CHAPTER 2

Literature Review

2.1 Introduction

Reducing noise has always been one of the standard problems of the image analysis and processing community. Often though, at the same time as reducing the noise in a signal, it is important to preserve the edges. Edges are of critical importance to the visual appearance of images. So, it is desirable to preserve important features, such as edges, corners and other sharp structures, during the denoising process. This chapter presents a review of some significant work in the area of image denoising.

Ideally denoising is all about filtering noise from the degraded image while keeping other details unchanged. Indeed, filtering is the most fundamental operation of image processing and computer vision, and it is used widely in various applications, including image smoothing and sharpening, noise removal, edge detection etc. In the broadest sense of the term “filtering”, the value of the filtered image at a given location is a function of the values of the input image in a small neighborhood of the same location [Gonzalez and Woods (2008), Jain and Tyagi (2013)].

The simplest form of filtering is an explicit Linear Translation Invariant (LTI) filtering, which can be implemented using a local neighborhood. For example, box filtering, also known as mean filtering or averaging [Gonzalez and Woods (2008)], is implemented by a local averaging operation where the value of each pixel is replaced by the average of all its neighbors. Although box filter gives the quickest filtering output, but its smoothing effect is often not sufficient.

Other LTI filters that do not involve the computation of the mean of a neighborhood are also often used for smoothing. Most common of these are the Gaussian smoothing filter [Shapiro and
Stockman (2001), Gonzalez and Woods (2008)] and Weiner filter [Jain (1989), Benesty et al. (2010)]. The weights for Gaussian filter are chosen according to the shape of a Gaussian function. Gaussian filter has been proved to be a good choice for removing noise drawn from a normal distribution and the multi-scale space representation of an image can be obtained easily by Gaussian smoothing with increasing variance. Wiener filters are a class of optimum linear filters which involve linear estimation of a desired signal sequence from another related sequence.

Although LTI filtering is the simplest form of filtering and is used widely in early vision processing, it also has some drawbacks. LTI filtering is the quickest approach for smoothing the noise but some important structures are also often get blurred along with noise. To reduce these undesirable effects of linear filtering, a variety of edge-preserving filtering techniques have been proposed during the last few years.

2.2 Evolution of Edge-Preserving Image Denoising Research

Though, traditional LTI filtering techniques like mean filtering [Gonzalez and Woods (2008)], Gaussian filtering [Shapiro and Stockman (2001), Gonzalez and Woods (2008)], Wiener filtering [Jain (1989), Benesty et al. (2010)] exist for a long time for their simplicity and are able to achieve significant noise removal when the variance of noise is low, they tend to blur sharp edges, destroy lines and other fine image details. To resolve the above issues, a variety of nonlinear filters such as median [Gonzalez and Woods (2008), Pitas and Venetsanopoulos (1990)], weighted median [Yang et al. (1995)], rank conditioned rank selection [Hardie and Barner (1994)], and relaxed median [Hamza et al. (1999)] have been developed.

Apart from above nonlinear median type filters, other edge preserving denoising methods have been introduced to resolve the issues arised with linear spatial filtering during past few years.
These methods are non-linear and can preserve the image details and local geometries while removing the undesired noise, because they consider local structures and statistics during the filtering process.

Most of popular denoising techniques in this class have been developed based on Partial Differential Equations (PDEs) and variation models. The nonlinear Anisotropic Diffusion (AD) [Perona and Malik (1990), Black et al. (1998), Weickert et al. (1998)] methods were suggested to overcome blurring issues of the Gaussian filter [Shapiro and Stockman (2001), Gonzalez and Woods (2008)] by smoothing the image only in the direction orthogonal to the gradient. The regularization methods based on Total Variation (TV) [Rudin et al. (1992), Chambolle (2004)] were given to smooth the homogenous regions of the image but not its edges. Similarly, another approach based on low level processing to provide better edge preserving denoising known as the Smallest Univalue Segment Assimilating Nucleus (SUSAN) [Smith and Brady (1997)] filter has been proposed that can average all pixels in the local neighborhood which are from the same spatial region as the central pixel.

Based on the work [Aurich and Weule (1995), Smith and Brady (1997)], Tomasi and Manduchi (1998) proposed a simple, non-iterative, local filtering method known as the bilateral filter which was further modified and improved in [Elad (2002)]. Although bilateral filter was non-iterative and had simple formulation, but its direct implementation was known to be slow. The brute force implementation has the time complexity $O(Nr^2)$, which is prohibitively high when the kernel radius $r$ is large. To speed up the evaluation of the bilateral filter, several techniques [Durand and Dorsey (2002), Paris and Durand (2006), Porikli (2008), Yang et al. (2009), Chaudhury et al. (2011), Chaudhury (2013)] have been proposed, fast implementation of these techniques is still a challenging problem. Another issue concerning the bilateral filter
[Durand and Dorsey (2002), Bae et al. (2006), Farbman et al. (2008)] is that it may have the gradient reversal artifacts in detail decomposition and High Dynamic Range (HDR) compression.

To resolve the issues raised with bilateral filter, He et al. (2010) proposed a new filter, called guided filter that can perform effective edge-preserving denoising by considering the content of a guidance image. One advantage of the guided filter over the bilateral filter was that it automatically had an $O(N)$ time exact algorithm. $O(N)$ time implies that, unlike the bilateral filter, the time complexity was independent of the window radius $r$, so they were free to use arbitrary kernel sizes in the applications. Unlike the bilateral filter, the guided filter avoided the gradient reversal artifacts that could appear in detail enhancement and HDR compression.

The past few years have witnessed substantial developments in the area of image denoising. Buades et al. (2005a) presented an excellent survey on image denoising algorithms and also proposed an algorithm (Non-Local Means) for improvements in denoising results. The main focus of the work was, first, to define a general mathematical and experimental methodology to compare and classify the classical image denoising algorithms, second, to propose an algorithm (Non Local Means) addressing the preservation of structure in a digital image. It soon became clear that self-similarity and nonlocality are the characteristics of natural images with by far the biggest potential for image denoising. The Non-local Means (NLM) [Buades et al. (2005b)] filter is the first one which makes use of the self-similarity in the whole image. With the NLM filter, a denoised patch can be obtained by weighted averaging all other patches in the same image. It is an extension of the bilateral filter [Tomasi and Manduchi (1998)] in the sense of replacing the Euclidean distance between two pixels with the weighted Euclidean distance between two patches.
One of the most powerful and effective extensions of the non local filtering approach is the BM3D image denoising algorithm [Dabov et al. (2006)]. Maggioni et al. (2013) recently presented an extension of the BM3D algorithm, namely BM4D, to volumetric data denoising.

Singular Value Decomposition (SVD) is also used in image noise filtering. Numerous approaches based on SVD filtering have been proposed in [Natarajan (1995), Konstantinides et al. (1997), Wongsawat et al. (2005), Orchard et al. (2008), Cai et al. (2010), Gu et al. (2014)].

Apart from edge-preserving filters mentioned so far, wavelets also gave superior performance in edge-preserving denoising due to properties such as sparsity and multiresolution structure. With wavelet transform gaining popularity in the last few decades, numerous algorithms for image denoising in wavelet domain have been developed.

Wavelet thresholding [Donoho and Johnstone (1994), Donoho and Johnstone (1995), Donoho (1995), Chang et al. (2000a), Chang et al. (2000b), Sendur and Selesnick (2002b), Silva et al. (2012), Jain and Tyagi (2014a)] is the key concept for wavelet domain denoising. The wavelet based methods exploit the decomposition of the data into a wavelet basis and modify the wavelet coefficients to denoise the data. The coefficients obtained through wavelet transform are modified according to thresholding rule applied. This process of obtaining a denoised image from given noisy image using wavelet thresholding is termed as wavelet shrinkage. Fodor and Kamath (2001) provided an empirical study on denoising using wavelet shrinkage.

Inspired by the SURE-LET method [Blu and Luisier (2007)] and the guided filter [He et al. (2010)], Qiu et al. (2013) presented a novel edge-preserving smoothing filter, called LLSURE filter which is based on a local linear model and the principle of Stein’s Unbiased Risk Estimate (SURE). The LLSURE filter has the edge-preserving smoothing property that can filter out noise.
while preserving edges and fine-scale details. In addition, it is quite simple and has an exact linear-time algorithm which can be applied to various image processing tasks.

2.3 Review of Edge Preserving Denoising Methods

The literature on edge-preserving denoising methods is vast and it is very tough to give attention on each method. Therefore, this section is devoted to few most promising edge-preserving denoising methods.

2.3.1 Total Variation (TV) based Filtering

Rudin et al. (1992) presented a nonlinear, constrained optimization type of numerical algorithm for image denoising. The total variation (TV) of the image is minimized subject to constraints involving the statistics of the noise. The constraints were imposed using Lagrange multipliers. The proposed algorithm was simple and relatively fast. Though, it provided good edge preservation but may oversmooth the image.

2.3.2 Bilateral Filtering

Tomasi and Manduchi (1998) proposed non-iterative, local and simple edge-preserving filtering method termed as “bilateral filtering”. As the name suggests, this filtering was the combination of two types of filtering, namely, domain filtering and range filtering.

Domain filtering enforced geometric closeness between two pixels based on their spatial vicinity by weighing pixel values with coefficients that fall off with distance. Similarly, range filtering which enforced photometric similarity between two pixels by averaging image values with weights that decay with dissimilarity. Range filters are non-linear because their weights depend on image intensity (for gray scale images) or color (for color images). Bilateral filtering has the following merits and demerits:
Merits:

- In contrast with filters that operate on the three bands of a color image separately, a bilateral filter can smooth colors and preserve edges in a way that is tuned to human perception.
- In contrast with standard filtering, bilateral filtering produces no phantom colors along edges in color images and reduces phantom colors where they appear in the original image.

Demerits:

- It has been noticed [Durand and Dorsey (2002), Bae et al. (2006), Farbman et al. (2008)] that the bilateral filter may have the gradient reversal artifacts in detail decomposition and HDR compression.
- Another issue concerning the bilateral filter is its efficiency. The brute force implementation is of $O(Nr^2)$ time which is excessively high when the kernel radius $r$ is large.

2.3.3 Guided Filtering

He et al. (2010) proposed a novel type of explicit edge preserving image filter i.e. guided filter. Derived from a local linear model, the guided filter produces the filtering result by considering the content of a guidance image. The guidance image can be the input image itself or another different image. The filtering result is locally a linear transform of the guidance image. It is also related to the matting Laplacian matrix [Levin et al. (2008)], so it is a more generic concept which can be applied in other applications alongwith smoothing. Guided filter has the following merits and demerits:

Merits:

- This filter has the edge-preserving smoothing property like the bilateral filter, but does not suffer from the gradient reversal artifacts.
• The guided filter has an $O(N)$ time (in the number of pixels $N$) exact algorithm for both gray scale and color images.

• The guided filter performs very well in terms of both quality and efficiency in a wide range of applications, such as noise reduction, detail smoothing/enhancement, HDR compression, image matting/feathering, haze removal and joint upsampling.

**Demerits:**

• As a locally based operator, the guided filter is not directly applicable for sparse inputs like strokes.

• It also shares a common limitation of other explicit filter - it may have halos near some edges. In fact, it is ambiguous for a low level and local operator to determine which edge should be smoothed and which should be preserved. Unsuitably smoothing an edge will result in halos near it.

2.3.4 **LLSURE Filtering**

Inspired by the SURE-LET method [Blu and Luisier (2007)] and the guided filter [He et al. (2010)], Qiu et al. (2013) recently proposed a novel approach for performing high quality edge-preserving image filtering. Based on a local linear model and using the principle of Stein’s Unbiased Risk Estimate (SURE) as an estimator for the mean squared error from the noisy image only, the authors derived a simple explicit image filter called LLSURE. In this approach, the filtered output in a local window is considered as a very simple affine transform of input signal in the same window and the optimal transform coefficients are determined by minimizing the SURE. This approach has following merits and demerits:
Merits:

- First, since using a local linear model, it is simple and can be computed efficiently with arbitrary kernel sizes in constant time.
- Second, it has the nice property of edge-preserving smoothing which can suppress noise while preserving important features of the original image.
- Third, it is completely automatic which doesn’t need to tune manually the parameters during the denoising process.
- Finally, it is flexible and is applicable in various image processing tasks including noise reduction, detail smoothing/enhancement, HDR compression and flash/no flash denoising.

Demerits:

- It may leave some noise in the vicinity of edges in denoising process.
- Application area is restricted to the above mentioned applications and presently not applicable in applications like image matting, haze removal and image colorization.

2.3.5 Transform Domain Filtering

Apart from the spatial domain denoising approaches discussed above, a wide research has also been devoted to transform domain methods. Though large number of variations have been documented in this category, such as Singular Value Decomposition (SVD) [Golub and Van Loan (1983)], Discrete Cosine Transform (DCT), wavelets [Mallat (2008)], wedgelets [Donoho (1999)], curvelets [Candes and Donoho (2002), Candes and Donoho (2003)], bandlets [Mallat and Pennec (2005), Pennec and Mallat (2005)], contourlets [Do and Vetterli (2003)] and steerable wavelets [Freeman and Adelson (1991), Mallat and Peyre (2008)], wavelets based methods are still dominant. Numerous transform domain edge-preserving denoising approaches have been reported in literature. Some well known approaches are discussed in this section.
The SVD based methods, in order to discriminate against noise, decompose the column space of given noisy matrix into the signal and noise subspaces. The ultimate goal is to truncate smaller singular values that usually are the part of noise subspace, while preserving large singular values that are part of the signal subspace. The crucial point is to determine a threshold which can separate the signal subspace from the noise subspace. Natarajan (1995) presented a procedure to determine the threshold that has to be used for truncation of the smaller singular values. Another important thing is the thresholding function used to apply a given threshold. Konstantinides et al. (1997) used a hard thresholding function to obtain the truncated singular values. However, this approach exhibits some discontinuities and may be unstable or more sensitive to small changes in the data.

To improve above technique, Cai et al. (2010) suggested to use soft thresholding function to determine the truncated singular values. This approach has also some issues to be resolved such as soft thresholding operator reduces each singular value with same amount which in turn will induce the deviation when the filtered results are reconstructed by the inverse SVD, and the technique does not consider any prior knowledge related to the relevance of the singular values.

To resolve the issues raised in above truncation procedure, Gu et al. (2014) recently proposed a new truncation procedure which generalizes the above procedure and greatly improves its flexibility. Instead of using a fixed threshold for the truncation of singular values, they associated some appropriate weight to each singular value by using the prior knowledge about singular values. Then, these weights are applied as thresholds for truncating the noisy singular values using soft thresholding function.

Other than the above discussed SVD domain techniques, some wavelet domain methods have also gained much attention of researchers. Bayesian procedures [Chipman et al. (1997),

Chang et al. (2000a) proposed an adaptive wavelet thresholding for denoising and compression. An adaptive and data-driven threshold, called *BayesShrink*, based on the Generalized Gaussian Distribution (GGD) modeling of subband coefficients, is proposed for image denoising via wavelet soft thresholding.

Portilla et al. (2003) proposed a well known wavelet domain denoising method called the Bayes Least Squares-Gaussian Scale Mixture (BLS-GSM) which performs denoising by modeling the wavelet coefficients of images as mixtures of Gaussians.

Choi and Baraniuk (2004) proposed a new image denoising algorithm that exploits an image’s representation in multiple wavelet domain. Projecting an image onto a Besov ball of suitable radius corresponds to a kind of wavelet shrinkage for image denoising. By defining Besov balls in multiple wavelet domains and projecting onto their intersection using the Projection Onto Convex Sets (POCS) algorithm, authors obtained an estimate that effectively combines estimates from multiple wavelet domains.

Blu and Luisier (2007) proposed a very appealing denoising approach to image denoising based on the principle of SURE. In order for this approach to be viable, the authors added another principle, that the denoising process can be expressed as a linear combination of elementary denoising processes—Linear Expansion of Thresholds (LET). The new approach was termed as the SURE-LET. This approach was initially developed for monochannel denoising but later been extended to color images [Luisier and Blu (2008)], video and mixed Poisson-Gaussian noise condition [Luisier et al. (2011)].
Choux et al. (2008) also focused on multichannel denoising. Authors were interested in multispectral image denoising in the wavelet domain and adopted a multivariate statistical approach in order to exploit the correlations existing between the different spectral components. The key contribution is the application of Stein’s principle to develop a new estimator for arbitrary multichannel images embedded in additive Gaussian noise.

Silva et al. (2012) presented a novel edge-preserving image denoising method based on wavelet transforms. The decomposition of an image is carried out by dividing it into a set of blocks and then transforming the data into the wavelet domain. An adaptive thresholding scheme based on edge strength is employed to effectively reduce noise while preserving important structures and fine details of the original image.

Ho and Hwang (2013) recently proposed a method that makes use of a hidden Bayesian network, constructed from wavelet coefficients, to model the prior probability of the original image. Authors have used the Belief Propagation (BP) algorithm, which estimates a coefficient based on all the coefficients of an image, as the Maximum-a-Posteriori (MAP) estimator to derive the denoised wavelet coefficients. They presented a constructive data-adaptive procedure that derives a hidden graph structure from the wavelet coefficients. The graph is then used to model the prior probability of the original image for denoising purpose.

Usually, the multiscale decomposition of an image by using wavelet transform is not adaptive i.e. local structures of image are not taken into account during decomposition. This issue is resolved by tetrolet transform (an adaptive Haar type wavelet transform) [Krommweh (2010)]. Tetrolets are Haar type wavelets whose supports are the shapes called tetrominoes. The tetrominoes are some geometric shapes made by connecting four equal sized squares. This tetrolet transform was not found suitable for the purpose of image denoising due to its non-
redundant nature, while redundant information is helpful for image denoising. Thus, it is very much needed to exploit redundancy by a denoising method based on tetrolet transform. Some denoising techniques have been reported in [Singh (2010), Li et al. (2010), Zhang et al.(2013), Dai et al.(2013)] based on tetrolet transform.

Most simple non-linear thresholding rules assume that the wavelet coefficients are independent. However, significant statistical dependencies can be observed among the wavelet coefficients of natural images. Mihcak et al. (1999a) proposed a powerful denoising scheme with low-complexity by exploiting the dependency of the local wavelet coefficients within each scale. The authors obtained reasonably good results by performing a Locally Adaptive Window based Maximum Likelihood (LAWML) estimate. Sendur and Selesnick (2002a) presented four jointly non-Gaussian models that exploit interscale dependencies between the coefficients. Further, in [Sendur and Selesnick (2002b)], a locally adaptive denoising algorithm has been presented which improved the performance by estimating model parameters in a local neighborhood.

Although the above discussed locally adaptive approaches are efficient in computation but all of them focus only on a fixed window size for the local neighborhood. Basically, it is a common thinking that the large size regions are more reliable in estimating the thresholding parameters. However, the locally i.i.d (independently and identically distributed) assumption becomes inaccurate as the size of the neighborhood grows [Mihcak et al. (1999b)]. This demands the presence of a proper neighborhood region for estimating the thresholding parameters. A number of methods [Eom and Kim (2004), Chang et al. (2000b), Michak et al. (1999b), Park et al. (1999), Boykov et al. (1998), Balan and Mather (2001)] based on variable sized local neighborhoods have been developed.
Although, various image denoising approaches based on either SVD or wavelet transform have been independently developed; however, their connection has rarely been addressed. In [Hou (2003)], a relation is investigated between wavelet transform and SVD. This approach: divides the detail subbands of wavelet transform domain into blocks; applies SVD based procedure to each block for checking the presence of edge structure in that block; associates different thresholds with edge present and edge absent blocks; applies these thresholds in SVD filtering using a soft shrinkage rule. Some problems are observed in this technique, such as the computation of thresholds does not consider the local statistics of the blocks; each threshold is applied to multiple blocks (matrices) while earlier, we have highlighted the problems with the case where even single threshold is used for filtering a single matrix.

2.4 Review of Image Denoising by Wavelet Shrinkage

In Section 1.4, we have briefly introduced the main stages of wavelet domain image denoising. In this section, these stages are elaborated in detail. Suppose that a given image \( f = \{f(x,y), x = 1, ..., M, y = 1, ..., N\} \) has been corrupted by additive noise as

\[
g = f + n,
\]

where \( n \) represents the noise and \( g \) the observed image. Most of the existing methods have an assumption that corruption in the original image is due to Additive White Gaussian Noise (AWGN). The obvious reason behind this assumption is that AWGN possesses statistical independence and identical distribution (i.i.d).

The decomposition of the image \( g \) into wavelet coefficients through a Discrete Wavelet Transform (DWT) \( W \) can be expressed as

\[
G = W(g)
\]
We can obtain low-pass (approximation) and high-pass (detail) coefficients at each scale by using Eqs. (2.3) and (2.4), respectively.

\[
W_\phi(j_0, r, s) = \frac{1}{\sqrt{MN}} \sum_0^{r-1} \sum_0^{s-1} g(x, y) \varphi_{j_0, r, s}(x, y)
\]  
\[
W_\psi^i(j, r, s) = \frac{1}{\sqrt{MN}} \sum_0^{r-1} \sum_0^{s-1} g(x, y) \psi^i_{j, r, s}(x, y)
\]

where \(W_\phi\) and \(W_\psi\) represent the approximation and detail (wavelet) coefficients, \(\varphi\) and \(\psi\) are the basis functions, index \(i\) identifies the horizontal, vertical and diagonal details, \(j\) represents the scale, \(j_0\) is an arbitrary starting scale and \(r, s\) are the position-related parameters [Gonzalez and Woods (2008)].

The successive applications of Eqs. (2.3) and (2.4) produce different frequency subbands labeled as \(\text{LL}_j\), \(\text{LH}_j\), \(\text{HL}_j\) and \(\text{HH}_j\), \(j = 1,2,\ldots,J\), where the subscript indicates the \(j\)-th resolution level of wavelet transform and \(J\) is the largest scale in the decomposition. These subbands restrain different details of the image. The lowest frequency \(\text{LL}_j\) subband corresponds to a coarsest approximation of the image signal while the \(\text{LH}_j\), \(\text{HL}_j\) and \(\text{HH}_j\) subbands correspond to horizontal, vertical and diagonal details of the image signal, respectively. The highest frequency \(\text{HH}_1\) subband is the noisiest subband among all the subbands obtained at each resolution level. The \(\text{LL}_{j-1}\) subband can be further recursively decomposed to form the \(\text{LH}_j\), \(\text{HL}_j\) and \(\text{HH}_j\) subbands. Fig. 2.1 illustrates the subband regions of a critically sampled wavelet transform.

Once the threshold \(\lambda\) is estimated, the wavelet coefficients are modified according to a shrinkage function \(T\), such that

\[
\hat{F} = T(G, \lambda)
\]

The last step in the wavelet shrinkage process is to transform the thresholded coefficients back to the original domain, expressed as
\[ \hat{f} = W^{-1}(\hat{F}) \]  

where \( W^{-1} \) denotes the Inverse Discrete Wavelet Transform (IDWT) and \( \hat{f} \) is the denoised image.

![Subband regions of critically sampled wavelet transform](image)

**Figure 2.1:** Subband regions of critically sampled wavelet transform

Given \( W_\varphi \) and \( W_\psi \) of Eqs. (2.3) and (2.4), the pixel value for the denoised image at the location \((x, y)\) can be obtained as follows:

\[
\hat{f}(x, y) = \frac{1}{\sqrt{MN}} \sum_{r} \sum_{s} W_\varphi(j_0, r, s) \varphi_{j_0, r, s}(x, y) + \frac{1}{\sqrt{MN}} \sum_{i=H,V,D} \sum_{j=j_0} \sum_{r} \sum_{s} W_\psi^i(j, r, s) \psi^i_{j, r, s}(x, y)
\]

(2.7)

### 2.4.1 Threshold estimation

Three threshold estimation criteria known as VisuShrink (non-adaptive), SureShrink (adaptive) and BayesShrink (adaptive) are described as follows:

VisuShrink [Donoho and Johnstone (1994)] is a non-adaptive thresholding scheme as it applies a single universal threshold to each subband, defined as

\[
\lambda_{VISU} = \hat{\sigma}_{\text{noise}} \sqrt{2 \log L}
\]

(2.8)

where \( \hat{\sigma}_{\text{noise}} \) is the estimated noise deviation and \( L = M \times N \) is the size of the image. As VisuShrink is a non-adaptive thresholding scheme, the threshold remains fixed for all levels of
decomposition. Although the resulting estimate is very smooth and has a pleasant visual appearance, it is known that VisuShrink tends to oversmooth the signal [Yuan and Buckles (2004)].

SureShrink [Donoho and Johnstone (1995)] is an adaptive thresholding scheme because it applies a threshold adaptive to a subband. A separate threshold is computed for each subband based on Stein’s unbiased risk estimate (SURE) as

\[
\lambda_{SURE} = \arg \min_{\lambda \geq 0} \text{SURE}(G_S, \lambda)
\]

(2.9)

where \( \arg \min \text{SURE}(G_S, \lambda) \) indicates the selection of a value for \( \lambda \) which minimizes the risk

\[
\text{SURE}(G_S, \lambda) = \hat{\sigma}_{noise}^2 - \frac{2 \hat{\sigma}_{noise}^2}{N_S} \#\{k: |y_k| < \lambda\} + \frac{1}{N_S} \sum \min(|y_k|, \lambda)^2,
\]

(2.10)

where \( G_S \) are the detailed coefficients from subband \( S \), \( \hat{\sigma}_{noise}^2 \) is the estimated noise variance, \( N_S \) is the number of coefficients \( y_k \) in \( G_S \), \( \lambda \) is the threshold, \( \#\{k: |y_k| < \lambda\} \) indicates the number of coefficients whose value is less than \( \lambda \). Donoho and Johnstone (1995) pointed out that if the coefficients are very sparse, universal threshold is applied, if not, SureShrink is applied.

BayesShrink [Chang et al. (2000a)] is also an adaptive thresholding scheme because it also applies subband adaptive threshold. It uses a Bayesian mathematical framework and assumes generalized Gaussian distribution (GGD) for the wavelet coefficients in each detail subband to find the threshold that minimizes the Bayesian risk [Chipman et al. (1997), Chang et al. (2000a)], expressed as

\[
\lambda_{Bayes} = \frac{\hat{\sigma}_{noise}^2}{\hat{\sigma}_{signal}} \text{, if } \hat{\sigma}_{signal} \text{ is non-zero.}
\]

(2.11)

Otherwise, it is set to some predetermined maximum value.

\[
\hat{\sigma}_{signal} = \sqrt{(\hat{\sigma}_{y}^2 - \hat{\sigma}_{noise}^2)_+}
\]

(2.12)
where $\hat{\sigma}_y^2 = \frac{1}{N_S} \sum_{k=1}^{N_S} y_k^2$ and $N_S$ is the number of wavelet coefficients $y_k$ on the subband under consideration and the function $(a)_+$ is defined as

$$(a)_+ = \begin{cases} 0, & \text{if } a < 0 \\ a, & \text{otherwise} \end{cases}$$

(2.13)

It can be observed from Eqs. (2.8) - (2.12) that most of the thresholding schemes require an estimate of noise variance to determine the desired threshold value.

As mentioned earlier, HH$_1$ (subband having highest frequency coefficients) is the noisiest subband among all the subbands at each level of decomposition. Using this fact, a robust estimator [Donoho and Johnstone (1994)] of noise variance which uses the median absolute value of the highest frequency wavelet coefficients, is defined as

$$\hat{\sigma}_{\text{noise}}^2 = \frac{\text{median}(|y_k|)}{0.6745}, y_k \in \text{HH}_1$$

(2.14)

It is assumed that the noise follows a Gaussian distribution with zero mean and variance $\sigma^2$.

### 2.4.2 Shrinkage rule

Once the threshold $\lambda$ is estimated using any of the above described threshold estimation criteria, it is applied to the wavelet coefficients $G$ according to a shrinkage function $T$, such that

$$\hat{F} = T(G, \lambda)$$

(2.15)

The shrinkage (or thresholding) rule defines the applicability of a threshold. Basically, each wavelet coefficient $y = y_k$ (say) is compared against a threshold $\lambda$ (say). If the coefficient is less than or equal to threshold, it becomes zero; otherwise, it is kept or modified. The motivation is that large coefficients are due to important signal features, while small coefficients can be thresholded without affecting the significant features of the image.

Manipulation of the coefficients is done by applying an estimated threshold according to a shrinkage rule. The shrinkage rule defines how the threshold must be applied. Some well-known
shrinkage rules are hard and soft thresholding, hyperbola function, firm thresholding, garrote thresholding, Smoothly Clipped Absolute Deviation (SCAD) thresholding. After application of a shrinkage rule, coefficients that are supposed to be affected by noise are replaced by zero or an adequate value.

In the shrinkage rule known as hard thresholding [Donoho (1995)], the coefficients that are less than or equal to the threshold are set to zero, while the remaining coefficients are left unchanged.

\[
T_{\text{hard}}(y, \lambda) = \begin{cases} 
  y, & \text{if } |y| > \lambda \\
  0, & \text{if } |y| \leq \lambda 
\end{cases}
\]  

(2.16)

A drawback of the hard thresholding is its abrupt discontinuity, which may cause artifacts in the reconstructed image. In soft thresholding [Donoho (1995)], the wavelet coefficients above the threshold are reduced by an amount equal to the value of the threshold

\[
T_{\text{soft}}(y, \lambda) = \begin{cases} 
  \text{sgn}(y)(|y| - \lambda)_+, & \text{if } |y| > \lambda \\
  0, & \text{if } |y| \leq \lambda 
\end{cases}
\]  

(2.17)

where \(\text{sgn}(x)\) function returns the sign of the parameter \(x\) and \((a)_+\) is defined in Eq. (2.13).

The main issue with the soft thresholding is its tendency to oversmooth the reconstructed image. Other thresholding schemes have been proposed, which are a compromise between hard and soft thresholding rules. The hyperbola function [Vidakovic (1998)] is continuous and attenuates large coefficients less than soft thresholding.

\[
T_{\text{hyper}}(y, \lambda) = \begin{cases} 
  \text{sgn}(y)\sqrt{y^2 - \lambda^2}, & \text{if } |y| > \lambda \\
  0, & \text{if } |y| \leq \lambda 
\end{cases}
\]  

(2.18)

The firm thresholding [Gao and Bruce (1997)] avoids the drawbacks of both hard and soft thresholding rules and provides less sensitivity to small perturbations in the data and smaller overall mean-squared error.
\[
T_{\text{firm}}(y, \lambda) = \begin{cases} 
  y, & \text{if } |y| > \lambda_2 \\
  \text{sgn}(y) \frac{\lambda_2(|y| - \lambda_1)}{\lambda_2 - \lambda_1}, & \text{if } \lambda_1 < |y| \leq \lambda_2 \\
  0, & \text{if } |y| \leq \lambda_1
\end{cases}
\] (2.19)

A drawback of the firm thresholding is that it requires two threshold values, \(\lambda_1\) and \(\lambda_2\), increasing the computational complexity of the estimation procedure. To overcome this drawback, the nonnegative garrote thresholding [Gao (1998)] was proposed.

\[
T_{\text{garrote}}(y, \lambda) = \begin{cases} 
  y - \frac{\lambda_2}{y}, & \text{if } |y| > \lambda \\
  0, & \text{if } |y| \leq \lambda
\end{cases}
\] (2.20)

The garrote thresholding offers advantages over hard and soft thresholding and is comparable to the firm thresholding, while requiring only one threshold value.

Antoniadis and Fan (2001) suggested the SCAD thresholding rule, defined as

\[
T_{\text{SCAD}}(y, \lambda) = \begin{cases} 
  \text{sgn}(y)(|y| - \lambda)_+, & \text{if } |y| \leq 2\lambda \\
  \frac{(a-1)y - \text{sgn}(y)\alpha\lambda}{\alpha-2}, & \text{if } 2\lambda < |y| \leq \alpha\lambda \\
  y, & \text{if } |y| > \alpha\lambda
\end{cases}
\] (2.21)

where \((a)_+\) is as given in Eq.(2.13) and the authors recommended the use of \(a = 3.7\).

**2.5 Formulation of Problems**

In this chapter, we have reviewed some state-of-the-art techniques for edge-preserving image denoising that have been reported in the literature. We have discussed various problems faced and their possible solutions as suggested by various researchers. From the study, it has been observed that there are many problems related to existing image denoising methods which require attention. Some of them are:

1. Usually, wavelet transforms are not adaptive i.e. local structures of image are not considered during decomposition.
2. Problem to exploit redundancy by a denoising method based on tetrolet transform.
3. Conventional thresholding schemes often used a global (universal) threshold to filter small wavelet coefficients. However, this procedure can also remove high frequency components, such as edges.

4. Noise variance which is often estimated from the diagonal detail coefficients at finest scale, is usually kept fixed through all resolution scales. However, the noise strength decreases with the raise in resolution scale.

5. In locally adaptive thresholding schemes, the role of each individual pixel (coefficient) is different when it involves in calculation of thresholds for each neighborhood around every member coefficient in its neighborhood. So the desired thresholded value this coefficient should be estimated by considering the cumulative effect of all the neighborhoods of the each member coefficient in its neighborhood.

6. The computation of thresholds used in block SVD filtering in wavelet domain does not consider the local statistics of a block. Apart from this, the undesirable blocking artifacts may also arise between boundaries of neighbor blocks.

7. The thresholds used in block SVD filtering in wavelet domain have been applied to multiple blocks (matrices), while some problems have been pointed out with the case where even single threshold is used for filtering a single matrix, such as the truncating operator reduces each singular value with same amount which in turn will induce the deviation when the filtered results are reconstructed by the inverse SVD and the prior knowledge related to the relevance of the singular values has not considered in SVD filtering.

8. In locally adaptive thresholding schemes, problems are identified with fixed size local windows that are used to estimate the thresholding parameters.

2.6 Organization of the Thesis

This thesis is organized as follows:
Chapter 1: This chapter provides the introduction of image noises, their sources and types. The basic model of an image denoising process is illustrated. A general classification of conventional image denoising techniques is provided with a brief explanation of wavelet-based image denoising.

Chapter 2: This chapter provides the detailed review of the state-of-the-art spatial and wavelet domain edge-preserving image denoising methods. Also a detailed review of the image denoising using wavelet shrinkage process is presented.

Chapter 3: This chapter presents an adaptive edge-preserving image denoising method based on tetrolet transforms. The method performs a subband-dependent thresholding for noise suppression in tetrolet coefficients by using an epsilon-median filtering. Another improved variation of this thresholding scheme is also proposed in this chapter, which is coefficient-dependent in nature and uses a bivariate shrinkage function for noise reduction.

Chapter 4: This chapter presents the locally adaptive patch-based thresholding scheme to achieve edge-preserving image denoising.

Chapter 5: This chapter presents an adaptive edge-preserving image denoising method using block SVD.

Chapter 6: Presents an adaptive edge-preserving image denoising method using patch based weighted SVD filtering.

Chapter 7: Presents an arbitrarily shaped locally adaptive windows based edge-preserving image denoising method.

Chapter 8: Concludes thesis and presents the scope of the future work.