CHAPTER-III
THE MEDIUM AND CERENKOV RADIATION

3.1 Introduction

The phenomenon of Cerenkov radiation which has applications in various branches of physics like optics, astrophysics, cosmic rays, biophysics etc. has been studied extensively both theoretically and experimentally. In the elementary description of this phenomenon, this radiation is produced by the passage of a relativistic particle through a dielectric medium with the condition that the velocity of the above particle called the Cerenkov particle be greater than the phase velocity of light in that medium. In the previous chapter an elementary discussion along with the classical theory of Frank and Tamm was given in brief. Recently Heintzmann and Nitsch (1979) have considered the phenomenon of Cerenkov radiation in a very strongly magnetized plasma in cold plasma approximation. For this, they have considered the linearized equation of motion of the plasma particles due to the perturbing fields (due to the interactions) in a constant magnetic field (background) with a phenomenological relaxation time and have obtained a dielectric tensor. They have applied this to the pulsar magnetosphere and have shown that the Cerenkov radiation is comparable to the curvature radiation emitted by the same particle gliding along the magnetic field lines. In all the above discussions Cerenkov radiation emerges as a collective mode due to the cooperative
effect of the dielectric medium interacting with the relativistic particle.

In a microscopic representation of a many particle system, the collective interactions occur in three different modes: transverse ($\lambda$), longitudinal ($\sigma^-$) and scalar ($\sigma^+$) modes as explained in the earlier chapter where it is shown that the interactions between the particles are through a radiation field in general. This radiation field is considered to be a set of harmonic oscillators. In general, the vibrations are fundamentally dependent upon the properties of the medium. When a high energy particle enters such a medium it excites the above collective oscillations. According to Klimontovich and Silin (1961) the excitation of the collective vibrations in the transverse mode leads to the emission of Cerenkov radiation and the excitation in the longitudinal and scalar modes gives rise to what is called the Bohr radiation or polarization radiation.

From the above discussion it is clear that the emission of Cerenkov radiation is a property of the medium in presence of a very high energy particle. Thus in the application of this natural phenomenon to various physical situations one needs to consider the modification of the radiation by the nature of the medium, i.e. its thermal and other dielectric properties. Also the introduction of a relativistic charged particle into an ambient medium would introduce density fluctuations which would couple the longitudinal and transverse modes. It is this problem that we are going to consider here. The questions we would like to answer are:
(1) How does the medium temperature affect the phenomenon of Cerenkov radiation?

(2) How is the energy loss suffered by the particle affected when we take into account the coupling of the longitudinal mode with the transverse mode?

(3) How does the coherence condition (2.2.1) and hence the Cerenkov cone get modified by the above effects?

The coupling of the longitudinal mode with the transverse mode has importance in the transfer of energy to the plasma in the form of heat.

Majumdar (1961) has considered the phenomenon of Cerenkov radiation i.e. the interaction in the transverse mode in a magnetically active cold plasma. He pointed out however that the presence of the external magnetic field will couple the transverse and longitudinal modes and thereby the radiation emitted by the particle will be modified. In his work the calculations are done in a phenomenological way, starting from Maxwell's equations and the equation of motion of plasma electrons as modified by fluctuating pressure gradient. The loss of energy by the moving particle is calculated and it is shown that the inclusion of this coupling would result in the particle emitting non-Cerenkov type of radiation in addition to the Cerenkov radiation. This mode of coupling through the magnetic field, however, is well-known (Ginzburg, 1964). But in the absence of a magnetic field in a general dielectric medium, the density fluctuations caused by the Coulomb interactions in the system will still couple the $\lambda$ and $\sigma$ modes. In fact
Field (1956) has shown that the inhomogeneity in a system without magnetic field will still produce the coupling of the transverse and longitudinal waves.

In the earlier work by Pratap on the Microscopic Theory of Cerenkov radiation in the framework of non-equilibrium statistical mechanics, the effects of the density fluctuations causing the coupling of the \( \lambda \) and \( \sigma' \) modes and the thermal state of the medium were not taken into account. A closer examination of relevant diagrams (see chapter II) reveals that a longitudinal (\( \sigma' \)) mode interacting with a transverse (\( \lambda \)) mode does not appear up to the fourth-order diagram. The coupling appearing in the fourth-order diagram gives a vanishing contribution when one sums over the polarization vectors and averages over all possible orientations (Pratap, 1967). Here we will show that this \( \lambda-\sigma' \) coupling does appear in the sixth-order diagrams. In this formulation the concept of the medium temperature is introduced through the initial distribution function for the medium particles. We get an effective temperature in the final state which will in general be a function of the parameters of the system.

3.2 One-particle Distribution Function

The formulation of the problem is given in chapter II. The system for the problem of Cerenkov radiation as in chapter II, consists of N\(_L\) harmonic oscillators forming the medium, a relativistic charged test particle and a radiation field with three different modes (Transverse (\( \lambda \)), longitudinal (\( \sigma' \)) and scalar (\( \sigma' \)) modes).
Now the one particle distribution function for the test-particle is given by (2.5.1) which is
\[ f(y, \omega_T; t) = \int \frac{d \omega_T}{\omega_T} J_\lambda d \omega \lambda J_f d \omega_f f(t) \] (3.2.1)

where \( f \) is the distribution function for the whole system given by the general solution (2.4.15) and (2.4.16) with the operators \( L_0 \) and \( \delta \mathcal{L} \) defined by the expressions (2.4.9) - (2.4.14). The test particle is considered as a probe which interacts with the rest of the system and one studies the response of the system in the presence of the probe. In the following we discuss the possibility of the coupling of the transverse interaction giving rise to Cerenkov radiation with other modes in different orders in \( \varepsilon \), obtained in chapter II.

Second Order in \( \varepsilon \).

The contribution in this order is given by the expression (2.5.14) in general. This, because of the integration over the field variables gives no coupling. Since we are interested only in the \( \lambda \)-interaction giving rise to the phenomenon of Cerenkov radiation we write the contribution in \( \varepsilon^2 \) as
\[
\varepsilon^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \int dJ_\lambda d\omega_\lambda \exp \left\{ -i L_0^T (t-t_1) \right\} \int \mathcal{A}_\lambda^T \exp (-i \mathcal{L}_0^T t_{12} - i \mathcal{L}_0^L t_{12}) \mathcal{B}_\lambda^T \exp (-i \mathcal{L}_0^T t_2) \times f(y, \omega_T; 0) \mathcal{J}_\lambda \\
\] (3.2.2)
Fourth Order in $\varepsilon^4$.

The contributions to (3.2.1) in this order is given by the expressions (2.5.25). In the diagram representation of (3.2.1) as shown in figure 11, the first figure corresponds to the Born approximation for the transverse interaction (Henin, 1963) and the other two diagrams correspond to the contributions beyond Born approximation and represent the collisions between the test particle and the medium oscillators. Thus, in Born approximation (which is important in our collective interaction) we consider the figure 7 for the contribution in $\varepsilon^4$ which can be written as

$$
\left[ \sum_{t_1} e^{4 \int^t_0 \sum_{t_2} \int^t_0 d\omega j \int^t_0 dJ j \int^t_0 d\omega l} \right] \times \exp \left[ -i \sum_{t_1} T T + \sum_{t_2} T T - \sum_{t_3} T T + \sum_{t_4} T T \right] \times \prod_{j=1}^{\infty} \times \exp \left( -i \sum_{t_3} T T \right) \int (o)
$$

(3.2.3)

since the interaction with the test particle and the medium starts in the $\lambda$-mode in the present problem. When $j$ in (3.2.3) stands for $\lambda$, as can be seen in Appendix A, we get a non-zero contribution. But when $j$ stands for $\sigma$ or $0\sigma$, we write (3.2.3) as

$$
\sum_{t_1} e^{4 \int^t_0 \sum_{t_2} \int^t_0 d\omega j \int^t_0 dJ j \int^t_0 d\omega l} \times \exp \left[ -i \sum_{t_1} T T + \sum_{t_2} T T - \sum_{t_3} T T + \sum_{t_4} T T \right] \times \prod_{j=1}^{\infty} \times \exp \left( -i \sum_{t_3} T T \right) \int (o)
$$

(3.2.3)

respectively. When we substitute the expressions for the operator $A's$ and $B's$, we get a term
\[ \sum (\hat{e}_\lambda \cdot \mathbf{a}_\ell)(\hat{e}_\sigma \cdot \mathbf{a}_\ell) = \{ (\hat{e}_{\lambda_1} \cdot \mathbf{a}_\ell) + (\hat{e}_{\lambda_2} \cdot \mathbf{a}_\ell) \} (\hat{e}_\sigma \cdot \mathbf{a}_\ell) \]  

which on integration over all possible orientations gives a vanishing contribution. Thus the coupling of \( \lambda \)-mode with other modes of interaction is not possible in fourth order contribution in the present situation.

**Sixth Order in \( \epsilon' \):**

Now the diagram in the 6th-order contributing to the transverse interaction in Born approximation is as given in the figure 13. The irreducible matrix element corresponding to the above diagram will involve the product of the operators

\[
A_{\lambda}^T B_{\lambda}^\ell A_{\lambda'}^\ell' B_{\lambda''}^L = A_{\lambda}^T B_{\lambda}^\ell (A_{\lambda'}^\ell' B_{\lambda''}^\ell + A_{\lambda'} B_{\lambda''}^\ell' + A_{\lambda'} B_{\lambda''}^\ell') \times A_{\lambda''}^T B_{\lambda''}^T
\]

(3.2.5)

The first term \( A_{\lambda}^T B_{\lambda}^\ell A_{\lambda'}^\ell' B_{\lambda''}^\ell' B_{\lambda''}^T \) in (3.2.5) is the term considered by Pratap (1967) in his work. In the present formulation, our interest is to see the effect of the second and third terms i.e., the terms involving the products \( A_{\alpha}^T B_{\lambda}^\ell B_{\sigma}^\ell' A_{\lambda'}^\ell'' B_{\lambda''}^T \) and \( A_{\lambda}^T B_{\lambda}^\ell A_{\alpha} B_{\sigma}^\ell' A_{\lambda'}^\ell'' B_{\lambda''}^T \)

As explained earlier the longitudinal part of the interaction vector potential \( \hat{A} \) i.e., \( \hat{A} \) has a polarization vector whereas
Fig. 13
the scalar potential \( \Phi \) does not contain any polarization. Since
the Cerenkov radiation is due to the polarization of the medium
particles, one expects the modification of the transverse polariza-
tion by the coupling of the longitudinal polarization. Thus we
consider the modification of the phenomenon which is due to the
\( \lambda \)-interaction due to the \( \lambda - \sigma \) coupling. A 0\( \sigma \) mode generated by
\( \Phi \) will affect the radiation only in a secondary manner through \( \Phi \). We
therefore neglect the coupling of \( \lambda \) and 0\( \sigma \) modes and consider
only the term \( A_\lambda^T A_\sigma^l A_\sigma^l' A_\lambda''^l B_\lambda''^T \).

As shown in Appendix A the term \( A_\lambda^T A_\sigma^l A_\sigma^l' A_\lambda''^l B_\lambda''^T \)
leads to the conservation of wave vectors in different mode, i.e.

\[
{k_\lambda} = {k_\sigma}, \quad {k_\sigma} = {k_\lambda''}
\]  

(3.2.6)

Now the transverse and longitudinal nature of \( \lambda \) and \( \sigma \) are
expressed by the conditions

\[
\hat{e}_\lambda \cdot {k_\lambda} = 0, \quad \hat{e}_\sigma \times {k_\sigma} = 0
\]  

(3.2.7)

obtained from (2.3.3) and (2.3.4). The conditions given by (3.2.7)
imply that \( {k_\lambda} \) is normal to \( \hat{e}_\lambda \) and \( {k_\sigma} \) is in the direction of \( \hat{e}_\sigma \)
and hence the condition (3.2.6) can be satisfied without any ambi-
guity. Also to get a non-zero contribution from the term
\( A_\lambda^T A_\sigma^l A_\sigma^l' A_\lambda''^l B_\lambda''^T \) one has to assume that all the
medium oscillators are identical.

Thus in the Born approximation using the previous discussions
on the thermodynamic assumption and the ring approximation, we
represent the contribution to the test-particle distribution function (3.2,1) for the present problem in the form of a diagram given in figure 14. (also see the figure 11). In figure 14 the wavy line as given by figure 15

\[ \sum = \lambda + \lambda \ell \lambda \lambda \ell \gamma \ell \lambda \]

represents the propagator which takes into account the modification of the field propagator due to the collective interactions in \( \lambda \) mode with the inclusion of the \( \lambda-\sigma \) coupling. The other diagrams in figure 11 which we have neglected in Born approximation correspond to the collision between the test particle and the medium particle and hence no analogue in the phenomenological theory. Thus in the collective description of the phenomenon these diagrams are of no importance and should give correction to the results obtained from the figure 14. But in the non-relativistic approximation for the particles constituting the ambient medium this correction will be negligibly small (Pratap, 1967).

Note that the loop has a very different meaning from that of the self-energy diagram of quantum field theory. It represents a correlation between the particle and the electromagnetic field modified by the collective interactions in the system instead of the emission or absorption of a photon.

In order to calculate the matrix elements contributing to the diagram given in figure 14, we define the Laplace transform of a function \( F(\tau) \) as
\[ F(z) = \int_0^\infty d\tau \exp(iz\tau) F(\zeta) \]  
(3.2.9)

where \( z \) is the Laplace variable. The notation in (3.2.9) is different from the traditional one. The complex variable \( iz \) is usually denoted by \(-S\). This implies some trivial changes of sign and of factors \( i \). The singularities of \( F(z) \) will appear on the real axis instead of along the imaginary axis of the complex plane as in the traditional notation. Making use of the evolution theorem in Laplace transform, we evaluate each contribution of the series (2.4.15) to (3.2.1) in the foregoing approximation and write (3.2.1) as

\[
\begin{align*}
\mathcal{F}(\zeta; T; t) &= \exp(-i\mathcal{L}_0 T) \mathcal{F}(\zeta; T; 0) + \mathcal{W}_{\lambda T}^{\lambda T}(t) \\
&+ \mathcal{W}_{\lambda T}^{\lambda T}(t) + \mathcal{W}_{\lambda T}^{\lambda T}(t) + \mathcal{W}_{\lambda T}^{\lambda T}(t) + \cdots
\end{align*}
\]

(3.2.10)

The number of letters indexing each term in (3.2.10) represents the order of the matrix element contributing to (3.2.1). It should be noted that the \( \lambda - \sigma \) coupling appears in the term \( \mathcal{W}_{\lambda T}^{\lambda T} \). After evaluating each term in (3.2.10) (as shown in Appendix-A) one can write (3.2.10) explicitly as

\[
\begin{align*}
\mathcal{F}(\zeta; T; t) &= \exp(-i\mathcal{L}_0 T) \mathcal{F}(\zeta; T; 0) \\
&+ \frac{8\pi e^{\frac{z}{m_T}}}{m_T} \int_0^t dt_1 \int_0^t dt_2 \sum_{k=1}^{\infty} \frac{1}{k^\lambda} \\
&\times \exp \left\{ -i\mathcal{L}_0 (t-t_1)^2 \right\} \times
\end{align*}
\]
As shown in Appendix-A

\[ \mathcal{E}_{\ell,\ell}^\lambda(z) = \left[ \mathcal{E}_{\ell}^\lambda(z) \right]^2 \]
\[ \mathcal{E}_{\ell,\ell}^{\sigma'}(z) = \mathcal{E}_{\ell}^\lambda(z) \cdot \mathcal{E}_{\ell}^{\sigma'}(z) \]  \hspace{1cm} (3.2.12)

where \( \mathcal{E}_{\ell}^\lambda(z) \) and \( \mathcal{E}_{\ell}^{\sigma'}(z) \) are the response functions corresponding to the interactions in transverse and longitudinal modes respectively. In the approximation (see Appendix-A) that the medium particles are non-relativistic

\[ \left( \frac{k_{\lambda} \cdot \alpha_{\ell}}{\mathcal{V}_{\ell}} \right)^2 \frac{k_B T}{m_\ell} \ll 1 \]  \hspace{1cm} (3.2.13)

The expressions for \( \mathcal{E}_{\ell}^\lambda(z) \) and \( \mathcal{E}_{\ell}^{\sigma'}(z) \) are

\[ \mathcal{E}_{\ell}^\lambda(z) = \frac{2}{3} \omega^2_{\ell} \mathcal{V}_{\ell}^2 \mathcal{F}_2 \left( \frac{1}{2}, \frac{1}{2}; 1, \frac{5}{2}; -2A \right) \frac{Z^2 - \gamma_{\ell}^2}{(Z^2 - \gamma_{\ell}^2)(Z^2 - \gamma_{\lambda}^2)} \]  \hspace{1cm} (3.2.14)

\[ \mathcal{E}_{\ell}^{\sigma'}(z) = \frac{1}{3} \omega^2_{\ell} \mathcal{V}_{\ell}^2 \mathcal{F}_2 \left( \frac{1}{2}, \frac{3}{2}; 1, \frac{5}{2}; -2A \right) \frac{Z^2 - \gamma_{\ell}^2}{(Z^2 - \gamma_{\ell}^2)(Z^2 - \gamma_{\lambda}^2)} \]  \hspace{1cm} (3.2.15)
where
\[ 2F_2 \left( \alpha, \beta \mid \lambda, \delta ; -2A \right) \]
\[ = 1 + \frac{\alpha \cdot \beta}{\lambda \cdot \delta} (-2A) + \frac{\alpha (\alpha + 1) \beta (\beta + 1)}{\lambda (\lambda + 1) \delta (\delta + 1)} (-2A)^2 + \cdots \]

is a hypergeometric function, and

\[ A = A(T) = \left( \frac{m_e^2}{\rho_e^2} \right) \frac{K_B T}{m_e c^2} \]

(3.2.17)

Now the sum in (3.2.11) in square brackets is
\[ 1 + \varepsilon_\ell^\lambda (z) + \varepsilon_\ell^\sigma (z) + \cdots \]
\[ = 1 + \varepsilon_\ell^\lambda (z) \left\{ 1 + (\varepsilon_\ell^\lambda (z) + \varepsilon_\ell^\sigma (z)) + \cdots \right\} \]
\[ = \frac{1 - \varepsilon_\ell^\sigma (z)}{1 - \varepsilon_\ell^\lambda (z) - \varepsilon_\ell^\sigma (z)} \]

(3.2.18)

In getting (3.2.18) we have assumed that
\[ \left| \varepsilon_\ell^\lambda (z) + \varepsilon_\ell^\sigma (z) \right| < 1 \]

(3.2.19)

But as seen in (3.2.14) and (3.2.15) the response function is
dependent on the density of medium particles through the plasma
frequency \( \omega_p \). Hence the assumption (3.2.19) corresponds to the
low density approximation. For \[ \left| \varepsilon_\ell^\lambda (z) + \varepsilon_\ell^\sigma (z) \right| > 1 \] one can
analytically continue and thereby obtain the high density approximation (Balescu, 1963).

The main interest in this chapter is to calculate the rate at which the test-particle loses energy due to the collective interactions in the system. This is done by evaluating

\[
\frac{d \mathcal{C}}{dt} = \frac{d}{dt} \int d\omega \frac{dQ_T}{Q_T} \mathcal{F}_T
\]

Now commuting the time derivative with the integral sign and remembering the fact that \( \frac{dH_T}{dt} = 0 \) we have to evaluate \( \frac{df}{dt} \) which is given by

\[
\frac{df}{dt} = \frac{8\pi e_T^2 c}{m_T \nu} \int_0^t dt_2 \sum_{\lambda} \frac{1}{\gamma_\lambda^2} \frac{\partial}{\partial u} \left\{ \frac{\lambda}{c} \cos k_{\lambda} \cdot \beta c (t-t_2) \right. \\
\times \int dz \exp \left\{ -iz(t-t_2) \right\} \frac{iz}{z^2 - \gamma_\lambda^2} \left( \frac{1 - \varepsilon_\lambda(z)}{1 - \varepsilon_\lambda(z) - \varepsilon_\lambda(z)} \right) \\
\left. - \beta \times \{ k_{\lambda} \times \hat{e}_\lambda \} \sin k_{\lambda} \cdot \beta c (t-t_2) \right. \\
\times \int dz \exp \left\{ -iz(t-t_2) \right\} \frac{\gamma_\lambda}{z^2 - \gamma_\lambda^2} \\
\times \left( \frac{1 - \varepsilon_\lambda(z)}{1 - \varepsilon_\lambda(z) - \varepsilon_\lambda(z)} \right) \}
\]

\[
(3.2.20)
\]
where we have used the sum (3.2.18).

3.3 Energy loss by the test-particle

The rate of average energy lost by the test-particle because of its electromagnetic interaction with the medium is

\[
\frac{d\overline{E}}{dt} = \frac{d}{dt} \left\langle \mathcal{H}^T \right\rangle = \frac{d}{dt} \int du \, dq_T \, \mathcal{H}^T \gamma (u, q_T, t)
\]

\[
= \frac{8\pi e^2 c^3}{V} \int_0^t dt_2 \sum_{\lambda} \frac{i}{\gamma_{\lambda}} \int du \, dq_T \, (1 + \mu^2)^{1/2}
\]

\[
\times \frac{2}{\mu} \left\{ \frac{\hat{e}_{\lambda \gamma}}{c} \cos k_{\lambda \gamma} \cdot \beta c (t - t_2) \right\} dz \exp \left\{ -iZ(t - t_2) \right\}
\]

\[
\times \frac{iz}{Z^2 - \gamma_{\lambda}^2} \left( \frac{1 - \varepsilon_{\lambda}(z)}{1 - \varepsilon_{\lambda}(z) - \varepsilon_{\gamma}(z)} \right) - \frac{\lambda}{\gamma_{\lambda}} \times \left( k_{\lambda} \times \hat{e}_{\lambda} \right)
\]

\[
\times \sin k_{\lambda \gamma} \cdot \beta c (t - t_2) \right\} dz \exp \left\{ -iZ(t - t_2) \right\} \frac{\gamma_{\lambda}}{Z^2 - \gamma_{\lambda}^2}
\]

\[
\times \left( \frac{1 - \varepsilon_{\lambda}(z)}{1 - \varepsilon_{\lambda}(z) - \varepsilon_{\gamma}(z)} \right) \right\} \left( \beta \cdot \hat{e}_{\lambda} \right) \exp(-i\mathcal{Q}^T t_2) \gamma (u, q_T, t)
\]

Equation (3.3.1) consists of two terms. The second term can be written as
The first term of (3.3.2) reduces to a surface integral which vanishes at the boundary. The vanishing of the second term is quite obvious from the properties of the vectors. Hence only the first term in (3.3.1) would survive. Summation over the \( k \) vectors can be changed to an integration using the relation

\[
\frac{8\pi^3}{V} \sum_{k^\lambda} \rightarrow \int dk^\lambda
\]

for large systems. After summing over the polarization vectors, (3.3.1) can be written as

\[
\frac{d\bar{E}}{dt} = -\frac{e^2 c^2}{\hbar^2} \int du \sum_{\alpha} \int \frac{dk^\lambda \ (k^\lambda \times \beta)^2}{k^\lambda} \int_0^t dt_2 \cos \left( k^\lambda \cdot \beta c (t-t_2) \right) \exp \left( -iE_\alpha \hbar^2 (t-t_2) f(c_\alpha, \gamma_{\alpha,\gamma}; 0) \right) \exp \left( -iE^\lambda \hbar^2 \right) \left( \frac{1 - \varepsilon_\lambda^\sigma(z)}{\varepsilon_\lambda^\sigma(z) - \varepsilon_\sigma^\nu(z)} \right)
\]
Substituting the expressions (3.2.14) and (3.2.15) in (3.3.4) and using the recurrence relation

\[
\begin{align*}
{\binom{2}{2}}_2 F_2 \left( \frac{1}{2}, \frac{1}{2}; 1, 5/2; -2A \right) &= \frac{3}{2} \left[ {\binom{2}{2}}_2 F_2 \left( 1, \frac{1}{2}; 1, 3/2; -2A \right) \\
- \frac{1}{3} {\binom{2}{2}}_2 F_2 \left( 1, 3/2; 1, 5/2; -2A \right) \right] 
\end{align*}
\]

(3.3.5)

and taking the inverse Laplace transformation, (3.3.4) becomes

\[
\frac{dE}{dt} = -\frac{e^2 c^2}{3\pi^2} \int d\mu d\varphi_p d\lambda \int_0^t dt_2 \left( \frac{k_\lambda \gamma \beta^2}{k_\lambda \gamma} \right) \int_0^t dt_2
\]

\[
\times \cos \left( k_\lambda \gamma \beta \cos (t-t_2) \right) \exp \left( -i \int_0^t \int_0^t dt_2 \right)
\]

\[
\times \int \phi_1 (-2A) \cos (\gamma_\lambda (t-t_2)) + \phi_2 (-2A) \left( 1 + \frac{k_\lambda \gamma^2}{\gamma} \right)
\]

\[
\times \cos \left( \left( \frac{k_\lambda \gamma^2 + \gamma^2}{\gamma} \right) \right)^{1/2} (t-t_2) \right) \right] + \phi_2 (-2A)
\]

\[
\times \left( 1 - \frac{k_\lambda \gamma^2}{\gamma} \right) \cos \left( \left( \frac{k_\lambda \gamma^2 + \gamma^2}{\gamma} \right) \right)^{1/2} (t-t_2) \right) \right] \]

(3.3.6)

where

\[
\phi_1 (-2A) = \frac{\binom{2}{2}}_2 F_2 \left( \frac{1}{2}, \frac{3}{2}; 1, 5/2; -2A \right)
\]

\[
\frac{\binom{2}{2}}_2 F_2 \left( 1, \frac{1}{2}; \frac{3}{2}; 1, 3/2; -2A \right)
\]

(3.3.7)
\[ \Phi_2 (-2A) = \frac{2 F_2 \left( \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; 1, \frac{5}{2}; -2A \right)}{2 F_2 \left( \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; 1, \frac{3}{2}; -2A \right)} \] (3.3.8)

and

\[ \chi^2 = \left( \gamma^2 - \gamma^2 \right)^2 + 4 \omega^2 \Gamma^2 F_2 \left( \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; -2A \right) \] (3.3.9)

In (3.3.6) the condition that the arguments of the cosine terms be real demands

\[ \bar{\gamma}^2 + \gamma^2 \pm \chi \geq 0 \] (3.3.10)

Use of the expression (3.3.9) reduces the inequality condition (3.3.10) to

\[ \bar{\gamma}^2 \geq \omega^2 \left[ F_2 \left( \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; -2A \right) - 1 \right] \] (3.3.11)

The integrations in (3.3.6) can be carried out easily by assuming the test-particle to be moving along the Z-axis with a velocity \( U_0 \), so that

\[ f(u_x, q, 0; 0) = \delta(u_x) \delta(u_y) \delta(u_z - U_0) \delta(q) \] (3.3.12)

and so
\[
\exp(-i \mathcal{L}_0 T \mathbf{t}_2) \mathcal{F}(u, \mathbf{u}_T; 0) = \delta(u_x) \delta(u_y) \delta(u_z - u_0) \delta(q_T - \beta c t_2 \mathbf{z})
\]

(3.3.13)

where \( \mathbf{z} \) is the unit vector along the Z-axis of the co-ordinate system.

In the asymptotic limit in time \((t \to \infty)\) and with (3.3.13), equation (3.3.6) after integrations becomes

\[
\frac{d\mathbf{E}}{dt} = \frac{e^2 c^2 \beta_0^2}{3} \int k_\lambda^2 dk_\lambda \int dx (1-x^2) \left[ \phi_i(-2A) \right]
\]

\[\times \left\{ \delta(x k_\lambda \beta_0 c + \gamma_\lambda) + \delta(x k_\lambda \beta_0 c - \gamma_\lambda) \right\}
\]

\[+ \phi_2(-2A) \left( 1 + \frac{\gamma_\lambda^2 - \gamma_\perp^2}{x} \right) \left\{ \delta(x k_\lambda \beta_0 c + \gamma_\lambda) + \delta(x k_\lambda \beta_0 c - \gamma_\lambda) \right\}
\]

\[+ \phi_2(-2A) \left( 1 - \frac{\gamma_\lambda^2 - \gamma_\perp^2}{x} \right) \left\{ \delta(x k_\lambda \beta_0 c + \gamma_\lambda) + \delta(x k_\lambda \beta_0 c - \gamma_\lambda) \right\}
\]

\[+ \phi_2(-2A) \left( 1 - \frac{\gamma_\lambda^2 + \gamma_\perp^2 - x}{x} \right) \left\{ \delta(x k_\lambda \beta_0 c + \gamma_\lambda) + \delta(x k_\lambda \beta_0 c - \gamma_\lambda) \right\}
\]

(3.3.14)

where

\[x = \cos \Theta \]

(3.3.15)
Here $\Theta$ is the angle between the direction of the propagation
vector $\mathbf{k}_\lambda$ and that of the test particle.

The $x$-integration between the limits $-1$ and $+1$ allows only
the third curly bracketed terms to give non-zero contribution
because of the presence of the Dirac $\delta$-function. Thus with
d$\mathbf{l} = \beta \mathbf{c} \mathbf{dt}$, we write (3.3.14) as

$$
\frac{d\mathbf{E}}{dt} = - \frac{2e^2}{3c^2} \int \gamma^2 \gamma^2 \phi_2 (-2\lambda) \left( 1 - \frac{\gamma^2}{\lambda} \right) \times \left( 1 - \frac{\gamma^2 + \gamma^2 - \chi}{2 \gamma^2 \beta_0^2} \right) \tag{3.3.16}
$$

It may be noted that the range of $x$ viz. $-1 \leq x \leq 1$ puts limit on the
range of $\gamma^2$.

In the linear approximation in $T$ (since we consider the case
when $\frac{k_B T}{m_L c^2} \ll 1$, the expressions (3.3.8) and (3.3.9) can be written
respectively as

$$
\phi_2 (-2\lambda) = 1 + \frac{2}{15} \frac{k_B T}{m_L c^2} \frac{K_B T}{m_L c^2} \tag{3.3.17}
$$

and

$$
\chi^2 = (\gamma^2 - \gamma^2) + 4 \omega^2 \gamma^2 \left( 1 - \frac{1}{3} \frac{\gamma^2}{\gamma^2} \frac{k_B T}{m_L c^2} \right)
= \chi_0 - \frac{2}{3} \omega^2 \frac{\gamma^2}{\chi_0} \frac{k_B T}{m_L c^2} \tag{3.3.18}
$$
where

\[ \chi_0 = \chi(T=0) = \left[ (\gamma^2_{\lambda} - \gamma^2_{\ell})^2 + 4 \omega^2_{pl} \gamma^2_{\ell} \right]^{1/2} \quad (3.3.19) \]

Thus the $\gamma_{\lambda}$ integration in (3.3.16) can now be carried out by defining the variable

\[ \omega^2_0 = \frac{1}{2} \left( \gamma^2_{\lambda} + \gamma^2_{\ell} - \chi_0 \right) \quad (3.3.20) \]

The expression (3.3.20) defines the effective frequency of the radiation coming out of the medium (Pratap, 1967) as $T \to 0$. To carry out the $\gamma_{\lambda}$ integration in (3.3.16) conveniently we derive certain parameters using the definitions (3.3.20) and (3.3.19) as

\[ \gamma^2_{\lambda} = \frac{\omega^2_0 \left( \gamma^2_{\ell} + \omega^2_{pl} - \omega^2_0 \right)}{\gamma^2_{\ell} - \omega^2_0} \quad (3.3.21) \]

\[ \gamma^2_{\ell} = \frac{\omega^2_{pl} \gamma^2_{\ell} + \omega^2_0 (\gamma^2_{\ell} - \omega^2_0)}{\gamma^2_{\ell} - \omega^2_0} = \omega^2_0 + \frac{\omega^2_{pl} \gamma^2_{\ell}}{\gamma^2_{\ell} - \omega^2_0} \quad (3.3.22) \]

\[ \chi_0 = \frac{\gamma^4_{\ell} + \gamma^2_{\ell} \omega^2_{pl} - \omega^2_0 \gamma^2_{\ell} + \omega^2_0}{\gamma^2_{\ell} - \omega^2_0} \quad (3.3.23) \]

and
\[ d\gamma^2_\lambda = 2 \gamma^2_\lambda d\gamma \]
\[ = \left( \frac{\gamma^4_\lambda + \gamma^2_\lambda \omega^2_p - 2 \omega^2_0 \gamma^2_\lambda + \omega^4_0}{(\gamma^2_\lambda - \omega^2_0)^2} \right) d\omega^2_0 \]

(3.3.24)

With the above expressions the result of the \(\gamma_\lambda\)-integration in (3.3.16) in linear approximation in temperature \(T\) is

\[\frac{d\bar{E}}{dl} = -\frac{2 e^2 T}{3 c^2} \left[ (\omega^2_{OM} - \omega^2_{OM}) \left( 1 - \frac{1}{\beta^2_0} \right) - \frac{\omega^2_p}{\beta^2_0} \right. \]
\[ \times \ln \left( \frac{\gamma^2_\lambda + \omega^2_p - \omega^2_{OM}}{\gamma^2_\lambda + \omega^2_p - \omega^2_{OM}} \right) \left. \right] - \frac{2 e^2 T}{3 c^2} \frac{K_B T}{m_c c^2} \]
\[ \times \left[ \frac{\omega^4_{OM} - \omega^4_{OM}}{15 \gamma^2_\lambda} \left( 1 - \frac{1}{\beta^2_0} \right) - \frac{2 \omega^2_p}{15 \gamma^2_\lambda} \right. \]
\[ \times (\omega^2_{OM} + \omega^2_{OM}) + \frac{\omega^2_p}{3} \ln \left( \frac{\gamma^2_\lambda - \omega^2_{OM}}{\gamma^2_\lambda - \omega^2_{OM}} \right) \]
\[ - \frac{1}{3} \omega^2_p \left( \frac{\omega^2_{OM} \gamma^2_\lambda - \omega^2_p \gamma^2_\lambda - \gamma^4_\lambda (1 - \gamma^2_\lambda) - \omega^2_{OM} \omega^2_p}{\gamma^2_\lambda + \omega^2_p \gamma^2_\lambda - 2 \gamma^2_\lambda \omega^2_{OM} + \omega^4_{OM}} \right) \]
\[ + \frac{1}{3} \left( \frac{\omega^2_{OM} \gamma^2_\lambda - \omega^2_p \gamma^2_\lambda - \gamma^4_\lambda (1 - \gamma^2_\lambda) - \omega^2_{OM} \omega^2_p}{\gamma^2_\lambda + \omega^2_p \gamma^2_\lambda - 2 \gamma^2_\lambda \omega^2_{OM} + \omega^4_{OM}} \right) \]

(3.3.25)
where \( \omega_{OM} = \omega_0 \mid_{\text{maximum}} \) and \( \omega_{OM} = \omega_0 \mid_{\text{minimum}} \). The terms in the second square bracket are the additional ones due to the consideration of the temperature of the medium. Expression (3.3.25) would have got modified in the absence of the \( \chi - \sigma \) coupling.

From (3.3.25) we can define a modified temperature as

\[
T_m = T_{\text{modified}}
\]

\[
= T \left[ \frac{\omega_{OM} - \omega_{OM}}{15 \gamma_l^2 \omega_{pe}^2} \left( 1 - \frac{1}{\beta_0^2} \right) - \frac{2}{15 \gamma_l^2} (\omega_{OM} - \omega_{OM}) \right. \\
+ \frac{1}{5} \ln \left\{ \left( \frac{\gamma_{OM}^2 - \omega_{OM}^2}{\gamma_l^2 - \omega_{OM}^2} \right) \right\} \\
- \frac{1}{3} \frac{(\omega_{OM} \gamma_{OM}^2 - \omega_{OM}^2 \gamma_l^2 - \gamma_l^4)(1 - \frac{1}{\beta_0^2}) - \omega_{OM}^2 \omega_{OM} \omega_{pe}^2}{\gamma_l^4 + \omega_{pe}^2 \gamma_l^2 - 2 \gamma_l^2 \omega_{OM}^2 + \omega_{OM}^4} \\
+ \left. \frac{1}{3} \frac{(\omega_{OM} \gamma_{OM}^2 - \omega_{OM}^2 \gamma_l^2 - \gamma_l^4)(1 - \frac{1}{\beta_0^2}) - \omega_{OM}^2 \omega_{OM} \omega_{pe}^2}{\gamma_l^4 + \omega_{pe}^2 \gamma_l^2 - 2 \gamma_l^2 \omega_{OM}^2 + \omega_{OM}^4} \right] \\
(3.3.26)
\]

Thus the (3.3.25) can be written as

\[
\frac{dE}{dl} = - \frac{2 e_T^2}{3 c^2} \left[ (\omega_{OM}^2 - \omega_{OM}^2) \left( 1 - \frac{1}{\beta_0^2} \right) - \frac{\omega_{pe}^2}{\beta_0^2} \right. \\
\times \ln \left( \frac{\gamma_{OM}^2 + \omega_{pe}^2 - \omega_{OM}^2}{\gamma_l^2 + \omega_{pe}^2 - \omega_{OM}^2} \right) - \left( \frac{2 e_T^2 \omega_{pe}^2}{3 c^2 \text{m}_e c^2} \right) K e T_m \\
(3.3.27)
\]
\( T_m \) defined in (3.3.26) is seen to be a function of the medium parameters like \( \gamma_l \), the density \( \zeta \) through the plasma frequency and the test particle velocity. This may be considered as a renormalized temperature.

In this analysis the effective frequency which is the frequency of the dressed particles at the medium temperature \( T \) is defined as

\[
\omega_{\text{eff}}^2 = \omega^2(T)
\]

\[
= \frac{1}{\delta} \left( \gamma_\lambda^2 + \gamma_l^2 - \chi \right)
\]

\[
= \omega_0^2 + \frac{1}{3} \frac{\omega_{pl}^2 \gamma_\lambda^2 K B T}{\chi_0 m_k c^2} \quad > \omega_0^2
\]

(3.3.28)
in the linear approximation in \( T \), where \( \omega_0^2 \) is obtained by setting \( \chi \) to \( \chi_0 \) i.e. without temperature dependence.

The effective index of refraction of the system is now defined as

\[
\eta^2(\omega_0, T) = \frac{\gamma_\lambda^2}{\omega_{\text{eff}}^2}
\]

(3.3.29)
or

\[
\frac{1}{\eta^2(\omega_0, T)} = \frac{1}{\eta^2(\omega_0, 0)} + \frac{1}{3} \frac{\omega_{pl}^2 K B T}{\chi_0 m_k c^2}
\]

(3.3.30)
where

\[ \eta^2(\omega_0, 0) = \frac{\gamma_\lambda^2}{\omega_0^2} = 1 + \frac{\omega_p^2}{\gamma_\ell^2 - \omega_0^2} \]  

(3.3.31)

Equation (3.3.31) is the modified form of the result obtained by Pratap (1967) for the case of $T \to 0$. This modification is due to the consideration of $\gamma_\lambda$ given by

\[ \gamma_\lambda^2 = \gamma_\lambda^2 + \omega_p^2 \]  

(3.3.32)

instead of $\gamma_\lambda$ in the formulation given in chapter II. Using the expression (3.3.23) for $\chi_0$, (3.3.30) can be written as

\[ \frac{1}{\eta^2(\omega_0, T)} = \frac{1}{\eta^2(\omega_0, 0)} + \frac{\omega_p^2 (\gamma_\lambda^2 - \omega_0^2)}{3(\gamma_\lambda^4 + \omega_p^2 \gamma_\ell^2 - 2\omega_0^2 \gamma_\lambda^2 + \omega_0^4)} + \frac{K_B T}{m_\ell c^2} \]  

(3.3.33)

The above expression (3.3.33) gives the modification of the refractive index due to the thermal state of the system. This implies that the increase in $T$ reduces the refractive index of the system. The expansion is valid for small temperature changes. In the high temperature region, this expansion is not valid. Decrease in the refractive index of the system corresponds to the absorption of the radiation by the medium. It may however be noted that if the effective frequency resonates with the oscillator frequency ($\omega_0 = \gamma_\ell$), we do not have any temperature effect on the Cerenkov threshold. Also a numerical calculation for (3.3.25) shows a gradual decrease with the rise in temperature. This
absorption is in the lower frequency side as obvious from the
definition for $\omega_{\text{eff}}$ (3.3.28). Now the coherence condition for
the radiation coming out is obtained from (3.3.14) and (3.3.15)
because of the Dirac $\delta$-function as

$$
\cos \Theta = \left( \frac{-\gamma^2 + \gamma^2_c - \chi}{2 \gamma^2 \beta_0^2} \right)^{\frac{1}{2}}
$$

(3.3.34)

or

$$
\cos^2 \Theta = \left( \frac{-\gamma^2 + \gamma^2_c - \chi}{2 \gamma^2 \beta_0^2} \right) / 2 \gamma^2 \beta_0^2
$$

$$
= \frac{\omega_{\text{eff}}^2}{\gamma^2 \beta_0^2} = \frac{1}{\eta^2(\omega_0, T) \beta_0^2}
$$

(3.3.35)

Equations (3.3.33) and (3.3.35) imply that as $T$ increases $\cos \Theta$
increases and thus $\Theta$ decreases and hence with the increase in
medium temperature the Cerenkov cone starts shrinking. In other
words the medium temperature decreases the coherence. From
(3.3.35) it can be concluded that the threshold velocity for the
Cerenkov particle (test particle) which is given by

$$
\beta_{\text{min}} = \frac{1}{\eta^2(\omega_0, T)}
$$

(3.3.36)

is increased with the increase in temperature $T$. 
3.4 Conclusion

In this microscopic theory we have considered the effect of
of the longitudinal mode on the transverse interaction and studied
the role of the medium temperature in the energy output of the
Cerenkov radiation. As seen in Heitler (1954), in Lorentz gauge
the $\sigma^-$-part of the vector potential corresponds to the polarization
dependent part of the Coulomb interaction which is responsible for
the density fluctuations in the medium. Since the basic mechanism
for the phenomenon of Cerenkov radiation is the polarization of the
medium particles, we consider only the effect of the $\sigma^-$-mode but
not of $0\sigma^-$-mode which is the scalar part of the Coulomb inter-
action. Thus the inclusion of $\sigma^-$-mode will correspond to the
coupling of the density fluctuation in the medium particles with
the excitation of the transverse collective interaction leading to
the Cerenkov radiation. The inclusion of $\lambda - \sigma^-$ coupling appears
for the first time in the sixth-order diagram. This leads to the
conservation of wave vectors in different modes i.e. $k_{\lambda} = k_{\sigma^-}$.
It is seen that the inclusion of $\lambda - \sigma^-$ coupling in the $\lambda$
interaction changes the test particle distribution function as in
(3.2.11) (also see (3.2.18) and (3.2.20)) as compared to the earlier
work by Pratap (1967). The change appears through the summation
given in (3.2.18) which would have been

$$1 + \mathcal{E}_{\lambda}^\lambda(z_i) + \mathcal{E}_{\lambda}^\lambda(z_i) \mathcal{E}_{\lambda}^\lambda(z_i) + \ldots$$

$$= \frac{1}{1 - \mathcal{E}_{\lambda}^\lambda(z_i)}$$

(3.4.1)
It can be seen that the expression for the energy loss suffered by the test-particle in absence of the $\lambda - \sigma'$ coupling would be

$$
\frac{dE}{dl} = -\frac{eT}{c^2} \int d\gamma_\lambda \gamma_\lambda \left( 1 - \frac{\gamma_\lambda^2 - \gamma_e^2}{\chi'} \right)
\times \left( 1 - \frac{\gamma_\lambda^2 + \gamma_e^2 - \chi'}{2 \gamma_\lambda^2 \beta_0^2} \right)
$$

instead of the expression (3.3.16) where now $\chi'$ in the linear expression in $T$ is

$$
\chi' = \left[ \left( \gamma_\lambda^2 - \gamma_e^2 \right) + \frac{8}{3} \gamma_e^2 \omega_{pt}^2 \left( 1 - \frac{\gamma_e^2}{5 \gamma_e^2} \left( \frac{K_B T}{m_e c^2} \right) \right) \right]^{1/2}
$$

$$
= \chi_0' - \frac{4}{15} \omega_{pt}^2 \frac{\gamma_\lambda^2}{\chi_0'} \frac{K_B T}{m_e c^2}
$$

(3.4.3)

with

$$
\chi_0' = \left[ \left( \gamma_\lambda^2 - \gamma_e^2 \right) + \frac{8}{3} \gamma_e^2 \omega_{pt}^2 \right]^{1/2} = \chi'(T \to 0)
$$

(3.4.4)

The $\gamma_\lambda$ integration in (3.4.2) can be carried in a similar way as explained in section (3.3) by defining the variable

$$
\omega_{0'}^2 = \frac{1}{\chi_0} \left( \frac{\gamma_\lambda^2}{\gamma_e^2} + \gamma_e^2 - \chi_0' \right)
$$

(3.4.5)
which defines the effective frequency of the radiation coming out of the medium as \( T \to 0 \) in absence of the \( \lambda - \sigma \) coupling. Instead of the expressions (3.3.21) - (3.3.24) we use the following expressions for the respective parameters which are

\[
\gamma_{\lambda}^2 = \frac{\omega_0^4 - \omega_0^2 \omega_{pl}^2 - \omega_0^2 \gamma_{l}^2 + \frac{2}{3} \omega_{pl} \gamma_{l}^2}{\omega_0^2 - \gamma_{l}^2}
\]

(3.4.6)

\[
\gamma_{\lambda}^2 = \frac{\omega_0^4 - \omega_0^2 \gamma_{l}^2 - 2/3 \omega_{pl} \gamma_{l}^2}{\omega_0^2 - \gamma_{l}^2}
\]

(3.4.7)

\[
\chi_0' = \frac{\omega_0^4 - 2 \omega_0^2 \gamma_{l}^2 + 2/3 \omega_{pl} \gamma_{l}^2 + \gamma_{l}^4}{\gamma_{l} - \omega_0^2}
\]

(3.4.8)

and

\[
d \gamma_{\lambda}^2 = 2 \gamma_{\lambda} \, d \gamma_{\lambda} = \frac{\gamma_{l} - 2 \omega_0^2 \gamma_{l}^2 + 2/3 \omega_{pl} \gamma_{l}^2 + \gamma_{l}^4}{(\omega_0^2 - \gamma_{l}^2)^2}
\]

(3.4.9)

and carry out the \( \gamma_{\lambda} \)-integration in (3.4.2) which in linear approximation in \( T \) is

\[
\frac{dE}{dl} = -\frac{eT}{c^2} \left[ (\omega_{om}^2 - \omega_{om}^2) \left( 1 - \frac{1}{\beta_0^2} \right) - \frac{\omega_{pl}^2}{x \beta_0^2} \right.
\]

\[
\times \ln \left( \frac{\omega_{om}^4 - \omega_{pl}^2 \omega_{om}^2 - \gamma_{l}^2 \omega_{om}^2 + 3 \omega_{pl}^2 \gamma_{l}^2}{\omega_{om}^4 - \omega_{pl}^2 \omega_{om}^2 - \gamma_{l}^2 \omega_{om}^2 + 3 \omega_{pl}^2 \gamma_{l}^2} \right)
\]

\[
- \frac{\omega_{pl}^2 + \frac{1}{3} \omega_{pl} \gamma_{l}^2}{2 \beta_0^2} \ln \left\{ \frac{\chi_M + X}{\chi_M - X} \right\} \]

\[
\times \left[ \frac{\chi_M - X}{\chi_M + X} \right]
\]
- $\frac{2e^2}{15c^2} \frac{K_BT}{m_e c^2} \omega_{pl} \left\{ \ln \frac{\gamma_e^2 - \omega_{OM}^2}{\gamma_e^2 - \omega_{OM}^2} \right\}$

\[
\left. \begin{array}{l}
\frac{(\gamma_e^4 - \omega_{OM}^2 \gamma_e^2)(1 - \beta_0^2) + \omega_{OM}^2 \omega_{pl}^2 (\frac{1}{3} - \frac{2}{3} \beta_0^2)}{\omega_{OM}^4 - 2 \omega_{OM}^2 \gamma_e^2 + \frac{2}{3} \omega_{pl}^2 \gamma_e^2 + \gamma_e^4} \omega_{pl} \gamma_e^2 \\
\end{array} \right\}
\]

(3.4.10)

with

\[
\chi = \sqrt{\left( \omega_{pl}^4 + \gamma_e^4 + \frac{2}{3} \omega_{pl}^2 \gamma_e^2 \right)}
\]

(3.4.11)

\[
\gamma_m = \gamma_e^2 + \omega_{pl}^2 - 2 \omega_{OM}^2
\]

(3.4.12)

The modification obtained by the inclusion of the $\lambda - \sigma'$ coupling in the transverse interaction can be noticed from the form of the expressions (3.3.16), (3.3.25), (3.4.2) and (3.4.10).

For comparison we make a quantitative estimation of the results (3.3.25) and (3.4.10) for which one needs to determine $\omega_{OM}$, $\omega_{OM}'$ and $\omega_{OM}, \omega_{OM}'$. This can be done by considering the coherence condition (3.3.35) which can be written as

\[
\eta^2 \beta_0^2 > 1
\]

(3.4.12)

where for $T \to 0$
The first relation of (3.4.13) with (3.4.12) gives a condition
\[ \gamma_{\ell}^2 - \frac{\beta_0^2 \omega_{pl}^2}{1 - \beta_0^2} < \omega_0^2 < \gamma_{\ell}^2 \]  
(3.4.14)
for the $\lambda$ -interaction when $\lambda$ - $\sigma$ coupling is taken into account.
For the pure $\lambda$ -interaction, the second relation of (3.4.13) itself gives a condition
\[ \gamma_{\ell}^2 \geq \omega_{0}^{'2} \geq \frac{1}{3} \gamma_{\ell}^2 \]  
(3.4.15)
but with the condition (3.4.12), it gives an inequality
\[ \omega_0^4 - \omega_0^{'2} \left( \gamma_{\ell}^2 - \frac{\beta_0^2 \omega_{pl}^2}{1 - \beta_0^2} \right) - \frac{\beta_0^2 \omega_{pl}^2 \gamma_{\ell}^2}{3 (1 - \beta_0^2)} \geq 0 \]  
(3.4.16)
The $\omega_0'$ which satisfies the equality condition of (3.4.16) will give $\omega_{0m}'$ and the $\omega_0$ satisfying the equality condition of (3.4.14) will give $\omega_{0m}$. From (3.4.14) the condition that $\omega_0 > 0$ we need
\[ \gamma_{\ell}^2 > \frac{\beta_0^2 \omega_{pl}^2}{1 - \beta_0^2} \]  
(3.4.17)
For a typical dielectric medium \( \gamma_l = 6 \times 10^{15} \) per second (Jelley, 1958). Satisfying the above conditions, we choose

\[
\omega_p^2 = 8 \times 10^{30}, \quad \omega_m^2 = 1.895 \times 10^{30}, \quad \omega_m^2 = 30 \times 10^{30}
\]

\[
\omega_m^2 = 21.2 \times 10^{30}, \quad \omega_m^2 = 30 \times 10^{30}
\]

Substituting the above values of the different frequencies, in the two expressions (3.3.25) and (3.4.10) we evaluate \( \frac{d\bar{E}}{dl} \lambda - \sigma \) and \( \frac{d\bar{E}}{dl} \lambda \) respectively and then evaluate the following ratios

\[
\left[ \frac{(\frac{d\bar{E}}{dl})^{T \to 0} - (\frac{d\bar{E}}{dl})^{T \to 0}}{(\frac{d\bar{E}}{dl})^{T \to 0}} \right] \cong 0.5545 \frac{K_B T}{m_l c^2} \]

and since \( K_B T / m_l c^2 \approx 1.683 \times 10^{-10} \), we see that the effect of the temperature on the radiation coming out of medium is very small for \( T \) up to the order of \( 10^{10} \). However the ratio

\[
\frac{(\frac{d\bar{E}}{dl})_{\lambda - \sigma} - (\frac{d\bar{E}}{dl})_{\lambda}}{(\frac{d\bar{E}}{dl})_{\lambda}} \cong 1.415
\]

\[
\frac{(\frac{d\bar{E}}{dl})_{\lambda - \sigma} - (\frac{d\bar{E}}{dl})_{\lambda}}{(\frac{d\bar{E}}{dl})_{\lambda - \sigma}} \cong 0.586
\]

(3.4.20)
Hence the inclusion of the density fluctuation in the evaluation of the Cerenkov radiation becomes significant as we include the transfer of energy from the longitudinal mode to the transverse mode. However, the effect of temperature becomes significant only at high temperatures ($\sim 10^{10}$ K). We have also shown that the inclusion of the $\sigma^-$-mode into the radiation problem modifies the effective collective frequency (see (3.3.20) and (3.4.5)).

The increase in $\left(\frac{d\xi}{dl}\right)$ because of the inclusion of $\sigma^-$-mode in the pure transverse interaction is obvious from the fact that when the relativistic charged particle enters the medium it excites both the collective transverse and longitudinal oscillations. Due to the coupling of the longitudinal mode with the transverse mode, the energy will be transferred from $\sigma^-$-mode to the $\lambda^+$-mode which should ultimately be coming from the moving particle exciting them. This fact is also pointed out by Majumdar (1961).

The effect of the temperature can be seen in the following manner. For a reasonable set of parameters (given in (3.4.18)), $T_m$ as defined in (3.3.26) becomes negative definite, which in turn makes the last term in (3.3.27) positive definite. Thus the energy loss is reduced and this implies that the energy coming out of the system is also reduced. We thus have the phenomenon of Cerenkov radiation which vanishes for a critical temperature and hence the collective modes disappear. This can also be interpreted physically following Bohm and Pines (1951, 1952) that the characteristic interaction length becomes smaller than the Debye length. It may however be noted that this critical temperature is very
high and at this temperature this formulation breaks down, since one has to consider the whole system to be constituted of relativistic particles and the higher terms in $T$ which have been neglected will give nonvanishing contribution. Hence while the inclusion of the longitudinal mode in the evaluation of the distribution function increases the energy, the thermal state of the system would reduce it and becomes significant only at very high temperatures. This is why the temperature effect is not observed in the usual laboratory systems. This confirms that Cerenkov radiation is not a form of the phenomenon of luminiscence (Cherenkov, 1960). The temperature effect may however be a significant factor in a very high temperature system such as the pulsar atmosphere or in a fusion reactor.