CHAPTER V

THE EFFECT OF LANDAU DAMPING ON CYCLOTRON RESONANCE IN A UNIFORM MAGNETOPLASMA

5.1 INTRODUCTION:

In the last chapter we studied how the effects of the Landau resonance and the gyroresonance would combine together to give enhanced probabilities for emissions at half the equatorial gyrofrequency. A question naturally arises as to whether this is the unique mechanism that supports or favours emissions at this particular frequency, or there are other mechanisms which also contribute to this phenomenon. An attempt has been made in this chapter to answer this question. We take up one of the very important features of the whistler mode dispersion relation already mentioned in the last chapter, namely that the Landau resonant $V_{\parallel}$ has a maximum value that corresponds to a frequency $(1/2)\,\Omega_{eq}$. Before studying the consequences of this, we would like to investigate the properties of a distribution function distorted through a Landau resonant process by the whistler mode wave. In particular, we would be interested in studying how the gyro resonant perturbations would be affected by such a modified distribution and whether the growth of perturbations is likely under such situations. We would apply the results of our study to the specific case of whistler mode Landau resonance, which exhibits the unusual occurence of a maximum in the value of the resonant $V_{\parallel}$, to see if such a process is likely to contribute to emissions at half the equatorial gyrofrequency.

5.2 LANDAU DAMPING AND CYCLOTRON RESONANCE IN UNIFORM MAGNETOPLASMA:

We consider a homogenous collisionless hot plasma immersed in a uniform static magnetic field $\vec{B}_0$ acting along z direction. We also consider a spectrum of electron plasma waves propagating in the negative z direction. This is our zero order system and our interest lies in finding how a wide band VLF perturbation propagating in the whistler mode in positive z direction would be affected by this system.

The Landau resonance of the electron plasma waves would lead them to damping. The growth rate for such an interaction is given by (Vedenov, 1963)

$$\gamma_L = \frac{\pi}{2}\left\{\omega_p^3 / k|k|\right\}\left\{\partial f/\partial v_{\parallel}\right\}\Big|_{V_{\parallel}=V_{res}} \quad ---(5.1)$$

where γ_L is the Landau growth rate, ω_p the electron plasma frequency and, $\vec{k}$, $\vec{v}$ and the subscript '$\parallel$' are used to refer to the wave vector, the particle velocity and the parallel component of a vector respectively, as also used in earlier chapters. However, here we use the symbol V_{res} for the phase velocity $\frac{\omega_p}{k}$ of the electron plasma waves. Since $\vec{k}$ for these waves has been assumed to be negative, V_{res} would also be negative. 'f' represents the normalised distribution function of plasma electrons with respect to $V_{\parallel}$ and $V_{\perp}$, unless otherwise specified through the use of proper arguments, e.g. $f(\alpha, v, t)$. $f(v_{\parallel}, v_{\perp})$ satisfies the following condition:

$$\iint f(v_{\parallel}, v_{\perp})\, v_{\perp}\, dv_{\perp}\, dv_{\parallel} = 1$$

To study the dynamics of the resonant interactions in the system, we would like to apply quasilinear theory. The theory considers that the plasma is in thermal equilibrium and the waves cause only a perturbation to its equilibrium state. Furthermore, it takes into account only the wave particle interactions and neglects both the wave-wave interactions and the particle-particle interactions. Therefore, the validity of the theory is subject to the restriction that the wave energy density in the system is much smaller compared to the thermal energy density of the particles and at the same time it is much larger compared to the energy density associated with the Coulomb interaction of the particles in the system. We will assume that the system under our consideration fulfills this condition.

The basic idea of quasilinear theory is that the growth or damping of plasma waves caused by a particular distribution function would slowly modify the distribution itself and then this modified distribution, in turn, modifies the growth or damping of the waves. The process continues unless some stable equilibrium state between the waves and the plasma is reached.

To apply quasilinear theory we divide the distribution function of the resonant particles into two parts: one rapidly oscillating and another slowly varying with time. The coupling of the mean square of the oscillating part with the slowly varying part leads us to the fact that the behaviour of the slowly varying part, $\bar{f}$, of the distribution can be described by the following equation of diffusion in phase space (Vedenov, 1963)

$$\frac{\partial \bar{f}}{\partial t} = \frac{\partial}{\partial v_{\parallel}} \left\{ \frac{4\pi e^2}{m^2 v_{\parallel}} \frac{E_k^2}{8\pi} \frac{\partial \bar{f}}{\partial v_{\parallel}} \right\} \quad \text{---} (5.2)$$

where t represents time, e and m the electron charge and mass and $E_k^2/8\pi$ the energy density of the waves. The growth rate γ_L of plasma waves will also vary with time as given by equation (5.1) where $\overline{f}$ is considered as the nonoscillating part of the distribution, varying slowly with time.

The growth rate γ_c for the whistler mode perturbations as a result of cyclotron resonance with electrons is given by (Kennel and Petschek, 1966).

$$\gamma_c = \pi\Omega\left(1-\omega/\Omega\right)^2 \eta(V_R)\left\{A(V_R) - \omega/(\Omega-\omega)\right\} \qquad (5.3)$$

where ω and Ω are the wave frequency and the electron gyrofrequency, V_R, is the cyclotron resonant velocity which is negative in magnitude and is given by the relationship

$$\omega - kV_R = \Omega$$

also

$$\eta(V_R) = 2\pi\left\{\frac{\Omega-\omega}{k}\right\}\int_0^\infty v_\perp dv_\perp\, f\left(v_\perp, v_\parallel = V_R\right) \qquad (5.3a)$$

and

$$A(V_R) = \left[\frac{\left\{ \int_0^\infty v_\perp \, dv_\perp \left(v_\parallel \frac{\partial f}{\partial v_\perp} - v_\perp \frac{\partial f}{\partial v_\parallel} \right) \frac{v_\perp}{v_\parallel} \right\}}{\left\{ 2 \int_0^\infty v_\perp \, dv_\perp f \right\}} \right]_{V_\parallel = V_R} \qquad (5.3b)$$

The cyclotron growth of the perturbation would affect the particle pitch angle distribution function. A quasilinear treatment of this process would lead us to the following expression for the time evolution of the slowly varying part of the distribution function (Kennel and Petschek, 1966).

$$\frac{\partial f(\alpha, v, t)}{\partial t} = \frac{\pi e^2}{m^2 \sin\alpha} \frac{\partial}{\partial \alpha} \left\{ \frac{\sin\alpha \, . \, B_K^2}{|v \cos\alpha|} \frac{\partial f}{\partial \alpha} \right\} \qquad (5.4)$$

Here α is the pitch angle of the particles and $B_k^2/8\pi$ the magnetic energy density of the whistler mode waves. The expression has been derived under the assumption $\omega \ll \Omega$. $v_\parallel$ and $v_\perp$ of the particles are related to V and α through the relations

$$v_{\parallel} = v \cos \alpha$$

and,
$$v_{\perp} = v \sin \alpha$$

If, in our system, the range of velocities which the Landau resonance takes place has an overlap with the range of velocities having gyroresonance, the change in the distribution function over the overlapping range of $V_{\parallel}$ would be simultaneously affected by the two types of resonant interactions. Under these circumstances the time evolution of the distribution function can be obtained by solving equations (5.2) and (5.4) together as a pair of simultaneous partial differential equations.

An analytical solution of these two equations is quite complicated and the numerical solution would also be highly time consuming even on a high speed computer. Therefore here we would try to analyse the problem in a somewhat qualitative manner so as to understand how the system would behave as the time passes.

The gyroresonant growth rate γ_c as defined in equation (5.3) is a function of $\partial f/\partial v_{\parallel}$ which itself changes with time as a result of the Landau resonant interaction. This change in γ_c affects the pitch angle distribution of the particle which then also starts varying with time in accordance with equation (5.4). The role of this equation (derived with the assumption $\omega \ll \Omega$) is to isotropize the distribution function in pitch angle. In general, the distribution function becomes constant along the diffusion curves in velocity space. Or, in other words, the diffusion curves become the contours of constant f. Thus the whole pitch angle distribution f(α) is reshaped. Now f(α) is a function of the distribution with respect of $V_{\parallel}$ and distribution with respect to $V_{\perp}$. Thus a change in f(α) and $\frac{\partial f}{\partial \alpha}$ would cause a corresponding change in $f(V_{\parallel})$ and $\frac{\partial f}{\partial v_{\parallel}}$ respectively. This change in $\partial f/\partial v_{\parallel}$ would again modify the Landau resonant growth rate γ_L. Thus a coupling is established between the tow types of growths.

In order to simplify the situation, we will introduce here certain restrictions. We presume that to start with, the whistler mode perturbations have amplitudes sufficiently small such that any change in the particle distribution caused by them is negligible compared to that caused by the Landau resonant interaction. With time the perturbations will grow and after some time they might become strong enough to react back and modify the particle distribution. This situation would be quite complicated to study, and therefore, we will not consider it here. We will restrict our attention only to that stage of interaction where the growth of the perturbations is insufficient to react back on the distribution function and cause any significant modification therein. This restriction will enable us to consider only the effect of the electron plasma waves on the growth of whistler mode perturbations neglecting the reciprocal effect of the whistler mode perturbations on the growth of electron plasma waves.

It is well known that the quasilinear diffusion process due to the Landau resonance of electron plasma waves leads to the formation of a ledge in the electron distribution function with respect to $V_{\parallel}$ over the range of resonant velocities (see fig.5.1). 'k' being negative, the ledge forms over negative values of $V_{\parallel}$ and the distribution f becomes constant over the length of the ledge. Therefore, $\partial f/\partial v_{\parallel}$ would become zero over the entire range of resonant $V_{\parallel}$ except at the boundaries of this region where steep gradients of f with $V_{\parallel}$ will be formed and the value of $\partial f/\partial v_{\parallel}$ shall become very large as shown in figure 5.1.

Under the assumption $\omega \ll \Omega$ equation (5.3) can be rewritten as

$$\gamma_c = \pi \Omega \eta(V_R) A(V_R)$$

where $\eta(V_R)$ and Ω are always positive and only the positive or negative sign of the value of $A(V_R)$ will decide whether the waves are growing or damping. Taking a distribution in which f monotonically decreases with increase in $|v_{\parallel}|$ and $v_{\perp}$, and noting that V_R is negative, we can reduce equation (5.3b) to the following form:

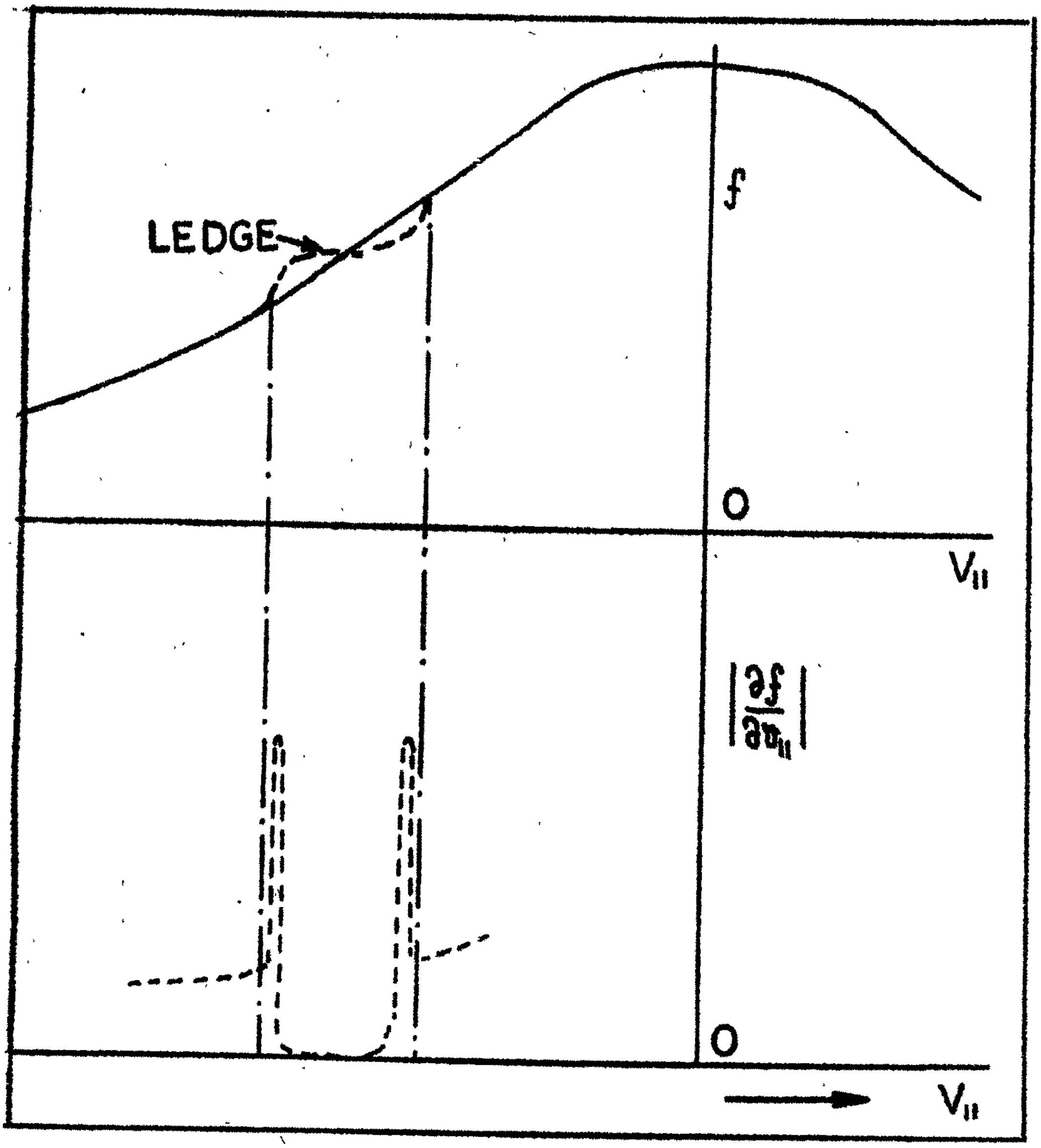

FIG 5.1

f AND $\frac{\partial f}{\partial V_{\parallel}}$ PLOTTED AGAINST $V_{\parallel}$ DURING THE LEDGE FORMATION. STEEP RISE IN f AT THE BOUNDARIES OF THE LEDGE GIVES THE PEAKS IN $\partial f/\partial V_{\parallel}$

$$A(V_R) = \left[\int_0^\infty v_\perp^2 dv_\perp \left\{ \frac{v_\perp}{|v_\parallel|} \left| \frac{\partial f}{\partial v_\parallel} \right| - \left| \frac{\partial f}{\partial v_\perp} \right| \right\} \Big/ \left\{ 2 \int_0^\infty v_\perp dv_\perp f \right\} \right]_{v_\parallel = V_R} \quad \text{---(5.5)}$$

Now, if we assume initial distribution to be isotropic i.e.

$$\left.\frac{\partial f}{\partial \alpha}\right|_{t=0} = 0 \quad \text{or,} \quad v_\perp \left.\frac{\partial f}{\partial v_\parallel}\right|_{t=0} = v_\parallel \left.\frac{\partial f}{\partial v_\perp}\right|_{t=0}$$

$$\text{or,} \quad \frac{v_\perp}{|v_\parallel|} \left| \frac{\partial f}{\partial v_\parallel} \right|_{t=0} = \left| \frac{\partial f}{\partial v_\perp} \right|_{t=0}$$

$A(V_R)$ becomes zero at $t = 0$ and therefore, the growth rate would be zero to start with. $\partial f / \partial v_\perp$ will, however, not be affected by the Landau resonant interaction.

Thus

$$\left| \frac{\partial f}{\partial v_\perp} \right|_t = \left| \frac{\partial f}{\partial v_\perp} \right|_0 = \frac{v_\perp}{v_\parallel} \left| \frac{\partial f}{\partial v_\parallel} \right|_{t=0}$$

Substituting this into equation (5.5), we get

$$A(V_R) = \left. \frac{\int_0^\infty \frac{v_\perp^3}{|v_{\parallel}|} dv_\perp \left\{ \left| \frac{\partial f}{\partial v_\parallel} \right|_t - \left| \frac{\partial f}{\partial v_\parallel} \right|_0 \right\}}{2 \int_0^\infty v_\perp \, dv_\perp \, f} \right|_{v_\parallel = V_R} \qquad (5.6)$$

As time goes, $\left| \frac{\partial f}{\partial v_\parallel} \right|$ goes on diminishing at the centre of the disturbed region and increasing at the boundaries because of the Landau resonant interaction. Hence the value of γ_c would become negative in the central part of the ledge and positive at its boundaries. So the VLF perturbations would damp or grow depending upon whether they resonate with the particles lying at the centre of the ledge or at the boundary of the ledge in the distribution function.

Thus we see that the growth or the damping of the perturbations depends upon how the ledge develops with time in the distribution and consequently it depends upon how $\partial f(v_\parallel, v_\perp, t)/\partial v_\parallel$ would vary with t.

The maxima occuring in $\partial f(v_{\parallel}, v_{\perp}, t)/\partial v_{\parallel}$ would correspond to the frequencies of maximum growth at time t and we would be mainly interested in the growth of the waves at these frequencies.

The evolution of these peaks can be qualitatively studied from fig.(5.2) which shows the different stages of ledge formation.

For a time period over which the assumptions made earlier in this chapter remain valid, the behaviour of the peaks in $\partial f(v_{\parallel}, v_{\perp}, t)/\partial v_{\parallel}$ with $V_{\parallel}$ and t can be stated as below:

i) They grow in height with time.

ii) Their width decreases with time i.e. they become sharper, with time, and

iii) Their position with respect to $V_{\parallel}$ changes with time - they move more and more towards the boundaries of the disturbed region.

Ideally speaking, as time progresses, the peaks would tend to become infinitely narrow and infinitely

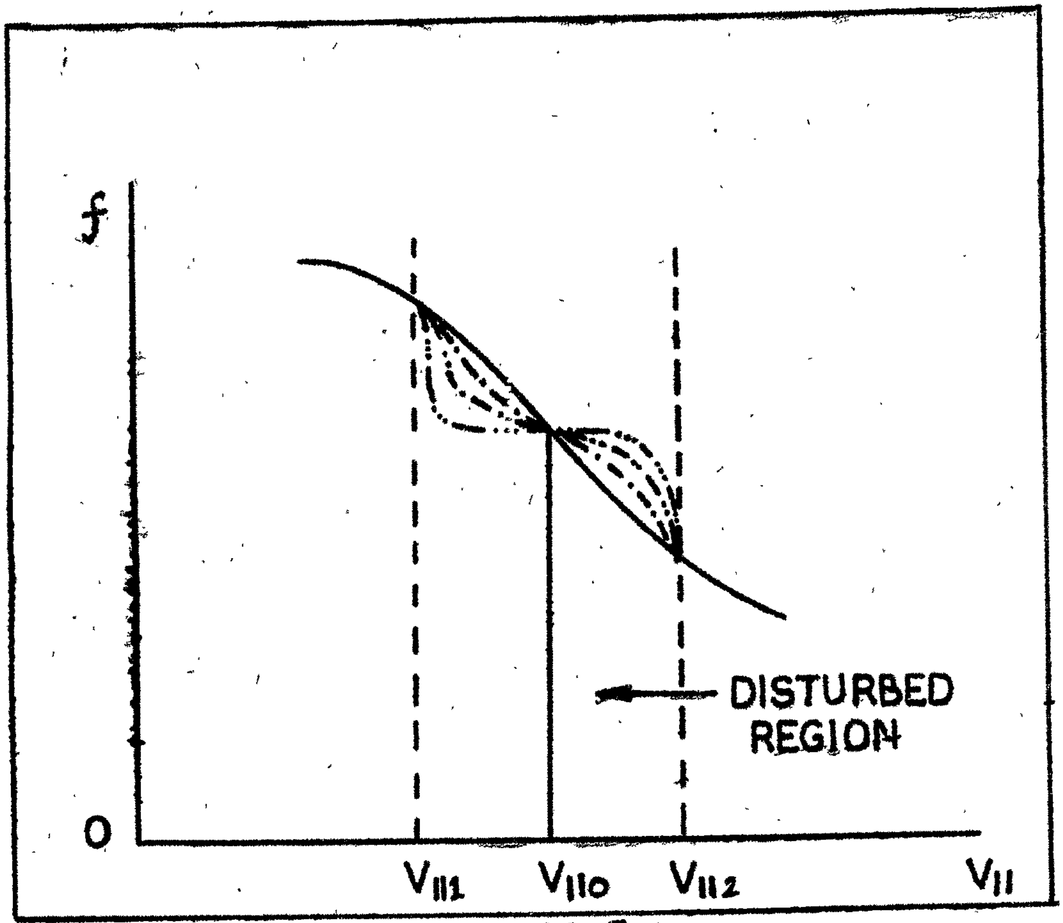

FIG 5.2

THE DIFFERENT STAGES DURING THE DEVELOPMENT OF THE LEDGE IN THE DISTURBED REGION OF THE VELOCITY DISTRIBUTION ARE INDICATED BY SINGLE DOTTED, DOUBLE DOTTED AND TRIPLE DOTTED LINES ARRANGED RESPECTIVELY IN CHRONOLOGICAL ORDER large and their position would tend to stabilise close to the edges of the disturbed region (see fig.5.2 and 5.3).

This shows that the growth of a wave at a certain frequency would be time dependent. Initially, the growth of the VLF perturbations will be at frequencies that correspond to the V_R lying in the central part of the disturbed region. But as time passes, the frequency of maximum growth would drift to values that correspond to velocities closer and closer to the edges of the disturbed region. Simultaneously the width in frequency of the growing waves would go on narrowing down and the magnitude of the growth rate will increase (fig.5.3).

The speculative picture presented above is linear in the sense that the amplitude of VLF wave is assumed not to have grown to an extent that can significantly react back on the particle distribution.

The whole treatment carried out so far is under the assumption that the wave frequency ω is very small compared to the particle gyrofrequency Ω . Under this condition, the initial growth rate becomes

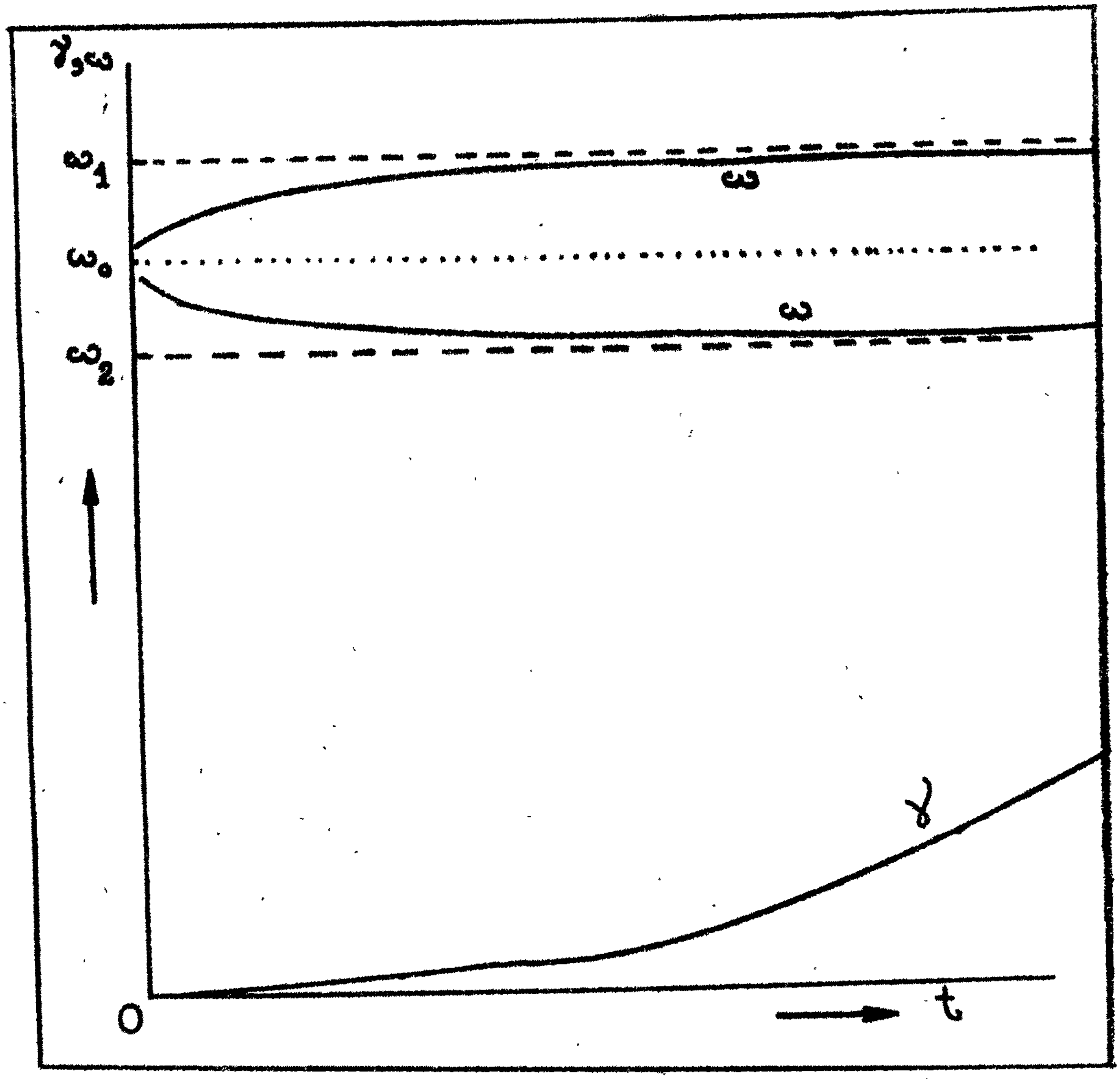

FIG 5.3

BEHAVIOUR OF GROWING WAVES WITH TIME. UPPER PART SHOWS THE TIME VARIATION OF THE FREQUENCY (ω) OF MAXIMUM GROWTH AND THE LOWER PART IS A SKETCH OF THE VARIATION OF CORRESPONDING GROWTH RATE WITH TIME. ω_0 CORRESPONDS TO THE CENTRE OF THE DISTURBED REGION AND ω_1 AND ω_2 TO THE EDGES OF THE LEDGE zero. However, if ω is not very small compared to Ω, the initial growth rate will be negative and the waves would damp in the beginning. This damping will continue unless, as equation (5.3) suggests, $A(V_R)$ exceeds the quantity $\omega/(\Omega-\omega)$ during the evolution of the ledge. This means that in the initial stage of the interaction, Landau damping would be accompanied by cyclotron damping and only after a certain time when the quantity $\left\{A(V_R) - \omega/(\Omega-\omega)\right\}$ in equation (5.3) becomes positive, the cyclotron growth would come into picture. This suggests that the growth will occur with a certain time delay.

5.3 CONSIDERATION OF THE EFFECT OF THE OCCURENCE OF A MAXIMUM IN THE LANDAU RESONANT $V_{\parallel}$ OF THE PARTICLES:

Our aim has been to study the possible mechanisms that contribute to the favourable triggering of emissions at half the equatorial gyrofrequency. In the last chapter, we considered the effects of the equality of the Landau resonant and the gyro resonant speeds at $\omega = \frac{1}{2}\Omega$ coupled with the equality of the group velocity of the wavepacket and the phase velocity of the central wave of the wave packet. Here, in this chapter, we started with an idea to study the effects of the maximum occuring in Landau resonant $V_{\parallel}$ of the particles.

Obviously, this maximum Landau resonant $V_{\parallel}$ denoted by $V_{\parallel L\ max}$ hereafter will divide the entire velocity space into two regions - one for which $V_{\parallel} < V_{\parallel L\ max}$ where the particles are susceptible to resonance if the waves of suitable frequencies exist in the system and the other for which $V_{\parallel} > V_{\parallel L\ max}$ where the particles will have no Landau resonance with any whistler mode wave having any frequency whatsoever (see fig. 5.4).

Now, if we assume a wide spectrum of oblique whistler turbulence (Helliwell, 1969) for a period of the order of quasilinear relaxation time for f due to the Landau resonant interaction with the waves, a ledge is likely to form over the entire disturbed region in the velocity space. The distribution in the undisturbed region will however, remain unchanged.

At the boundary of the two regions, f will develop a steep negative gradient with respect to $V_{\parallel}$ (see fig.5.4). The application of the mechanism discussed in this chapter seems to be quite promising to favour growth of perturbations at a frequency corresponding to this $V_{\parallel}$.

The whistler mode dispersion relation

$$\frac{c^2k^2}{\omega^2} \approx \frac{\omega_p^2}{\omega(\Omega\cos\theta - \omega)}$$

shows that the maximum $V_{\parallel}$ that a Landau resonant particle can have is given by

$$V_{\parallel L max} = c\Omega/2\omega_p$$

It may be noted that a particle with $V_{\parallel} = c\Omega/2\omega_p$ would gyroresonate with a whistler mode wave with frequency $\omega = \Omega/2$ propagating antiparallel to B_0 .

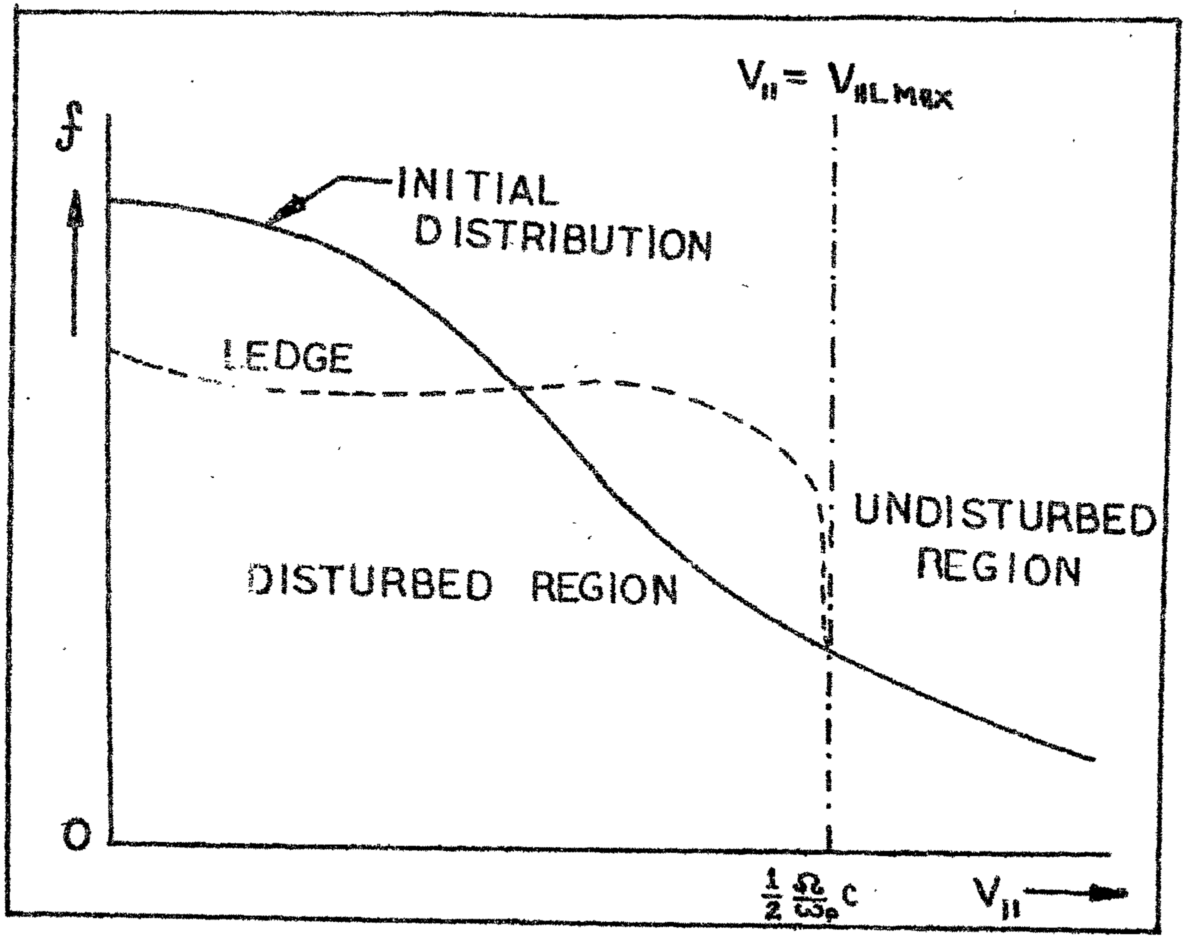

FIG 5.4

THE DISTURBED AND THE UNDISTURBED REGIONS IN VELOCITY SPACE RESULTING FROM THE LANDAU RESONANCE OF WIDE BAND WHISTLER MODE NOISE. THE STEEP GRADIENT OF f WITH $V_{\parallel}$ DEVELOPED NEAR $V_{\parallel} = V_{\parallel LMAX}$ INCREASES PITCH ANGLE ANISOTROPY LEADING TO WHISTLER GROWTH.

5.4 CALCULATIONS OF GROWTH RATE AT $\omega = \frac{1}{2}\Omega$ AND CONCLUSIONS

The steep gradient in $f(V_{\parallel})$ formed at $V_{\parallel} = c\Omega/2\omega_p$ would be unstable to whistler mode perturbations at the frequency $\omega = \frac{1}{2}\Omega$ propagating in the negative z direction as equation (5.6) suggests and the amplification of the waves is expected at this frequency. However, this amplification (see eq.3) would be delayed as in this case $\omega/(\Omega - \omega)$ is not negligible and so the growth rate is initially negative for a finite period and then it becomes zero and then positive. The nature of these results will not be altered even if the initial distribution were assumed to have an anisotropy in pitch angle.

To calculate the growth rate we use the following equation which is equivalent to equation (5.3) and has already been used in the last chapter for similar calculations:

$$\gamma = -\frac{\pi k^2 V_R^2}{n\Omega}\int_0^\infty \left[v_\perp \frac{\partial f}{\partial v_\parallel} - (v_\parallel - v_p)\frac{\partial f}{\partial v_\perp}\right]_{V_\parallel = V_R} v_\perp^2 \, dv_\perp \qquad (5.7)$$

When the gradient $\partial f/\partial V_{\parallel}$ is sufficiently steep, the following inequality will hold·

$$\frac{\partial f}{\partial v_{\parallel}} >> \frac{v_{\parallel}-v_{p}}{v_{\perp}} \quad \frac{\partial f}{\partial v_{\perp}}$$

The steep gradient in f formed at $V_{\parallel} = V_{\parallel L\ max}$ because of the Landau resonant interaction allows to make use of the above inequality and neglect the second term in the integral of eq.(5.7). Thus we get

$$\gamma \doteq -\frac{\pi k^2 V_R^2}{n\Omega}\left[\int_0^{\infty} v_{\perp}^2 \frac{\partial f}{\partial v_{\parallel}} \, v_{\perp}\, dv_{\perp}\right]_{V_{\parallel}=V_R}$$

or,

$$\gamma \doteq -\frac{\pi k^2 V_R^2}{n\Omega} \quad \frac{2}{m}\left[\frac{\partial}{\partial v_{\parallel}}\int_0^{\infty}\frac{1}{2} m v_{\perp}^2 f \, v_{\perp} dv_{\perp}\right]_{V_{\parallel}=V_R}$$

replacing $\frac{1}{n}\left[\int_0^\infty \left(\frac{1}{2}\, m v_\perp^2 f\right) v_\perp\, dv_\perp\right]_{v_\parallel = V_R}$ by $T_\perp(V_R)$, the average perpendicular energy per particle at $V_\parallel = V_R$, we get

$$\gamma = -\frac{2\pi k^2 V_R^2}{m\Omega} \left.\frac{\partial T_\perp}{\partial v_\parallel}\right|_{V_\parallel = V_R}$$

Approximating $(\partial T_\perp / \partial v_\parallel)_{V_\parallel = V_R}$ by a delta function $-\Delta T_\perp(V_R)\,\delta(V_\parallel - V_R)$, the above equation leads to:

$$\gamma = \frac{2\pi k^2 V_R^2}{m\Omega}\, \Delta T_\perp(V_R)\, \delta(v_\parallel - V_R)$$

Integrating γ over a narrow range around $v_\parallel = V_R$,

$$\int_{V_R-\epsilon}^{V_R+\epsilon} \gamma\, dV_R = \left[\frac{2\pi k^2 V_R^2}{m\Omega}\, \Delta T_\perp(V_R)\right] = \frac{\pi\Omega}{2m}\, \Delta T_\perp(V_R) \qquad \text{---(5.8)}$$

Since V_R at $\omega = \frac{1}{2}\Omega$ is equal to $-\left.\frac{\Omega}{2k}\right|_{\omega=\frac{1}{2}\Omega}$

To determine the step $\Delta T_{\perp}$ at $V_{\parallel} = V_R$ the following procedure can be adopted if the initial distribution is known (see fig.5.5).

The curves shown in the figure are contours of constant $V_{\perp}$. For each $V_{\perp}$, a separate ledge will be formed because $V_{\perp}$ does not change during the interaction and therefore the height of the ledge, f_c, would be a function of $V_{\perp}$. This f_c can be calculated from the following expression

$$f_c(v_{\perp}) = \frac{1}{v_{\parallel L\,max}} \int_0^{V_{\parallel L\,max}} f(v_{\parallel}, v_{\perp})\, dv_{\parallel}$$

The step Δf at $V_{\parallel} = V_R = V_{\parallel L\,max}$ would be given by

$$\Delta f(v_{\perp})\Big|_{V_{\parallel} = V_{\parallel L\,max}} = f_c(v_{\perp}) - f(v_{\perp})\Big|_{V_{\parallel} = V_{\parallel L\,max} + \epsilon}$$

in the limit as $\epsilon \to 0$.

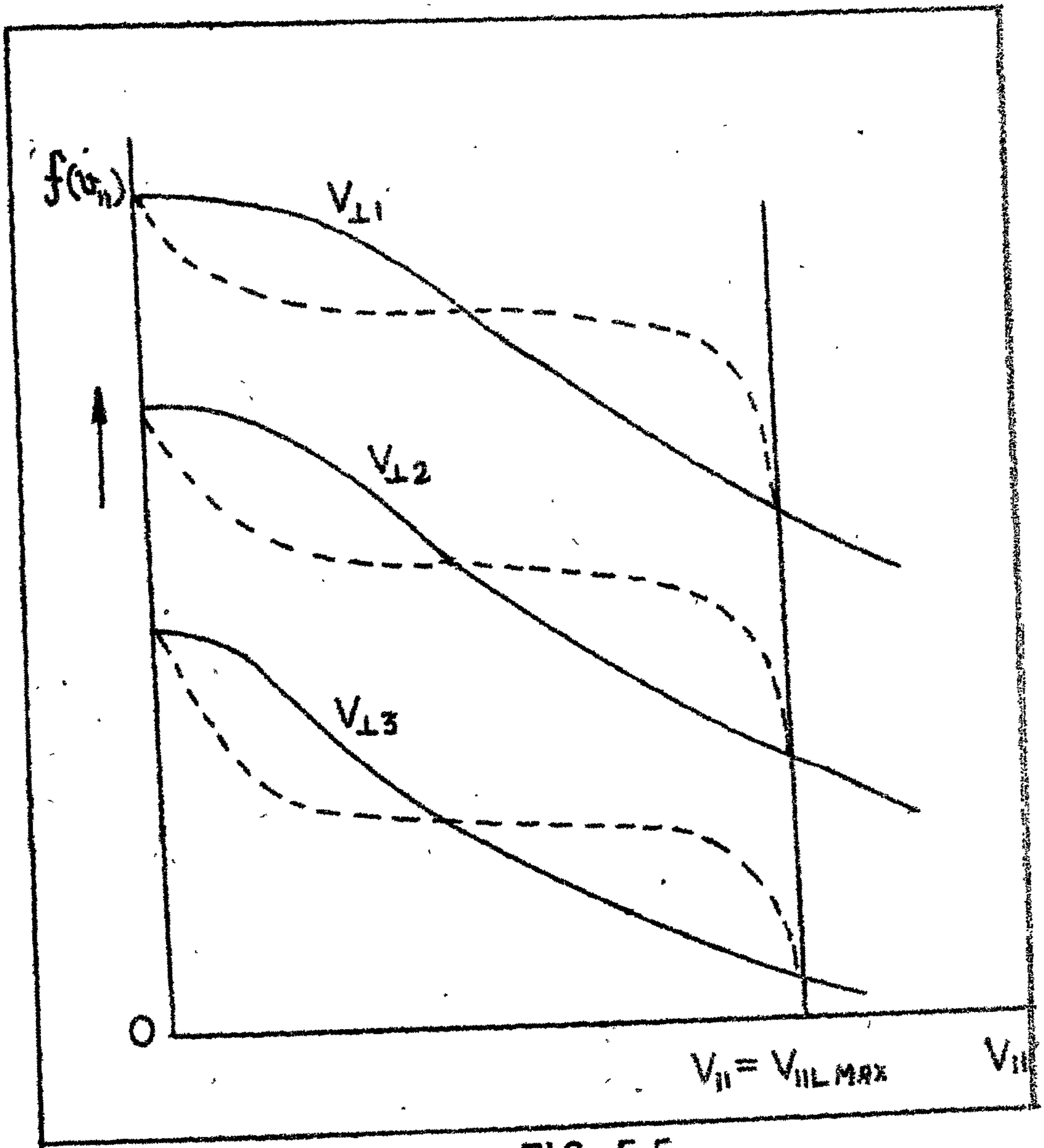

FIG 5.5

AS $V_{\perp}$ DOES NOT CHANGE DURING LANDAU RESONANCE, $f(V_{\parallel})$ FORMS DIFFERENT LEDGES FOR DIFFERENT VALUES OF $V_{\perp}$. HOWEVER, $\int f(V_{\parallel})\, dV_{\parallel}$ FOR A GIVEN $V_{\perp}$, BETWEEN $V_{\parallel} = 0$ AND $V_{\parallel} = V_{\parallel L\,MAX}$ WILL ~~EVALUATE~~ REMAIN UNCHANGED BEFORE AND AFTER THE LEDGE FORMATION DUE TO THE CONSERVATION OF THE NUMBER OF PARTICLES IN THE RESONANT REGION

The step in $T_\perp$ can be easily calculated from this as:

$$\Delta T_\perp \Big|_{V_\parallel = V_{\parallel L max} = V_R} = \int_0^\infty \frac{1}{2} m v_\perp^2 \{\Delta f(v_\perp)\}_{V_\parallel = V_R} v_\perp \, dv_\perp \qquad \text{--- (5.9)}$$

The large negative value of $\partial f / \partial v_\parallel$ at $v_\parallel = v_{\parallel L max} = V_R \big|_{\omega = \frac{1}{2}\Omega}$ would give large growth rates for perturbations at $\omega = \frac{1}{2}\Omega$ propagating antiparallel to $\vec{B}_o$.

Throughout this chapter the ambient static magnetic field has been assumed to be uniform. While applying the results obtained here to the emissions from the magnetosphere, we recall that the resonant interactions occuring in the equatorial regions are much more important than those occuring in the non-equatorial regions as explained in the earlier chapters. The field in the equatorial regions can, as a first approximation, be assumed to be uniform and the theory presented here can be applied to this situation. However, we must replace our Ω by Ω_{eq}, the equatorial gyrofrequency and then the mechanism discussed here would exhibit a definite favour to emissions at $\omega = \frac{1}{2} \Omega_{eq}$. (Vyas and Das, 1975).

It has been experimentally observed (Helliwell, 1969) that the VLF emissions tend to occur more often at half the equatorial gyrofrequency and it is very likely that the mechanism described here is one that favours emissions at that frequency.