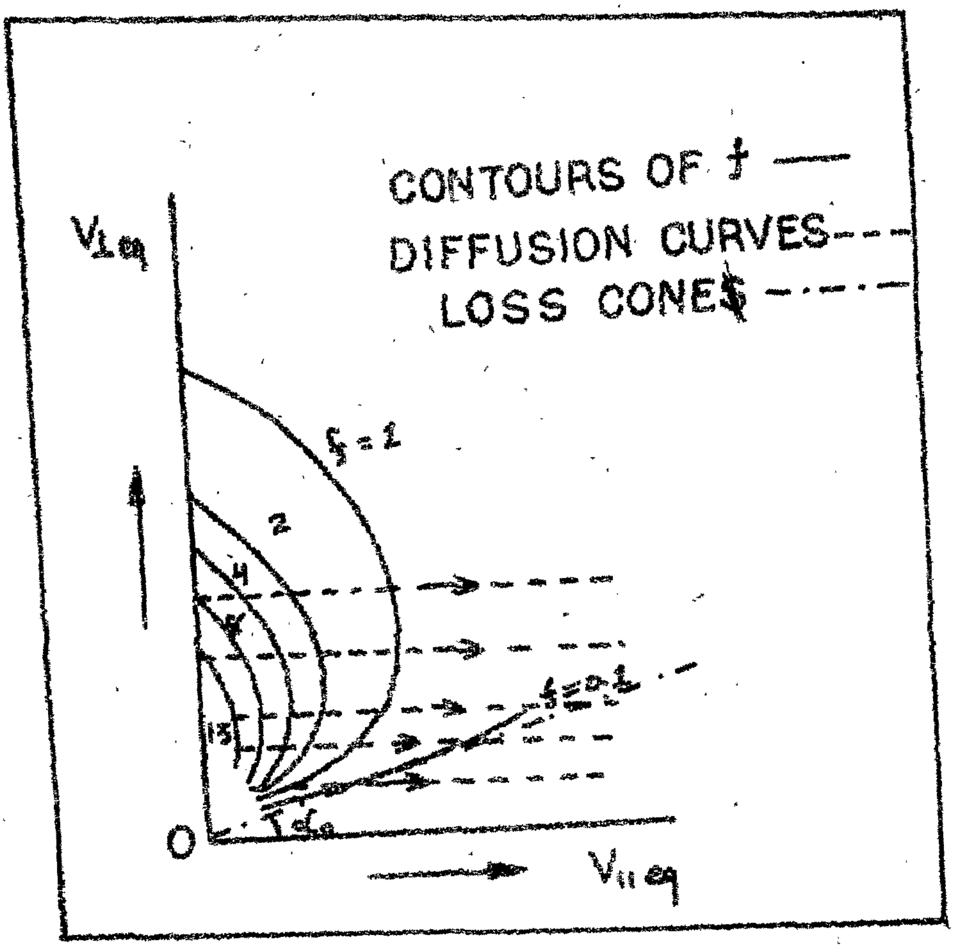

FIG 4.6

CONTOURS OF CONSTANT f AND DIFFUSION CURVES FOR EQUATORIAL LANDAU RESONANCE. THE DIRECTION OF DIFFUSION IS INDICATED BY ARROWS.

(Contours of f are after Ashour Abdalla, 1972)

CALCULATION OF GROWTHRATE:

The slot is formed in the neighbourhood of $\vartheta_{\parallel} = \vartheta_{\parallel GLR}$ (Group Landau resonant velocity). The distribution $f(\vartheta_{\parallel}, \vartheta_{\perp})$ for $\vartheta_{\perp} \approx \vartheta_{\parallel GLR} \tan \alpha_0$ suffers a steep fall in its value with a slight increase in $\vartheta_{\parallel}$ at the lower edge of the slot. This behaviour of f may be approximated by a step of f at $\vartheta_{\parallel} = \vartheta_{\parallel GLR}$ and therefore the partial derivative $\frac{\partial f}{\partial \vartheta_{\parallel}}\Big|_{v_{\parallel} = v_{\parallel GLR}}$ may be represented by $-\Delta f_{\parallel}\, \delta(\vartheta_{\parallel} - \vartheta_{\parallel GLR})$ where $-\Delta f_{\parallel}$ is the step and the symbol δ represents the Dirac δ function. The quantity $\Delta f_{\parallel}$ has been considered to be positive and the negative sign has been attached to it to indicate that the step is negative.

During Landau resonance, the changes in $\vartheta_{\perp}$ are very small and therefore the temporal charg es in the partial derivative of f with $\vartheta_{\perp}$ will be negligible. Again during the gyroresonance the counters of constant f (Ashour Abdalla, 1972 , see fig.4.7) at the slot exhibit the gradient direction to be parallel to the $\vartheta_{\parallel}$ axis.

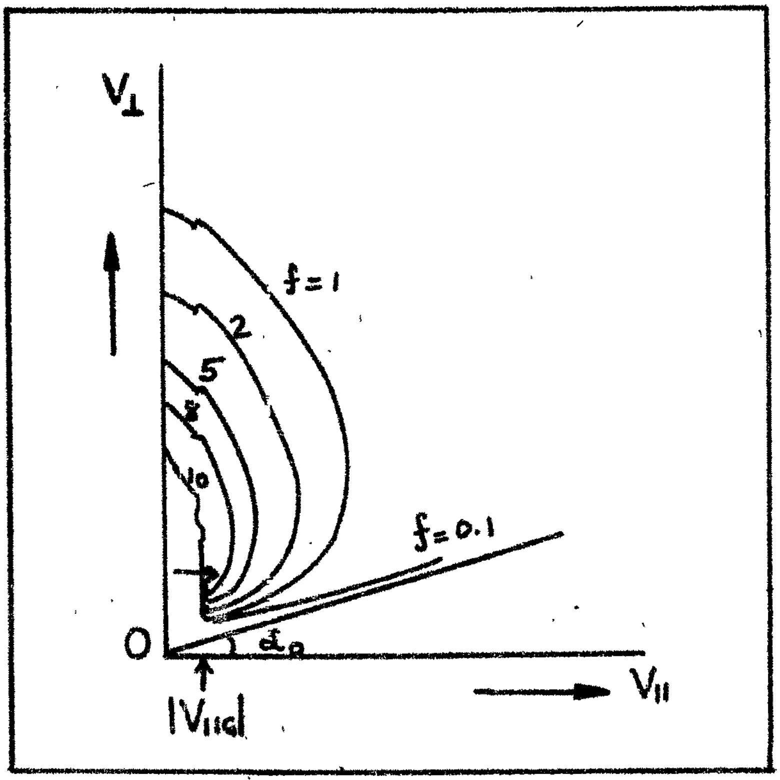

FIG 4.7

TYPICAL CONTOURS OF CONSTANT f IN A DISTRIBUTION MODIFIED BY GYRORESONANT DIFFUSION. VERTICAL ARROW INDICATES THE POSITION OF SLOT AND THE HORIZONTAL ARROW POINTS TO THE DIRECTION OF GRADIENT OF f .

(Contours of f are after Ashour Abdalla, 1972)

However, because of the presence of the loss cone, f will have a step at all $v_\perp$'s given by $v_\perp = v_\parallel \tan\alpha_0$. This step will be positive and we will denote its value at $v_\parallel = v_{\parallel GLR}$ by $+\Delta f_\perp$ and replace $\frac{\partial f}{\partial v_\perp}$ by $\Delta f_\perp \, \delta(v_\perp - v_\parallel \tan\alpha_0)$ for the sake of approximate growth rate calculations.

The gyroresonant growth rate γ at the edge of the slot in the distribution function for a VLF whistler mode perturbation in the system under consideration is given by (Vedenov et al. 1972):

$$\gamma = -\frac{\pi k^2 v_{res}^2}{n\Omega} \int_0^\infty \left[v_\perp \frac{\partial f}{\partial v_\parallel} - (v_\parallel - v_p) \frac{\partial f}{\partial v_\perp} \right]_{v_\parallel = v_{res}} v_\perp^2 \, dv_\perp \quad (4.10)$$

Here n represents the electron concentration and v_{res} the gyroresonant velocity $v_{\parallel G}$ for the perturbation. Since the resonance is taking place at $v_\parallel \simeq v_{\parallel GLR}$, which occurs at $\omega \simeq \frac{1}{2}\Omega$, $v_{\parallel G}$ and $v_{\parallel GLR}$ will be roughly equal to each other. Thus we have the relationship

$$v_{\perp GLR} \simeq v_{res} \quad \text{---(4.11)}$$

Other symbols in equation (4.10) have their usual meanings as defined earlier.

Substituting $\frac{\partial f}{\partial v_{\parallel}}$ and $\frac{\partial f}{\partial v_{\perp}}$ in eq.(4.10) by their approximately equivalent δ function representations, $-\Delta f_{\parallel}\ \delta(v_{\perp}-v_{\parallel}\tan\alpha_0)$ and $\Delta f_{\perp}\ \delta(v_{\perp}-v_{\parallel}\tan\alpha_0)$ respectively in accordance with our previous discussion, we get;

$$\gamma = \frac{\pi k^2 v_{res}^2}{n\Omega}\int_0^{\infty} v_{\perp}^2 \left[v_{\perp}\Delta f_{\parallel} - (v_{\parallel}-v_p)\Delta f_{\perp}\right]_{v_{\parallel}=v_{res}} \cdot \delta(v_{\perp}-v_{\parallel}\tan\alpha_0)\, dv_{\perp} \quad \text{---(4.12)}$$

which, on integration gives:

$$\gamma = \frac{\pi k^2 v_{res}^2 v_{\perp}^2}{n\Omega}\left[v_{\perp}\Delta f_{\parallel} - (v_{\parallel}-v_p)\Delta f_{\perp}\right]_{v_{\parallel}=v_{res}} \quad \text{---(4.13)}$$

Using eq. (4.11) in conjunction with the following relations

(i) $v_\perp \simeq |v_{res}| \tan\alpha_0$ (at the slot)

and (ii)

$$\omega = \frac{1}{2}\Omega$$

$$k\Big|_{\omega=\frac{1}{2}\Omega} = \frac{\omega_p}{c}$$

$$v_{res}\Big|_{\omega=\frac{1}{2}\Omega} = -\frac{1}{2}\frac{c\Omega}{\omega_p}$$

$$v_p\Big|_{\omega=\frac{1}{2}\Omega} = \frac{1}{2}\frac{c\Omega}{\omega_p}$$

For the gyro-resonant perturbation assumed to be propagating $\parallel$ B_o

we can reduce eq. (4.13) to

$$\gamma = \frac{\pi}{16}\frac{\Omega^4 c^3 \tan^2\alpha_0}{n\omega_p^3}\left\{\frac{1}{2}\tan\alpha_0\,\Delta f_\parallel + \Delta f_\perp\right\} \qquad (4.14)$$

which gives the growth rate at the edge of the slot. Evidently, γ increases with $\Delta f_\parallel$, the step in f at the slot which, in turn, depends upon the strength of the slot-forming mechanism.

In the light of what has been discussed so far in this chapter, we can make the following concluding remarks:

The slots in the distribution will be formed in the regions in velocity space where the probability for the particles to be lost into the loss cone due to the diffusion by the waves is large compared to the remaining areas. A slot region would be strong if the probability for the loss of the particles is large as is the case with the group Landau resonance or with the gyro resonance. A slot region would be weak if the probability of loss of particles from there is small as is the case with ordinary Landau resonance.

In general, for a whistler mode wave-packet, there are two slot regions, one strong due to the gyroresonance and the other weak due to the Landau resonance. Also, they are located away from each other. However, if $\omega = \frac{1}{2}\Omega$, both the slot regions are strong and at the same time they overlap each other thereby greatly enhancing the probability of the loss of particles into the loss cone. This would lead to the formation of a steep edge and consequently to the development of a highly negative gradient of f with $|v_{\parallel}|$ at the lower $v_{\parallel}$ boundary of the slot. The large negative step thus formed should favour emissions at half the equatorial gyrofrequency (Vyas and Das, 1975b) which is supported by the observations (Helliwell, 1969).