CHAPTER 1
INTRODUCTION
1.1 Introduction:

Every organization holds some stock of goods and materials to meet future demand which is called stock. For example retailer buys goods from wholesaler and keeps them in stock until it sells to the customer, similarly a factory keeps raw material for its production and bank holds some cash with them to meet day to day transactions. Management of movement and storage of these materials held in stock is overall called Logistics. Inventory Management is merely one of the tasks of this broader function logistics. Thus Inventory Management takes decisions regarding activities policies and procedures to make sure that right amount of each item held in stock at any time.

Inventory act as a cushion between supply and demand. It allows mismatches between best rate of supply and actual rate of demand. It helps to meet situation of shortages, emergencies and delays. It is very much beneficial in decoupling adjacent operations of production.

We have already discussed that inventory plays a very vital role in smooth running and functioning of any organization. But at the same time if stock is not managed properly then inventory carrying cost increases which decreases total profit of the organization. For example, if retailer orders less than the actual demand then it will be the case of stock – out which may result into (i) the loss of gain which could have achieved by selling those stock – out units and (ii) repeated practice of stock – out lead to loss of good will which is a long term loss in the business. On the other hand, if retailer orders for more units then the actual demand than it will be a case of over stocking which will lead to more
inventory carrying cost which does not sounds in favor of the retailer. Thus in this scenario three very basic questions relevant to inventory management arises as follows:-

(1) What items should be kept in stock?
(2) When orders should be placed?
(3) How much should be ordered?

Type of Inventories on the basis of their use:-

1. **Decoupling Inventory** – When various manufacturing process operates successively then failure of any one can interrupt whole production. To overcome this stocking point of the inventory takes place between adjacent stages.
   - Raw material and component parts.
   - Work in process Inventory.
   - Finished goods Inventory.
   - Spare parts Inventory.
   - Consumables. Eg. Oil, stationary, cleaners.

2. **Lot size Inventory** – This inventory exists when there is more production and less immediate demand. Amount of inventory depend upon storage, space limitation, economical shipment quantity, investment, resources etc.

3. **Safety – Buffer Stock** – This stock is created to meet any uncertainties of demand. This helps to meet any unpredictable shortage or emergency.
4. **Pipeline (or Transit) Inventories** – This inventory is required for shipment of inventory items from production units to distribution centers and further to customers.

5. **Seasonal Inventory** - This inventory is created to meet demand which is caused due to seasonal variation in demand.

6. **Anticipation Inventory** – This inventory is simply maintained to make sure that no customer remains disappointed by not getting the desired item at any instant of time.

### 1.2 Basic Concepts and Terminology

Following are the basic concepts and terminology associated with an inventory system.

- **Demand**: Rate of demand is defined as number of units required per unit time. Nature or pattern of demand is either deterministic or probabilistic. When number of unit required over a period of time is known with certainty then demand is said to be of deterministic pattern. It is fixed and static for a period of time. At the same time it can vary (dynamic) from one period to another. In the case of Probabilistic demand requirements are not known in advance with certainty but can be described by some probability distribution. Demand can be satisfied in two ways either instantaneously at the beginning of the cycle or uniformly during the period.

- **Lead Time**: The time gap between placing an order and its actual receipt is called lead time. Again it can be deterministic and probabilistic. It has
got two types of components as administrative lead-time from initiation of procurement action until the placing of an order, and the delivery lead time from placing of an order until the delivery of the ordered material.

- **Planning Horizon**: This is defined as time period over which a particular inventory level is maintained. This can be finite or infinite depending on nature of the demand.

- **Reorder Level**: This is level at which ordering activities must be initiated as the on hand stock just cover the demand until ordered units arrive in the inventory.

- **EOQ (Economic Order Quantity)**: EOQ is size of order which gives minimum total inventory cost.

- **Order Cycle**: The time period between placements of two successive orders is referred to as an order cycle. The orders may be placed on the basis of the following two types:

  1. **Continuous Review**: In this review, a fixed re-order level is set for each item and inventory level is supervised continuously. A fixed order quantity is ordered when stock fall down a specific level. Any variation in demand is allowed by changing time between orders.

  2. **Periodic Review**: In this system, the inventory levels are reviewed at equal intervals of time. Orders of variable size are placed at regular intervals of time. Variation in demand can be satisfied by changing order size.
• **Deterioration:** Deterioration is defined as decay, evaporation, obsolescence, loss of utility or marginal value of the commodity that results into the reduction of usefulness from the original condition. It may happen due to the following reasons.

1. The items may have a fixed life and may be of no use if stored after the life period, e.g., medicines, photo films, bulbs, etc.

2. Deterioration can take place due to unsatisfactory, poor or inadequate storage conditions, e.g., dairy products.

3. Deterioration can also result from poor handling in the store, e.g., fruits and vegetables, crockery items.

• **Salvage Value:** Some Salvage value is associated with the deteriorated units it simply means that deteriorated units is not losing their complete utility or we can say marginal value of the commodity. For example if a vendor sells his fresh tomatoes at the rate of Rs10 / Kg and in the evening when tomatoes are not that much fresh to attract customers he will not be able to sell it at same price. If he takes back that tomatoes may be next day it will get deteriorate completely and he will not get any profit out of it. In evening itself if he sells those tomatoes at the rate of Rs 6 / Kg then he will get 60 % salvage value associated with it at the same time he is able to satisfy the customers who want to get vegetables at cheaper rate and ready to compromise with its quality may be hotel or restaurant people.
- **Ordering Cost**: This cost is associated with the ordering of raw material for the production purposes. It includes advertisement cost, stationary, postage, telephone charges, telegrams, rents etc.

- **Purchase cost**: This is referred as cost price charged by supplier to retailer for one unit of item. It plays an important role when some discount on purchase price / unit given on purchasing some specific units.

- **Inventory Carrying (Holding) cost**: Cost associated with smooth running of the inventory is called holding or carrying cost. It includes costs due to warehouse, rent, heat special package, refrigeration, computer updates, electricity bill, insurance and opportunity cost of capital tied up in the inventory.

- **Shortage cost**: Shortages means when immediate demand of customers are not satisfied then there are two type of losses-

  There is a loss of profit which could be gained by retailer by selling those articles to customer. If this shortages occurs frequently than there is a loss of good will of the retailer which is considered as long term in business.

- **Opportunity Cost**: The gain or profit from an alternative offer, e.g., investment decision that is forgone in favor of another.

- **Inflation**: The phenomenon of sustained increase in the prices of goods and services in general is known as inflation. It may arise due to less supply and more demand which is known as 'demand – pull inflation'. Inflation also results due to increase in cost of some critical input, such as steel or petroleum, which then triggers off a gradual rise in overall cost
which is known as 'cost-push inflation'. When inflation increases, it results into the decline in the purchasing power of money.

- **Present Value**: The discounted value of future cash-flows is known as present value of that particular future cash-flow. For example, present value of $1000 expected two years from today is $826.45, if the discounting rate is 10%, i.e. \( \frac{1000}{(1+0.10)^2} \)

- **Discounting Rate**: The interest rate used in calculating the present value of future cash-flows is known as discounting rate. The interest rate is often considered to be the inflation rate. The procedure for calculating the present value of future cash-flows by applying appropriate rate of discounting rate is known as discounting. This approach is also known as Discounted-cash-flow (DCF) approach. This is based on the concept of time value of money which differentiates between dollars offered today and one expected at some point of time in future.

- **Advertisement and Stock display Cost**: Advertisements and display of products plays a vital role in attracting mass customer. The advertisement through electronic media, newspaper, internet, using innovative ways of display of product in the show room is the best tool for the promotion of a product. This attracted researcher to analyze the inventory problem when demand depends on stock displayed.
1.3 Literature Survey:

Since long time, researchers are engaged in analyzing inventory models for deteriorating items such as volatile liquids, blood, medicines, electronic components, fashion goods, fruits and vegetables etc. Whitin (1957) studied deterioration at the end of the storage period, for example, for the fashion goods industry. Berrotoni (1962) observed that both the leakage failure of the dry batteries and life expectancy of ethical drugs could be expressed in terms of Weibull distribution, in discussing the problem of fitting empirical data to mathematical distributions. In both cases, the rate of deterioration increases with time or the longer the time remains unused, the higher rate at which they fail. Ghare and Schrader (1963), first formulated a mathematical model with a constant deterioration rate. They classified the phenomena of inventory deterioration into three types, viz direct spoilage, physical depletion and deterioration. Covert and Philip (1973) derived an EOQ model for items with Weibull distribution deterioration.

The two parameters weibull density function given by Covert and Philip (1973) is

\[ f(t) = \alpha \beta t^{\beta-1} e^{-\alpha t^\beta} \]

Where, \( \alpha \) denotes the scale parameter, \( (\alpha > 0) \)
\( \beta \) denotes the shape parameter, \( (\beta > 0) \)
\( t \) denotes time of deterioration, \( (t > 0) \)

This distribution gives probability density function for time dependent deterioration.

Time dependent deterioration by weibull distribution is given by
\[ \theta(t) = \alpha \beta t^{\beta - 1}, \quad 0 \leq t \leq T, \]

When \( \beta > 1 \), deteriorating rate increases with time.

When \( \beta < 1 \), deteriorating rate decreases with time.

When \( \beta = 1 \), deteriorating rate remain constant.

\( \alpha \) and \( t \) are directly proportional to each other, if \( \alpha \) is increased the distribution gets stretched out to the right and its height decreases while maintaining its shape and location. If \( \alpha \) is decreased, the distribution reaches to 0 and its height increases. Since then Misra (1975), Shah (1977), Love (1979), Dave and Patel (1981), Höller Mak (1983), Aggarwal and Jaggi (1989) Heng et al. (1991), Hariga (1996), Jaggi and Aggarwal (1996), and Wee (1995) worked on deteriorating inventory systems. The review articles Raffat (1981), Shah and Shah (2000) and Goyal and Giri (2001) give a complete and up-to-date survey of published literature for the deteriorating inventory models. The most of the addressed articles assume that deterioration of units is a complete loss to the inventory system.

In classical economic order quantity (EOQ) model, it is tacitly assumed that the quantity received matches with the quantity requisition. Though in practice due to various reasons, viz. machines breakdown, worker’s strike, unavailability of raw material, delay in transportation etc. it is found that the quantity received does not necessarily matches with the quantity ordered, but may be a random variable with prescribed mean and variance.

Silver (1976) developed an EOQ model when the quantity received is uncertain. Kalro and Gohil (1982) extended Silver’s model to allow shortages.
Shah and Shah (1992) derived an EOQ model for items with constant rate of deterioration under the assumption that quantity received is uncertain. Gor and Shah (1994) allowed shortages in Shah and Shah's model. Yano and Lee (1995) gave an up-to-date review of the published articles on uncertainty of input.

Trade credit is the most effective tool of the supplier to encourage retailer to buy more and to attract more customers. Practically, it is not possible to settle the account as soon as retailer received the units therefore supplier use to give some credit period to settle the account. Obviously, supplier charges interest at some rate on the remaining amount which retailer not able to settle before given credit period. This type of offered period to settle the account by the supplier to the retailer is actually called trade credit. Goya (1985) derived an economic order quantity inventory model when supplier offers credit period to settle the retailer's account. Shah et al. (1988) extended Goyal's model by allowing shortages. Mandal and Phaujdar (1989) developed a mathematical model including interest earned from the sales revenue on the stock remaining beyond the settlement period. Shah (1993a) studied inventory model for constant deterioration of units in inventory under the scenario of permissible credit period. Jamal et al. (1997) developed an inventory model to allow for shortages under the permissible delay in payments. Shah (1993 c) analyzed inventory model when supplier offers credit period to settle the retailer's account by considering stochastic demand. Shah (1997) derived a probabilistic order-level system with lead-time when delay in payments is permissible. Jamal et al. (2000) formulated a mathematical model when retailer can settle the account either at
the end of the credit period or later incurring interest charges on the un-paid balance for the over-due period.


In progressive credit periods two successive time period is given to the retailer by the supplier to settle the account. Rate at which interest charged to the retailer after second credit period is more than the rate of interest charged after first credit period. It is considered as the best option to price discounts. Brigham (1995) defined credit period as "net 30", i.e. the supplier offers a retailer, the delay period of 30 days for settling the account.

Goyal (2007) explored the concept of progressive credit period and developed economic order quantity. Soni and Shah (2005) developed a mathematical model when units in inventory are subject to constant deterioration
under the scenario of progressive credit periods. Soni et al. (2006) studied effect of inflation in above stated model. Levin et al. (1972)'s quotation: "the presence of inventory has motivational effect on customer a-round it" is studied by Soni and Shah (2008). They developed a model in which demand is partially constant and partially dependent on the stock, and the supplier offers to retailer progressive credit period to settle the account.

Aggrawal and Jaggi (1995), Jamal et al. (2000), Sarkar et al. (2000a), formulated the inventory are subject to constant rate of deterioration under the condition of trade credit. Chang et al. (2002) extended above model for time dependent deterioration. Liao et al. (2000) and Sarkar et al. (2000b) analyzed the model with inflation. Jamel et al. (1997) and Chang and Dye (2001) developed model when demand varies linearly with time. Chen and Chung (1999) optimized a light buyer’s inventory policy under trade credit using discounted cash flow. Arcelus and Srinivasan (1993) for delay in payments for extraordinary purchases. Hwang and Shinn (1997) developed an inventory model for a retailer’s pricing and ordering policy for exponentially deteriorating items when delay in payments is permissible. Teng (2002), Chung et al. (2002) modified Goyal’s model (1985) under assumption that the selling price is different from the purchase price. Shinn and Hwang (2003), Gor and Shah (2006) optimized the retailer’s price and order size simultaneously under the condition of order size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer’s order size. Related articles are by Chung and Huang (2003), Salameh et al. (2003), Chang et al. (2003), Chung and Liao (2004),

The aforesaid articles assumed that the supplier would offer the buyer a trade credit to his customers. This is termed as one level of trade credit. In competitive world, this assumption is not practical. Huang (2003) gave the concept of two levels of trade credit in which buyer gets credit period from the supplier which is partly offered to the customers. Huang and Hsu (2008) extended above concept when buyer is a powerful decision maker. They assumed that the buyer offers the practical trade credit to the customer.

Practically, it is not possible for retailer to settle the account as soon as retailer received ordered units, therefore supplier can offer a cash discount and / or a permissible delay to the retailer if the outstanding amount is paid within the allowable fixed settlement period. For example, the supplier offers 3% discount off the unit purchase price if the payment is made within 15 days; otherwise full price of items is due within 30 days. This credit term is usually denoted as “3/15, net 30” (e.g, see Brigham (1995, p. 741)).

Eilon and Mallaya (1966) derived the pricing policy for perishable items under the assumption that the product has a maximum shelf life with no deterioration before the expiration date. Cohen (1977) analyzed inventory system with simultaneous price setting and ordering levels for exponentially deteriorating items. Kang and Kim (1983) extended Cohen's model for finite replenishment. Abad (1988) considered pricing and lot size decisions with incremental quantity discounts for non deteriorating items. Shah (1993 d) developed an EOQ model
for constant deterioration rate of units in inventory when short term discount is offered. Shinn (1990) formulated a mathematical model for exponentially deteriorating units with quantity discounts and partial backordering under a prescribed scheduling period. However, in his total cost function, the lost sale penalty cost was not considered. Wee (1995) developed joint pricing and replenishment policy for inventory with a constant rate of deterioration. In this study, an optimal inventory model with quantity discount, pricing and partial backordering is derived where units deteriorate with respect to time. The salvage value is incorporated to the deteriorated units.

Deterioration is assumed to be a function of time. In order to reduce loss due to deterioration, supplier is tempted to offer temporary price discount. The strategy of discount is also considered to enhance the demand. However, it may not always be advantageous for retailer to buy more units during the discounted period as he has to pay more for holding inventory and deteriorating cost. The buyer needs to formulate optimal ordering policy to take advantage of the temporary price discount offered by the supplier, in order to maximize total cost savings.

discount strategies for one-time-only sales. Wee and Yu (1995, 1997) studied ordering policy for non-deteriorating and deteriorating items the assumptions of Tersine (1994) and Martin (1994) with three additional assumptions as given below:

1. For the first regular order in case 1 (Fig. 7.2.2(a)) the economic order quantity $Q_0$ is derived using the discounted price $(c - d)$,

2. the number of replenishment must be integer,

3. the temporary price discounted order cycle $T_d$, is equal to the average of the regular order cycle times and the time period $t_1$ (Fig. 7.2.2(a)) and $t_2$ (Fig. 7.2.2(b)).

In this study, the optimal ordering policy is analyzed to maximize total cost savings for following two cases:

**Case 1:** A temporary price discount occurs at the regular inventory replenishment cycle time. (Fig. 7.2.2(a))

**Case 2:** A temporary price discount sale occurs at non-regular inventory replenishment cycle time (Fig. 7.2.2(b)).

Bhunia and Maiti (1997), Goyal and Gunasekaran (1995), Luo (1998). Pal et al. (2006) developed an inventory model for deteriorating items by taking into account the impact of marketing strategies viz pricing, advertisement and the displayed stock level of the demand rate of the system. These articles optimize total cost or net profit per time unit of an inventory system.

From a financial point of view, inventory bricks a capital investment and must compete with other assets for a form’s limited capital funds. Thus, the effect of inflation on the inventory system plays an important role. Buzacott (1975), Bierman and Thomas (1977), Misra (1977a) studied the inventory decisions under inflationary conditions for EOQ model. Misra (1979b) derived an inflation model for the EOQ, in which the time value of money and different inflation rates were considered. Gurunani (1983) gave the economic analysis of inventory systems and claimed that the discounting effects on EOQ were substantial. Chandra and Bahner (1985) studied the effects of inflation and time value of money on optimal ordering policies. Related articles are by Queyranne (1985), Roundy (1986), Federgruen et al. (1992), Atkins et al. (1992), Mitchell (1987), Atkins and Sun (1995), Sun and Atkins (1997), Shah et al. (2003, 2004).

1.4 Outline of the thesis:

The proposed thesis has been divided into eight chapters on the basis of the structure of the different models which are as under:

- **Chapter – 1:** Introduction
- **Chapter – 2:** Inventory model for deteriorating items with salvage value.
• **Chapter 3**: Lot – size model for the deteriorating items with Salvage value when units received are uncertain and shortages are allowed.

• **Chapter 4**: Ordering policies for Weibull distributed deterioration with associated Salvage value under progressive credit periods.

• **Chapter 5**: An EOQ model for deteriorating items under supplier credit period when demand is stock dependent.

• **Chapter 6**: EOQ models for stock dependent demand under buyer partial trade credit scenario.

• **Chapter 7**: Inventory models for Weibull distributed deterioration with Salvage value with price breaks and temporary price discount.

• **Chapter 8**: Present Value formulation of lot size inventory model for deteriorating items with advertisement and stock displayed dependent demand.

**Chapter 1** gives an overall introduction of different inventory models proposed in following chapters in the light of different assumptions.

**Chapter 2** discusses a basic inventory model when deterioration is time dependent with associated salvage value.

**Chapter 3** deals with lot – size for the deteriorating item with salvage value when units received are uncertain.

**Section 3.1** explains a model for obtaining optimal procurement when items are subjected to constant deterioration with some salvage value associated to these deteriorated units. Units received are uncertain which is a random variable distributed normally.
Section 3.2 is an extension of above mentioned model of sec 3.1 to allow shortages under the same assumptions of constant deterioration with associated salvage value when units received is uncertain.

Chapter 4 presents optimal ordering policies with time dependent deterioration with associated salvage value.

Section 4.1 gives optimal ordering policies for weibull distributed deterioration with associated salvage value when credit period is given by the supplier to the retailer to settle the account. This model assumes that demand rate is deterministic and constant, lead time is zero and replenishment rate is infinite and instantaneous.

Section 4.2 proposes a model with same basic assumptions as 4.1 but in this model progressive credit periods are offered from the supplier to retailer assuming that if retailer settle account after first credit period then retailer suppose to pay interest at some rate say (IC_1), if he settles account after second credit period then he is suppose to pay interest on the remaining amount at the rate of say (IC_2) (IC_2 > IC_1). Moreover it is assumed that as soon as retailer gets revenue from sale it is deposited in some interest bearing account.

Chapter 5 gives a model for deteriorating items when demand is stock dependent and deterioration is constant with associated salvage value. In this model retailer offers two credit periods for cash discounts and permissible delay respectively if outstanding amount is paid within the allowable fixed settlement period. After both the settlement periods, if account is not settled then interest will
be charged by the retailer. At the same time it is assumed that as soon as retailer
earns revenue from sales it is deposited into some interest bearing account.

Chapter 6 discusses EOQ models when demand is stock dependent under a
scenario of partial trade credits.

Section 6.1 is derived under two levels of trade credit in the environment of
supply chain. The supplier offers the credit period M – time units to the buyer. When T ≥ M, [0, M] the buyer off all units sold and keeps his profits and starts paying interest charges during [M, T] for the items in the stock with interest rate i. When T ≤ M, the account is settled at T = M and interest charges are zero. The buyer offers the partial trade credit to his customer. So his customers make a cash down payment to the buyer when the item is sold. Then his customer must do the remaining payment at the end of the trade credit; N offered by the buyer. The buyer can incur interest from his customer payment with rate i. Demand is stock dependent. In this section inventory is not subjected to any deterioration and shortages are also not allowed here.

Section 6.2 is an extension of above discussed model with two levels of trade
credits and stock dependent demand. But in this section it is assumed that units in inventory deteriorate with a constant rate of deterioration.

Section 6.3 is development of section 6.2 as a salvage value is associated with
each deteriorated units. Rests all assumptions have remain same in this model.

Chapter 7 proposes inventory models for Weibull distributed deterioration with
Salvage value with price breaks and temporary price discount.
Section 7.1 develops a deterministic inventory model with price breaks, pricing and partial backordering when the units in inventory are subject to Weibull distributed deterioration with respect to time. The salvage value is associated to the deteriorated units. The purchase cost of units is assumed to have quantity discounts, and demand is deterministic and decreasing function of selling price. The lead time is zero and partial backordering is allowed.

Section 7.2 explains a lot – size model for Weibull distributed deterioration of units when a temporary price discount is offered. The salvage value is associated to the deteriorated units. It is discussed for scenarios of regular and non – regular inventory replenishment.

Chapter 8 is about Present Value formulation of lot size inventory model for deteriorating items when demand depends on advertisement and stock displayed.

Section 8.1 gives optimal ordering policy of a constant deteriorating item when demand rate is dependent on displayed stock level and frequency of advertisement through media. Shortages, (if any), are allowed and completely backlogged. The objective is to minimize present value of all future cash out – flows.

Section 8.2 is extension of above proposed section. Only deterioration is time dependent which is weibull distributed deterioration instead of constant in this model. Rest all assumptions are same as sec 8.1.

The thesis ends with Bibliography.