6.1 Introduction

In oil recovery technology it is common practice to inject water into oil field at certain spots in an attempt to drive oil to other spots for pumping. But when a fluid flowing through porous media, is displaced by another fluid of lesser viscosity then, instead of regular displacement of the whole front, protuberance takes place which shoots through the porous medium at relatively very high speed. (as in figure 6.1.1) This phenomenon is called fingering/instability. In porous media of large extent the oil displacement by water injection often assumes the form

Fig 6.1.1 Instability phenomenon

of an advancing front, a moving zone of limited extent across which the saturation of water changes rapidly compared to regions nearly constant saturation upstream and downstream. In this process the phenomenon of fingering has been long identified. The water, which is intended to push the oil forward, tends to penetrate the oil through simultaneously formed
Figure 6.1.2 Cross-sectional view of fingers in cylindrical piece of porous media

multi-branched channels (fingers) (as in figure 6.1.2). In this chapter, fingering with capillary mean pressure effect has been statistically discussed in homogeneous porous media.

Many researchers have discussed this phenomenon with different points of view. Saffman and Taylor [1] derived a classical result for the shape of fingers in absence of capillary. Scheidegger [2] considered the average cross-sectional area occupied by the fingers while shape and size of the individual fingers were neglected. Most of earlier researchers, such as Scheidegger and Johnson [3], Saffman and Taylor [1] have neglected the capillary pressure. Verma [4] included capillary pressure in the analysis of the fingers in heterogeneous porous media. Joshi and Mehta [5] Found analytical solution of fingering using group theoretic approach. Patel, Mehta and Patel [6] found the power series solution of fingering phenomenon arising in fluid flow through homogeneous porous media.
In the present chapter, specific problem of fingering in homogeneous porous media with capillary pressure from the statistical viewpoint has been discussed. In this analysis homogeneous porous matrix with impermeable cylindrical surface has been considered and it has been assumed that Darcy-like formulation of the flow equations is sufficient for analysis. Specific results of the dependence of relative permeability on phase saturation have been assumed from standard literature [7]. The gradient of capillary pressure is assumed to be directly proportional to the gradient of saturation [8]. From statistical behaviour of the figures, mean pressure is assumed to be constant.

The mathematical formulation of this phenomenon yields a non-linear partial differential equation. We have applied DQM for solving this non-linear partial differential equation with initial condition. This method is a simple and efficient numerical technique, which approximates the derivative with respect to a coordinate direction at a grid point by a weighted linear sum of all the functional values in that direction. The key to DQM is the determination of weighting coefficients for any order derivative discretization. The numerical solutions which represent the average cross section are occupied by fingers.
6.2 Statement of the problem

Considering that, there is a uniform injection of less viscous fluid (water-injected fluid) into more viscous fluid (oil-native fluid) in saturated homogeneous porous medium of length L such that injected fluid shoots through the native fluid and gives rise to protuberances. This furnishes a well developed fingers flow. Since the entire native fluid (oil) at the initial boundary (x=0, x being measured positive in the direction of displacement) is displaced through a small distance due to injection, therefore it is further assumed that merely complete saturation of oil exist at the initial boundary.

For mathematical formulation, it has been assumed that Darcy’s law is valid for the investigated flow system and assumed further that the macroscopic behaviour of fingers is governed by a statistically treatment. Hence, only the average behaviour of two fluids involved is taken into consideration [2]. The treatment of the motion with the introduction of the concept of fictitious relative permeability becomes formally identical to the Buckley-Leverett description of two immiscible fluids flow in porous medium. Thus the saturation of displacing fluid in a porous medium represents the average cross-sectional area occupied by the fingers.
As mentioned above in introduction, in endeavouring to describe
the fingering process from a macroscopic standpoint, one is concerned
with average behaviour of the two fluids involved. Thus, considering a
liner displacement parallel to the x-direction which progresses with time
t, for certain values of x and t there will be some fingers present in the
porous medium. The problem is then to set up a determining equation for
the (saturation) function \( s_w(x,t) \) which is the average cross-sectional area
occupied by the fingers formed by injected fluid at the distance x at time t.

### 6.3 Mathematical Formulation

Assuming the validity of Darcy’s law for flow of two immiscible
fluids, the filtration velocity of water \( v_w \) and oil \( v_o \) may be written as [9],

\[
v_w = -\left(\frac{k_w}{\delta_w} k\right) \frac{\partial P_w}{\partial x}
\]

\( (6.3.1) \)

\[
v_o = -\left(\frac{k_o}{\delta_o} k\right) \frac{\partial P_o}{\partial x}
\]

\( (6.3.2) \)

where \( k \) is the permeability of the homogeneous medium, \( k_w \) and \( k_o \) are
the relative permeability of wetting-displacing fluid (water) and non-
wetting-displaced fluid (oil). \( P_w \) and \( P_o \) are pressures; \( \delta_w \) and \( \delta_o \) are the
constant kinematic viscosities of wetting fluid and non-wetting fluid
respectively.

Assuming that the phase densities are constant, the continuity
equation of two incompressible and immiscible fluids (water and oil) are
given by,
where $P$ is the porosity of the medium and $s_w$ and $s_o$ are saturation of injected fluid (water) and native fluid (oil) respectively.

\[ s_w + s_o = 1 \]  

(6.3.5)

An analytical expression between the relative permeability and phase saturation suggested by [3] as,

\[ k_w = s_w \]  

(6.3.6)

\[ k_o = 1 - \alpha s_w; \alpha = 1.11 \]  

(6.3.7)

The capillary pressure ($P_c$), defined as the pressure discontinuity of the flowing phases across their common interface at any point of the porous media. The pressure $P_w$ and $P_o$ are assumed to be related to each other via the capillary pressure $P_c$.

\[ P_c = P_o - P_w \]  

(6.3.8)

Hence, the two pressure gradients are related to each other by the gradient of the capillary pressure, [10]. The capillary pressure gradient was first introduced by [10].

\[ \frac{\partial P_c}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x} \]  

(6.3.9)

substituting equation (6.3.9) into equation (6.3.1) yields,
\[ v_w = -\frac{k_w}{\delta_w} k \left( \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} \right) \]  

(6.3.10)

Babchin and Nasr [11] suggested that when both the phases are continuous then the capillary pressure gradient can be written as,

\[ \Delta P_c = (\gamma_{os} - \gamma_{ws})s_v \Delta s_o \]  

(6.3.11)

where, \( s_v \) is the specific surface area of homogeneous porous media, \( \gamma_{os} \) and \( \gamma_{ws} \) are native fluid (oil) - solid and displacing fluid (water)- solid specific energies respectively. They added variable part in the surface energy of the system should coincide with the variable part of the capillary pressure versus saturation. Due to this reason, there is no fundamental reason for the capillary pressure in homogeneous porous media not to be a linear function of saturation as long as both phases allow a continuous passage within each phase. Mehta [12] suggested that capillary pressure is proportional to saturation of displacing fluid and it is in opposite direction. He suggested the expression

\[ P_c = -\beta s_o \]  

(6.3.12)

where \( \beta \) is proportionality constant.

Combining equation (6.3.11) and (6.3.12) gradient of capillary pressure may be written as

\[ \Delta P_c = \Delta s_o \]  

(6.3.13)
Since the flow is in x direction only and using equation (6.3.9) the capillary pressure gradient may be written as,

\[
\frac{\partial P_c}{\partial x} = -\beta \frac{\partial s_w}{\partial x}, \quad \text{where } \beta = (\gamma_{os} - \gamma_{wo}) s_v \tag{6.3.14}
\]

The equation of motion for saturation can be obtained by substituting the values of \( v_w \) and \( v_o \) from (6.3.1) and (6.3.2) into the equation (6.3.3) and (6.3.4) respectively and using equation (6.3.10) we get,

\[
P \frac{\partial s_w}{\partial t} = \frac{\partial}{\partial x} \left[ \frac{k_w}{\delta_w} k \left( \frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} \right) \right] 
\tag{6.3.15}
\]

\[
P \frac{\partial s_o}{\partial t} = \frac{\partial}{\partial x} \left[ \frac{k_o}{\delta_o} k \frac{\partial P_o}{\partial x} \right] 
\tag{6.3.16}
\]

combining equations (6.3.15) and (6.3.16) and using equation (6.3.5) gives,

\[
\frac{\partial}{\partial x} \left[ \frac{k_w + k_o}{\delta_w} k \frac{\partial P_o}{\partial x} \frac{k_w}{\delta_w} k \frac{\partial P_c}{\partial x} \right] = 0 
\tag{6.3.17}
\]

integrating (6.3.17) with respect to x, we get,

\[
\left[ \frac{k_w + k_o}{\delta_w} k \frac{\partial P_o}{\partial x} \frac{k_w}{\delta_w} k \frac{\partial P_c}{\partial x} \right] = -V 
\tag{6.3.18}
\]

where V is the constant of integration which can be evaluated later on. By simplifying (6.3.18) we get,

\[
\frac{\partial P_o}{\partial x} = \frac{V}{k_w \frac{k}{\delta_w} k \left(1 + \frac{k_k}{\delta_o \delta_w}\right)} + \frac{\partial P_c}{\partial x} \tag{6.3.19}
\]

from equation (6.3.15) and (6.3.19) it may written as,
\[ P \frac{\partial s_w}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\kappa w k}{\delta w} \frac{\partial P_c}{\partial x} + \frac{V}{1 + \frac{k_o \delta w}{k_w \delta o}} \right) = 0 \]  \hspace{1cm} (6.3.20)

The value of the pressure of oil \( P_o \) can be written as,
\[ P_o = \bar{P} + \frac{1}{2} P_c, \text{ where } \bar{P} = \frac{P_o + P_w}{2} \]

which is mean pressure and is constant.
\[ \frac{\partial P_o}{\partial x} = \frac{1}{2} \frac{\partial P_c}{\partial x} \] \hspace{1cm} (6.3.21)

It may be mentioned that the concept of mean pressure is justified in the statistical treatment of fingering [13]. Substituting the value of \( \frac{\partial P_o}{\partial x} \) from the equation (6.3.21) into the equation (6.3.18) gives the value of integration constant \( V \), as;
\[ V = \frac{1}{2} \left[ \frac{k_w k}{\delta w} - \frac{k_o k}{\delta o} \right] \frac{\partial P_c}{\partial x} \] \hspace{1cm} (6.3.22)

substituting the value of \( V \) from (6.3.22) into equation (6.3.20) gives us,
\[ P \frac{\partial s_w}{\partial t} + \frac{1}{2} \frac{\partial}{\partial x} \left[ \frac{k_w k}{\delta w} \frac{\partial P_o}{\partial x} \right] = 0 \] \hspace{1cm} (6.3.23)

using the linear relation between capillary pressure and saturation of injected fluid (water) equation (6.3.23) may be written as,
\[ P \frac{\partial s_w}{\partial t} + \frac{1}{2} \frac{\partial}{\partial x} \left[ \frac{k_w k}{\delta w} \frac{dP_o}{ds_w} \frac{\partial s_w}{\partial x} \right] = 0 \] \hspace{1cm} (6.3.24)

substituting the equation (6.3.6) and (6.3.12) into equation (6.3.24) we get,
\[ p \frac{\partial s_w}{\partial t} = \frac{k}{2\delta_w} \frac{\partial}{\partial x} \left[ \beta s_w \frac{\partial s_w}{\partial x} \right] \] (6.3.25)

The initial condition is taken as,

\[ s_w(x,0) = F(x) \ ; \ 0 < x < L \] (6.3.26)

equation (6.3.25) along with initial condition (6.3.26) is the desired non-linear partial differential formulation governing the fingering phenomenon.

Using the dimensionless variables

\[ X = \frac{x}{L} \] (6.3.27a)
\[ T = \frac{\beta k t}{2PL^2 \delta_w} \] (6.3.27b)

where \( 0 \leq x \leq 1 \) and \( 0 \leq T \leq 1 \) in equation (6.3.21) results in,

\[ \frac{\partial s_w}{\partial T} = \frac{\partial}{\partial X} \left( s_w \frac{\partial s_w}{\partial X} \right) \] (6.3.28)

The initial condition may be written in new variables from equations (6.3.26) as;

\[ s_w(0,T) = s_1(T > 0) \] (6.3.29)

The condition (6.3.29a) represents saturation of injected fluid at \( x = 0 \) and condition (6.3.29b) represents initial saturation of injected fluid where \( S_2 < S_1 \) has been considered for the given phenomenon.
6.4 Numerical Solution

The Differential Quadrature Method (DQM) was introduced by Bellman in the early 1970s and, since then, the technique has been successfully employed in finding the solutions of many problems in applied and physical sciences. Recently, the DQM are extensively used to analyze deflection, vibration and buckling of linear and nonlinear structural components by Bert’s group ([14],[15],[16]) and other researchers ([17],[18]). Many scholars have made important contributions to this method and its applications. Civan and Sliepcevich [19] proposed a linear transformation in the DQM to simplify the computing effort of the evaluation of the DQ weighting coefficients for high order derivatives. Bert et. al. [20] extended this method to multi-dimensional problems and integro-differential equations. Wang and Bert [21] applied the DQ M to fluid dynamic problems using parallel computation based on the multi-domain technique and gave a general recursion formula for computing the DQ weighting coefficients. Bellman, Kashaef and Casti [22] proposed a new and efficient approach in applying the DQM to high order boundary value problems. This method has been predicted by its proponents as a potential alternative to the conventional solution techniques such as the finite difference and finite elements methods. The basic idea of DQM is that the derivative of a function with respect to a
space variable at a given point is approximated as a weighted linear sum of the functional values at all discrete points in the domain of that variable.

The essence of the DQM is that the partial derivative of a function with respect to a variable is approximated by a weighted sum of function values at all discrete points in that direction. Its weighting coefficients do not relate to any special problem and only depend on the grid spacing. Thus, any partial differential equation can be easily reduced to a set of algebraic equations using these coefficients. More detailed descriptions on the DQ-type methods are given in [22],[23],[24],[25] etc.

The DQM is a new and different discretization technique to obtain accurate numerical solution with a small number of grid points. The method is based on assuming that the unknown functions can be basically approximated as polynomials. In this method, the first and second order partial derivatives of a function $S_u(X,T)$ at a point X are approximated by,

$$
(S_u)_x(X_i,T) \approx \sum_{j=1}^{N} a_{ij} S_u(X_j,T); i=1,2,\ldots,N
$$

(6.4.1)

$$
(S_u)_{xx}(X_i,T) \approx \sum_{j=1}^{N} b_{ij} S_u(X_j,T); i=1,2,\ldots,N
$$

(6.4.2)

where N is the number of grid points and $a_{ij}$ and $b_{ij}$ are the first and second order weighting coefficients respectively. The weight functions $a_{ij}$ and $b_{ij}$ are defined as,
\[ a_{ij} = \frac{M^{(1)}(X_i)}{(X_i - X_j)M^{(1)}(X_j)}, i \neq j \]  
\[ a_{ii} = -\sum_{j=1, j \neq i}^{N} a_{ij}, i = j \]  
\[ b_{ij} = 2a_{ij} \left( a_{ii} - \frac{1}{X_i - X_j} \right), i \neq j \]  
\[ b_{ii} = -\sum_{j=1, j \neq i}^{N} b_{ij}, i = j \]  
\[ M^{(1)}(X_i) = \prod_{j=1, j \neq i}^{N} (X_i - X_j) \]  

Using Chebyshev-Gauss-Lobatto grid points we have;

\[ X_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{i-1}{N-1} \right) \right] L_X, i = 1, 2, ..., N \]  

where \( L_X \) is the length of the interval.

Using equations (6.4.1) and (6.4.2) in

\[ \frac{\partial S_w}{\partial T} = \left( \frac{\partial S_w}{\partial X} \right)^2 + S_w \frac{\partial^2 S_w}{\partial X^2} \]  

we get the system of ordinary differential equations,

\[ \frac{d(S_w)}{dT} = \left( \sum_{j=1}^{N} a_{ij}S_w(X_j, T) \right)^2 + S_w \sum_{j=1}^{N} b_{ij}S_w(X_j, T) ; i = 1, 2, ..., N \]  

where \( (S_w)_i = S_w(X_i, T) \) and \( (S_w)_j = S_w(X_j, T) \)

We establish the initial condition as,

\[ S_w(X_i, 0) = \phi(X_i), a < X < b; i = 1, 2, ..., N \]

where \( L_X = 1 \) is the length of the interval and for \( N \) the total number of grid points.
Table (6a) Computational results of Fingering phenomenon for N=9

<table>
<thead>
<tr>
<th>X</th>
<th>$S_w$</th>
<th>$S_w$</th>
<th>$S_w$</th>
</tr>
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<td>$X_1=0$</td>
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<td>$S_{w1}=0.5177$</td>
<td>$S_{w1}=0.9239$</td>
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<td>$S_{w2}=0.0435$</td>
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<td>$S_{w3}=0.0258$</td>
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### Table (6b) Computational results of Fingering phenomenon for N=13

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<th>T=0.9</th>
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Table (6c) Computational results of Fingering phenomenon for N=17

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<td>X_13 = 0.853553391</td>
<td>$s_{w_{13}}=0.0238$</td>
<td>$s_{w_{13}}=0.0792$</td>
<td>$s_{w_{13}}=0.1345$</td>
</tr>
<tr>
<td>X_14 = 0.915734806</td>
<td>$s_{w_{14}}=0.0319$</td>
<td>$s_{w_{14}}=0.1195$</td>
<td>$s_{w_{14}}=0.2072$</td>
</tr>
<tr>
<td>X_15 = 0.961939767</td>
<td>$s_{w_{15}}=0.0531$</td>
<td>$s_{w_{15}}=0.2255$</td>
<td>$s_{w_{15}}=0.3980$</td>
</tr>
<tr>
<td>X_16 = 0.990392640</td>
<td>$s_{w_{16}}=0.1120$</td>
<td>$s_{w_{16}}=0.5200$</td>
<td>$s_{w_{16}}=0.9281$</td>
</tr>
<tr>
<td>X_17 = 1</td>
<td>$s_{w_{17}}=1.5995$</td>
<td>$s_{w_{17}}=7.9577$</td>
<td>$s_{w_{17}}=14.3159$</td>
</tr>
</tbody>
</table>
6.5 Conclusion

The numerical solutions demonstrate that the saturation of the injected fluid \( S_w(X,T) \) is increasing smoothly as the time and distance are increasing. It starts from the initial value \( S_w(0,T)=S_{w0} \) and will subsequently stabilize after a long time. It also observes from the numerical study that the saturation of the injected fluid is decreasing when any injected fluid comes into contact with the native fluid. Hence from the numerical table as well as from the graph (6a) it is also clear that the saturation will decrease for any time \( T \) for a fixed \( X \). It is consistent with the fingering phenomenon. The numerical and graphical presentation was obtained using MATLAB coding.
The use of a combination of polynomial-based DQM in space and a low-storage Runge-Kutta fourth stage method given by Pike and Roe in time has been proposed for the nonlinear partial differential equation, with high convergence, is also capable of producing highly accurate solutions with minimal computational effort for both time and space. It has been seen that the polynomial-based differential quadrature technique approximates the exact solution very well. Since the scheme is explicit, linearization is not needed. No requiring extra effort to deal with nonlinear terms, ease in use, and computational cost effectiveness has made the current method an efficient alternative method for modeling this nonlinear behaviour. For concrete problems where an exact solution does not exist, the present method is a very good choice to achieve a high degree of accuracy while dealing with the non linear problems.
Chapter-6
NUMERICAL SOLUTIONS OF FINGERING PHENOMENA IN A HOMOGENEOUS POROUS MEDIA

References


