CHAPTER I

INTRODUCTION

Classical dynamics is the oldest part of the present-day theoretical physics. It can be said that the modern science began from the pioneering work of Galileo and Newton on celestial mechanics. Their ideas of the motion of celestial bodies can be focussed into a single equation

\[ m \frac{d^2 \mathbf{r}}{dt^2} = \mathbf{F} \]  \hspace{1cm} (1.1.1)

which describes the trajectory of a test body with \( \mathbf{r} \) as its position co-ordinate and \( m \) being its mass. One can note that equation (1.1.1) involves time, since the "effect" of the force \( \mathbf{F} \) is an acceleration, the rate of change of velocity in time. The expression (1.1.1) is deterministic in character; if the position and the velocity of a particle are given at any time, the equations determine its position and velocity at any other time in the past or future. Further progress in the dynamics of Newton was made by the mathematical investigations of Laplace, Lagrange, Gauss, Hamilton and many others. Before the introduction of quantum mechanics, the advanced Newtonian dynamics seemed to form a closed universe by answering any question that is asked. Almost by definition, a question to which dynamics had no answer was discarded as a pseudoproblera. In the words of Prigogine (1980) "Dynamics thus seemed to give man access to ultimate reality. In
this vision, the rest appeared only as a kind of illusion, devoid of fundamental significance”.

After the hypothesis that matter is composed of ultimate and indivisible particles, the principal aim of physics became to identify the microscopic level, to which the known laws of macroscopic dynamics could be applied, in order to have a basis for explaining all observable phenomena. It is a well-known fact that the universal constants such as the Planck's constant
\[ h \approx 6 \times 10^{-27} \text{ erg sec.} \] and the velocity of light \[ c \approx 3 \times 10^{10} \text{ cm/sec.} \] limit the validity of the classical dynamics. This gave rise to the birth of quantum mechanics and relativistic dynamics. However these new forms of dynamics inherited the idea of Newtonian mechanics: as stated by Prigogine (1980) "a static universe, a universe of being without becoming”.

We see in dynamics the time parameter appears as motion and future and past appear on equal footing. In other words the classical equations of motion which are highly deterministic maintain a time symmetry. From the day-to-day changes in natural phenomena in association with the "evolution theory" of Darwin, we see that evolution is the inheritance of the natural processes. This arrow of time can be observed in physics, biology, chemistry and sociology with different meaning. In physics the study of heat flow in material bodies introduces this arrow of time, an essential difference between past and future, a definite 'flow of time' i.e. the flow of events in time. This branch of physics known as thermodynamics starts from the work of Joule, namely that the work necessary to transfer a system from one equilibrium state to another depends only on these two states, not on the process of
application of the work. This is the real content of the first law of thermodynamics which gave an empirical concept of temperature. But the definition of the absolute scale of temperature arose from the discovery of the second law of thermodynamics by Clausius and Lord Kelvin. In short this law can be stated as "heat can not be transferred completely into work nor can it be brought from a state of lower temperature to one of higher without any compensation". This established the abstract concepts of entropy and absolute temperature. Caratheodory was the first to give a satisfactory axiomatic formulation of the second law of thermodynamics. All these lead to the result that entropy increases monotonically until it reaches its maximum value, at the state of thermodynamic equilibrium. The use of thermodynamics gives a vast amount of knowledge not only in physics but also in borderland sciences of physico-chemistry, metallurgy, minerology etc. Most of it refers to equilibrium. In fact, the only dynamical statements possible are concerned with the irreversible transitions from one equilibrium state to another which are outside the scope of thermodynamics.

In the wave theory of light we see that the electromagnetic theory formulated by James Clerk Maxwell which was confirmed by Kohlrausch and Weber and finally by Hertz's discovery of electromagnetic waves, maintains a time symmetry. Thus here also there is a kind of reversibility very similar to that of classical dynamics as explained above. But here there is a difference, a change of sign of all current densities and the whole magnetic field (as done usually) is not as simple to perform as that of a finite set of velocities (Born, 1949). While considering the
emission of radiation from an accelerated charge, Maxwell's equations allow two types of waves: retarded wave (outgoing wave) and advanced wave (incoming wave) i.e. wave spreading out of the source and wave converging to the source respectively, whereas in experience we observe the retarded wave. The consideration of retarded waves really satisfied the "causality" condition according to which the effect must follow rather than precede the causes. The reason why we do not observe the advanced wave is explained by Wheeler and Feynman (1949), by considering the interaction of the radiating source with the universe. Hence here also we see an arrow of time. This electromagnetic arrow of time has a relation with the arrow of time in thermodynamics (Narlikar, 1978). But this electromagnetic time arrow in connection with the arrow of time in cosmology is still in ambiguity and becomes model dependent.

Thus we see that the dynamics which is highly deterministic explains the time symmetry of the natural phenomena. On the other hand thermodynamics which gives a unidirection of time leading the system to thermal equilibrium when the entropy becomes maximum and thus introduces the concept of irreversibility in the natural processes. The problem of linking this macroscopic observation of irreversibility in natural events with dynamics was the subject of investigation for a long time.

The new turn in physics was the introduction of atomistics and statistics. The concept of statistics entered in science through the theory of probability which actually sprang out not from the needs of natural science but from gambling and other more or less disreputable human activities (Born, 1949). The first use of the probabilistic considerations in science was made
by Gauss in his theory of experimental errors. Efforts to make a connection with mechanics and irreversibility started with the development of kinetic theory of gases from the works of Bernoulli, Maxwell and others (Born, 1949). With a statistical hypothesis, the "principle of molecular chaos" Maxwell derived his celebrated law of velocity distribution for an ideal gas. But the derivation was not free from objection, because the assumptions involved are too crude. The serious objection was that the formulation did not involve the interactions between the particles which are revealed by non-equilibrium phenomena: viscosity, conduction of heat, diffusion. The qualitative understanding of the mechanism of these phenomena gives a satisfactory result of the observation. But the problem is to derive a rigorous kinetic theory taking into account the interactions which is valid for both equilibrium and non-equilibrium states from which the hydrodynamical equations of visible motion together with the phenomenon of transformation and conduction of heat and, for a mixture, of diffusion can be derived.

The formulation of non-equilibrium theory of gases which gives the Maxwell's celebrated law for equilibrium velocity distribution in the asymptotic limit in time was due to Ludwig Boltzmann. Boltzmann made a basic assumption - the so-called "Stosszahlansatz" - concerning the change in $f$ (the one particle distribution function) due to the collisions between molecules. He writes the evolution equation for $f$ as

$$\frac{\partial f^{(1)}}{\partial t} + \mathbf{v} \cdot \frac{\partial f^{(1)}}{\partial \mathbf{x}} = \mathcal{Q}^{(1)} \quad (1.1.2)$$
in the absence of external forces. In the expression (1.1.2) \( \mathbf{v} \) is the velocity of the molecule and \( \mathcal{C}(1) \) represents the influence of the molecules on the particle \( i \). This \( \mathcal{C}(1) \) is called the collision integral. The interesting feature of the equation (1.1.2) can be obtained from a pure dynamical consideration where as \( \mathcal{C}(1) \) is deduced only from the probabilistic assumption about the number of collisions (Stosszahlansatz). Moreover, the collision term is estimated as if there were no flow, and the forces responsible for collisions are neglected in the flow term.

The equation (1.1.2) indicates an irreversible approach from any initial state to a homogeneous equilibrium state where the distribution becomes the Maxwellian. From the Boltzmann equation, one immediately derives the hydrodynamic equations and the transport co-efficients appearing in these equations are directly related to the intermolecular interactions of the gas. Also there is a remarkable agreement with the experiment in the calculation of these transport co-efficients for dilute gases. Methods of solving the equation (1.1.2) have been developed by Hilbert, Enskog, Chapman, Pidduck, Burnett, Grad and others. For a complete account of all developments of the theory up to about 1939, one can refer to an admirable book by Chapman and Cowling (1970) "The Mathematical Theory of Non-Uniform Gases".

But Boltzmann's formulation, for all its good points, did not have a smooth sailing, due to the intermixing of dynamics and statistics. One of the main objections was due to Loschmidt's reversibility paradox because the laws of mechanics are invariant
under time inversion \((t \rightarrow -t)\) i.e. to each process there corresponds a time reversal process. Also this seems to be in contradiction with the existence of irreversible processes. The other serious objection was due to Ernst Zermelo's recurrence paradox, which is based on a theorem of Poincare which states that every mechanical system is, if not exactly periodic, at least quasiperiodic. Finally these points were clarified by Paul Ehrenfest and his wife Tatjana in their well-known article in volume IV of the Mathematical Encyclopedia.

Then came the need for a general formulation covering gaseous, liquid and solid states under all kind of external forces, because Boltzmann's theory was applicable only for dilute gases. Even for dilute gases, Boltzmann's definition of entropy applies only for certain initial conditions.

It is because of such difficulties that Gibbs and Einstein worked out a much more general approach in terms of the ensemble theory. These ideas are very much helpful in understanding the connection between the dynamics and thermodynamics. In classical dynamics a system of \(N\)-bodies is uniquely specified by \(6N\) generalized state variables (\(3N\) co-ordinates \(q_i\) and \(3N\) canonically conjugate momenta \(p_i\)). The laws of classical dynamics may be expressed in terms of the Hamilton's equations:

\[
\frac{dq_i}{dt} = \frac{\partial \mathcal{H}}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial \mathcal{H}}{\partial q_i}
\]  

(1.1.3)

where

\[
\mathcal{H} = \mathcal{H}_0 + \lambda \mathcal{V}
\]  

(1.1.4)
is the Hamiltonian of the system of interest. This represents the total energy if the system is conserved; $\mathcal{H}_0$ is the Hamiltonian for a non-interacting system and $V$ gives the interactions amongst the particles of the system through the coupling parameter $\lambda$.

Imagine a space of $6N$ dimensions whose points are determined by the above $6N$ co-ordinates. This space is called the phase space or gas space ($\Gamma$-space). Then the state of the system is represented by a single point and in time this point traces out a trajectory in this space. If the initial state of the system is known uniquely, then this single trajectory can be followed in time. However it is not possible to know $6N$ variables at any given time. To overcome this difficulty, Gibbs made mental replicas of the given system with all possible initial states but with the same Hamiltonian and for each initial state, there would be a trajectory which are infinite in number. At any instant of time, the end points of the trajectories would form a cloud of points and one can then define a density of points in this space as

$$\mathcal{P} = \frac{L t}{\Delta v} \frac{\Delta N}{\Delta v} = \mathcal{P}(q_1, \ldots, q_{3N}, p_1, \ldots, p_{3N}; t)$$

where $\Delta N$ is the number of points contained in the volume element $\Delta v$, in this $6N$-dimensional $\Gamma$-space. This density function, giving the distribution of the points in phase space, is known as the Liouville density. The members of the ensemble do not interact and hence two states do not occupy the same space-time point. Thus the flow in phase space is just like an incompressible fluid. This is equivalent to saying that the divergence of the
flow in phase space vanishes. Indeed using Hamilton's equations (1.1.3) we have

$$\sum_{j=1}^{3N} \left[ \frac{\partial}{\partial q_j} \left( \frac{dq_j}{dt} \right) + \frac{\partial}{\partial p_j} \left( \frac{dp_j}{dt} \right) \right] = 0$$  \hspace{1cm} (1.1.6)

Since the trajectories do not intersect, there is no accumulation of the points in the $\mathbb{R}^3$-space. Taking this into account one can write the continuity equation

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u})$$

$$= \frac{\partial \rho}{\partial t} + \sum_{j=1}^{3N} \left[ \frac{\partial}{\partial q_j} (\rho \frac{dq_j}{dt}) + \frac{\partial}{\partial p_j} (\rho \frac{dp_j}{dt}) \right] = 0$$  \hspace{1cm} (1.1.7)

where $\mathbf{u}$ is the velocity of the fluid in phase space. Because of (1.1.6), from (1.1.7) we then get the Liouville equation

$$\frac{\partial \rho}{\partial t} + \sum_{j=1}^{3N} \left( \frac{\partial \mathcal{H}}{\partial p_j} \frac{\partial \rho}{\partial q_j} - \frac{\partial \mathcal{H}}{\partial q_j} \frac{\partial \rho}{\partial p_j} \right) = 0$$  \hspace{1cm} (1.1.8)

or

$$\frac{\partial \rho}{\partial t} + i \mathcal{L} \rho = 0 , \quad i = \sqrt{-1}$$  \hspace{1cm} (1.1.9)

where

$$i \mathcal{L} = \sum_{j=1}^{3N} \left( \frac{\partial \mathcal{H}}{\partial p_j} \frac{\partial}{\partial q_j} - \frac{\partial \mathcal{H}}{\partial q_j} \frac{\partial}{\partial p_j} \right)$$  \hspace{1cm} (1.1.10)

is the Liouville operator. The factor 'i' is introduced in the operator $\mathcal{L}$ in order to draw parallelism with quantum field theory.
so that one can use all the operator algebra developed for field 
theory in the study of the dynamics of correlations.

The power of Gibbs ensemble theory was only recognized around 
1930 when among many other advances Urry and Mayer applied it 
successfully to the problem of equation of state. But the ensemble 
theory was only limited to the equilibrium state of the system. No 
进一步 development was done in order to explain the evolution of 
matter by using the principles of dynamics. Using the Gibbs 
ensemble theory Yvon (1935), Born and Green (1946 and 1947), 
Kirkwood (1946) and Bogoliubov (1946) independently presented their 
pioneering work in this line. Their approach is as follows:

The physicist's interest is mainly to study the local proper­
ties like density, velocity field, energy density etc of a given 
system at a point q. This can be achieved by studying the one 
particle distribution function and take its various moments. The 
question mainly is then to obtain the one particle distribution 
function from the N-particle distribution function. Towards this 
end and in general one defines a reduced distribution function 
(for S-particles, S < N) as

\[ f_S(q_1, q_2, \ldots, q_{3S}, p_1, p_2, \ldots, p_{3S}) = \int dq_{3S+1} \ldots dq_{3N} dp_{3S+1} \ldots dp_{3N} f(q_1, p_1; t) \]  

One can obtain an evolution equation for \( f_S \) by integrating the 
Liouville equation (1.1.8) over the N-S particle variables. This 
could also be considered as a Liouville equation for an open 
system of S-particles which will interact with the outside 
world i.e. the rest of N-S particles of the system. Thus with
the Hamiltonian (1.1.4) with $V = V(q_i)$, the evolution equation for $f_s$ can be written as

$$\frac{\partial f_s}{\partial t} + \sum_{j=1}^{3s} \left( \frac{\partial H_0}{\partial p_j} \frac{\partial f_s}{\partial q_j} - \frac{\partial H_0}{\partial q_j} \frac{\partial f_s}{\partial p_j} \right) = \lambda \sum_{j=1}^{3N} \int dq_{3s+1} \cdots dq_{3N} dp_{3s+1} \cdots dp_{3N} \frac{\partial V}{\partial q_j} \frac{\partial f_s}{\partial p_j} \quad (1.1.12a)$$

But for a Hamiltonian of the form (e.g., in case of ordinary plasma)

$$\mathcal{H} = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \lambda \sum_{j<k} V_{jn} (q_j - q_k) \quad (1.1.13)$$

Equation (1.1.12a) becomes (with $H_0 = \sum_{j=1}^{N} \frac{p_j^2}{2m}$)

$$\frac{\partial f_s}{\partial t} + \sum_{j=1}^{S} \frac{p_j}{m} \frac{\partial f_s}{\partial q_j} = \lambda \sum_{j=1}^{N} \int dq_{3s+1} \cdots dq_{3N} dp_{3s+1} \cdots dp_{3N} \frac{\partial V}{\partial q_j} \frac{\partial f_s}{\partial p_j}$$

$$= \lambda \sum_{j<k} \int dq_{3s+1} \cdots dq_{3N} dp_{3s+1} \cdots dp_{3N} \frac{\partial V_{jn}}{\partial q_j} \left( \frac{\partial}{\partial p_j} - \frac{\partial}{\partial p_k} \right) f \quad (1.1.12b)$$

$$\frac{\partial V_{jn}}{\partial q_j} = - \frac{\partial V_{jn}}{\partial q_k} \quad (1.1.12c)$$

as $V_{jn}$ is symmetric. Carrying out the integration in r.h.s. of the equation (1.1.12b), it can be written as
\frac{\partial f_s}{\partial t} + \sum_{j=1}^{s} \frac{m_j}{\lambda} \frac{\partial f_s}{\partial p_j} = \lambda \sum_{j=1}^{s} \int dq_{j2} \frac{\partial V_{j2}}{\partial q_{j1}} \frac{\partial}{\partial q_{j1}} f_s(q_{j1}, p_{j1}; t) \frac{\partial f_{s+1}}{\partial q_{j1}} \frac{\partial f_{s+1}}{\partial p_{j1}} \quad (1.1.14)

Now the expression (1.1.14) gives a hierarchy of equations relating the evolution of $f_s$ to $f_{s+1}$. This set of equations is known as B-B-G-K-Y hierarchy. This hierarchy of equations form an open set and to solve this, one has to impose a closure condition. The first equation of the set is written as

\frac{\partial f_1(q_1, p_1; t)}{\partial t} = \frac{\partial f_1}{\partial q_1} + \frac{p_1}{m} \frac{\partial f_1}{\partial q_1} \quad (1.1.15)

It may be noted that the calculation of the one particle distribution function $f_1$ (which is the main interest) by using the above integro-differential equation, requires the knowledge of $f_2$ which in turn depends on the form of $f_3$ and so on. One method of effecting the closure of the hierarchy is by taking the two particle distribution function

\begin{align*}
 f_2(q_1, q_2, p_1, p_2; t) &= f_1(q_1, p_1, t) f_1(q_2, p_2, t) \quad (1.1.16)

this implies that the two particles $(p_1, q_1)$ and $(p_2, q_2)$ are uncorrelated. But this way of factorization for closing the hierarchy is not at all satisfactory.

But Bogoliubov's approach in closing the hierarchy was different. In this formulation there is a clear distinction between
the characteristic times involved. There are at least two time scales involved: the duration of interaction $t_{\text{int}}$ (time required for a collision) and the relaxation time (which for a dilute gas is of the order of the time between two successive collisions). Also the gas evolves differently in different time scales. Bogoliubov, following Hilbert and Enskog assumes that after a time interval of the order of time of collision $t_{\text{int}}$, the time in the many particle distribution function can be expressed as a functional of the one particle distribution function and the time dependence of this many particle distribution is completely determined once the one particle distribution function is known. This can be written as

$$f_s (q_1, \ldots, q_s, p_1, \ldots, p_s; t) = f_s (q_1, \ldots, q_s, p_1, \ldots, p_s | f_1(t)) \quad (1.1.17)$$

where $f_s$ is a functional of the quantities appearing on the right of the vertical bar.

In view of the ansatz of Bogoliubov, equation (1.1.15) can be written as

$$\frac{\partial f_1}{\partial t} + \frac{p_1}{m} \frac{\partial f_1}{\partial q_1} = \lambda \int dq_2 dp_2 \frac{\partial V_{12}}{\partial q_{12}} \frac{\partial}{\partial p_{12}} f_2 (q_1, q_2, p_1, p_2 | f_1(q_1, q_2, t)) \quad (1.1.18)$$

This assumption makes it possible to justify the use of the one-particle distribution function in the Boltzmann theory.

The assumption (1.1.17) alone does not close the equation (as seen from the equation (1.1.18)) since one has to give a form
for $f_s(f^2$ in the expression (1.1.18)). Bogoliubov, hence, introduces a second basic assumption as an initial condition

$$\mathcal{L}_{t=-\infty} f_s = \frac{S}{\prod_{j=1}^{n}} f_s (q_j, p_j; t)$$

(1.1.19)

where $\mathcal{L}_{t=-\infty}$ is an unitary translation operator which takes $f_s$ to the infinite past. The equation (1.1.19) implies that at infinite past, the system remains uncorrelated. This however allows the system to evolve in time by continuous creation and destruction of correlations in the system. This initial condition thus gives an arrow of time and introduces irreversibility (Wu, 1966). The clear existence of the two different time scales in his theory allows one to expand $f_s$ and $\frac{\partial f_s}{\partial t}$ in powers of density and thus Bogoliubov obtains a system of equations which could in principle be solved to all orders of density. When carried out to the first order in density, this theory yields Boltzmann equation and thus the higher order terms act as corrections to higher densities.

But the justification of the expression (1.1.17) from the point of view of dynamics, was not clear (Balescu, 1963) and also the density expansion method in the Bogoliubov's theory gives a divergence problem (Wu, 1966). Further progress in this formulation keeping the above difficulties in view, was done by Friemann and Sandri (Wu, 1966).

Prigogine and his co-workers developed still another theory to account for the irreversibility processes in gases (Prigogine, 1962, Balescu, 1963) which avoided some of these assumptions.
Here the Liouville equation

\[ \frac{\partial \rho}{\partial t} + i \mathcal{L}_0 \rho + \lambda (i \delta \mathcal{L}) \rho = 0 \]  

(1.1.20)

where \( \mathcal{L} = \mathcal{L}_0 - \lambda \delta \mathcal{L} \)  

(1.1.21)

with \( \mathcal{L}_0 \) and \( \delta \mathcal{L} \) as the Liouville operators corresponding to the Hamiltonian \( \mathcal{H}_0 \) and interaction potential \( V \) respectively is first solved formally. The equation (1.1.20) is a linear equation for \( \rho \) and hence can be solved by using the standard techniques of linear partial differential equations in mathematics. Several equivalent methods can be used as a starting point for this investigation, such as the direct iteration solution of the Liouville equation in the iteration representation first used in this problem by Brout and Prigogine (1956) or the resolvent formalism first applied to this problem by Resibois (1961) (Also see Henin et al, 1961). Both these methods are explained in Prigogine's monograph referred to above. Balescu in his monograph mentioned above, has used the latter method using the Green's function of the Liouville equation to the case of electrostatic interactions in plasma (Also see Prigogine and Balescu, 1959, 1960; Balescu 1960; Balescu and Kuezzell 1964).

The formulation of Prigogine and his co-workers is applied for explaining the field (electromagnetic) and particle interaction for the first time by Prigogine and Leaf (1959). Henin (1963) applied the same formulation in order to explain the Liouville equation for a relativistic charged particle interacting with the system through the radiation field in the presence of an external
electromagnetic field and has calculated the energy radiated away from the system of charged particles. In this framework Pratap (1967) has explained the phenomenon of Cerenkov radiation.

In this thesis the formulation of Prigogine and his co-workers is applied to radiation problems using the diagram technique given by Pratap (1967). Equation (1.1.20) can be solved formally and \( \mathcal{F}(t) \) can be written as

\[
\mathcal{F}(t) = \exp\left(-i\mathcal{L}_0 t\right)\mathcal{F}(0) \\
+ \lambda \int_0^t dt' \exp\left(-i\mathcal{L}_0 (t-t')\right) (i\delta \mathcal{L}) \mathcal{F}(t')
\] (1.1.22)

where \( \mathcal{L}_0 \) and \( \delta \mathcal{L} \) are not explicit functions of time. This equation can be iterated and the final solution can be written as

\[
\mathcal{F}(t) = \exp\left(-i\mathcal{L}_0 t\right) \times \\
\left[ 1 + \sum_{n=1}^{\infty} \lambda^n \int_0^t dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n \mathcal{O}_n \right] \mathcal{F}(0)
\] (1.1.23)

where

\[
\mathcal{O}_n = \exp\left(i\mathcal{L}_0 t_1\right) (i\delta \mathcal{L}) \exp\left(-i\mathcal{L}_0 t_2\right) (i\delta \mathcal{L}) \times \\
\exp\left(-i\mathcal{L}_0 t_3\right) \cdots (i\delta \mathcal{L}) \exp\left(-i\mathcal{L}_0 t_n\right)
\] (1.1.24)

We see that the solution (1.1.23) of the Liouville equation (1.1.20) is an exact solution for a given many-body system. In general, handling the solution (1.1.23) will be difficult. But the existence of different time scales: \( t_{\text{int}} \) the duration of an interaction, \( t_r \) the relaxation time and \( t_h \) the hydrodynamic time
(Balescu 1963, Prigogine 1962) when they are widely separated, simplifies the situation very much. The usual procedure is to use a diagram technique and select the diagrams giving dominant contributions to the solution (1.1.23), corresponding to the physical situation, pertaining to a particular time scale.

While considering a system of \( N \)-particles, we consider the interaction of these particles through a radiation field in general (Heitler, 1954), and write the Hamiltonian as

\[
\mathcal{H} = \sum_i \mathcal{H}_i + \sum_f \mathcal{H}_f \tag{1.1.25}
\]

where \( \mathcal{H}_i \) is the Hamiltonian for the \( i^{th} \)-particle and \( \sum_f \mathcal{H}_f \) is the Hamiltonian for the radiation field. Even though the part of \( \mathcal{H} \) corresponding to the particles is written as a sum of that corresponding to each particle \( i \), the interaction appears through the vector potential \( \vec{A} \) and the scalar potential \( \Phi \), which are functions of the variables corresponding to the particles and the radiation field appearing in \( \mathcal{H}_i \). Following Heitler (1954) (also see Landau and Lifshitz 1979), the \( \mathcal{H}_i \) for a non-relativistic particle with the field (internal) given by \( \vec{A}_i \) and \( \Phi_i \) is

\[
\mathcal{H}_i = \frac{1}{2m} \left( \vec{p}_i - \frac{e}{c} \vec{A}_i \right)^2 + e \Phi_i \tag{1.1.26}
\]

In general the contribution from \( \vec{A}_i^2 \) is neglected by most of the authors, while applying the formulation of Prigogine and co-workers, in considering the field-particle interaction in a many particle system. After making the Fourier expression for \( \vec{A}_i \), it is seen
that in random phase approximation (Bohm and Pines 1951) we get
a contribution from $A_{ij}^2$ as a shift in the frequency (Patro and
Pratap 1983). The inclusion of this contribution produces a
significant change in the final results.

In chapter-II a general formulation that is used in later
chapters in the light of the diagram technique devised by Pratap
(1967) is given with a special reference to the phenomenon of
Cerenkov radiation in an ordinary dielectric medium. In the formu-
lation the contributions from $A_{ij}^2$ are taken into account. This
implies a kind of superposition of the plasma waves with the
electromagnetic field. In order to have a better appreciation of
the present formulation, the classical theory of this phenomenon
based on the phenomenological electrodynamics by Frank and Tamm
is also given.

The formulation of statistical mechanics starts from the general
solution (1.1.23) where 'e', the electronic charge plays the role of
$\lambda$ the coupling parameter. Since the problem of interest here is to
put a test particle (relativistic or non-relativistic) in a system
of particles acting as the medium and to study its motion under
the influence of the medium. As such for this we explain the general
diagram technique explained by Pratap(1967). As seen the existence
of a time scale $t_\tau \sim 1/\omega_{pl}$ where $\omega_{pl}$ is the plasma frequency in the
system, allows one to select diagrams of order $e^2(e^2N_\tau/V)$ where
$N_\tau$ = number of particles constituting the medium and $V$ is the
volume (Balescu, 1963). This class of diagram is called the ring
diagrams. Also further simplification is done by using the thermo-
dynamic limit which is $N_\tau \to \infty, V \to \infty$ and $N_\tau = C_1 V$ a finite constant.
The phenomenon of Cerenkov radiation which has a wide application in various branches of physics has been studied extensively both theoretically and experimentally. The basic mechanism explained by many workers indicates that the phenomenon depends on the properties of the medium. But the effect of the thermal state of the medium on the phenomena which is a collective phenomenon still remains an open question. Also when we put a test particle (relativistic) in a medium which was initially homogeneous, one can expect density fluctuations in the system. These density fluctuations will couple the transverse electromagnetic interaction with the longitudinal and scalar interactions in the system (using the terminology of Heitler 1954). Since the phenomenon of Cerenkov radiation is due to the polarization of the medium, we consider the coupling of transverse and longitudinal interaction (Patro, 1982). In chapter III we consider the effect of the temperature of the medium and the coupling of the transverse and longitudinal interactions on the phenomenon of Cerenkov radiation, which is due to the pure transverse interaction (Klimontovich & Silin, 1961). The calculations are done in Born approximation. Finally it is seen that the effect of the medium temperature on the phenomenon is negligible in the limit when the medium particles are non-relativistic. This confirms Cerenkov's observation that the phenomenon is not a form of luminiscence. On the other hand, the effect of the transverse and longitudinal coupling has a significant effect on the coherence and also some longitudinal energy is converted into the transverse modes thereby increasing the outcoming radiation.
The phenomenon of Cerenkov radiation is explained in chapters II and III in an ordinary dielectric medium. For an ordinary plasma the situation is not the same. The coherence condition needed for the phenomenon of Cerenkov radiation is not satisfied. But when a magnetic field is applied a dramatic change takes place. For certain frequency regions the coherence condition is satisfied and a passband is available for the Cerenkov radiation. But it is well-known that the relativistic particle which is the source for the above mentioned phenomenon emits synchrotron radiation due to its gyration around the magnetic lines of force. It is generally considered while explaining the radiation from astronomical sources that in presence of both the magnetic field and the ambient medium, the radiation emitted by a charged particle is due to both synchrotron and Cerenkov processes. This is normally computed independently and then simply superposed to describe the overall radiation. This however has been questioned by Schwinger and his co-workers (1976). In chapter IV a new formulation of this problem of emission of radiation from a relativistic charged particle under the joint action of an external magnetic field and the ambient plasma (medium) (Patro, 1982, Patro and Pratap, 1983) is given in the framework of the above formulation. Considering the motion of the particle in the plane normal to the direction of the magnetic field, the power is calculated in the cold plasma approximation. It is seen that the power emitted is a function of time which is strictly due to the non-Markoffian nature of the problem. The result of Schwinger and his co-workers is recovered from our time dependent solution when the asymptotic limit in time is taken. For the sake of completeness some of the results of Schwinger and
his co-workers are shown in order to show the synergic nature of the radiation.

In chapter V a general theory of radiative transfer in a non-equilibrium medium is discussed. Here turbulence is defined as due to the presence of multiple time scales and hence multiple length scales in the system. The feature of turbulence appears through the transfer of energy from one mode of interaction to another. The general theory developed, is applied to study the propagation of electromagnetic radiations through such a medium. It is seen that a quasiparticle representation is possible and one obtains the induced emission with an effective frequency. The important feature of this theory is that one can define an effective Einstein's coefficient containing the signature of medium concerned, which is time dependent.

In chapter VI we have summarised our work and have pointed out the possible extensions of this work.