CHAPTER 4

OPTIMAL ORDERING AND PRICING POLICIES FOR

TWO STAGE CREDIT POLICIES
4.0 Introduction:

In this chapter, an optimal pricing and ordering policies for retailer when the supplier offers two progressive credit periods to settle the accounts is derived in section 4.1. The objective function is to optimize the net profit which is difference of gross revenue and all cash-out-flows. The extension of the model derived in section 4.1, when units in inventory are subject to constant rate of deterioration is carried out in section 4.2. The effects of deterioration of units in inventory and credit periods on objective function and decision variables are studied using hypothetical numerical example.

4.1 Optimal pricing and ordering policies when the supplier provides two-stage credit periods:

In this section, an EOQ model is developed in which demand is assumed to be decreasing function of selling price (a decision variable) when supplier offers two progressive credit periods, if the retailer could not settle his account by the end of the first credit period. The objective function to be optimized is considered as net profit which is difference of gross revenue and all cash-out-flows. The decision variables are selling price and ordering quantity (equivalently, cycle time). A flowchart is given to find the flow of optimal selling price and ordering policy. Analytic proofs are discussed to observe the effect of various parameters on an objective function.

4.1.1 Assumptions and Notations:

The following additional assumptions and notations other than those given in A.1 and N.1 are used to formulate the proposed model:

- The demand is \( R(P) = a - bP, \ (a, b > 0, \ a \gg b) \). \( a \) denotes the fixed demand, \( b \) termed as display factor and \( P \) denotes selling price of the item during the cycle time.
• $GR = $ Gross Revenue.

• $NP(P, T) = $ Net profit/cycle

$P$ and $Q$ (equivalently, $T$) are decision variables

4.1.2 Mathematical Formulation:

The on-hand inventory $Q(t), 0 \leq t \leq T$ depletes due to constant demand $R(P)$. The instantaneous state of inventory $Q(t)$ at any instant of time $t, 0 \leq t \leq T$ is governed by the differential equation

$$\frac{dQ(t)}{dt} = -R(P) = -(a-bP), \quad 0 \leq t \leq T \tag{4.1.2.1}$$

with initial condition $Q(0) = Q$ and boundary condition $Q(T) = 0$. Consequently, the solution of (4.1.2.1) is given by

$$Q(t) = R(P)(T-t), \quad 0 \leq t \leq T \tag{4.1.2.2}$$

and the order quantity is

$$Q = R(P)T = (a-bP)T \tag{4.1.2.3}$$

The cost components per unit time are as follows:

- Ordering cost; $OC = \frac{A}{T}$ \tag{4.1.2.4}

- Inventory holding cost;

$$IHC = \frac{h}{T} \int_{0}^{T} Q(t) dt = \frac{h(a-bP)T}{2} \tag{4.1.2.5}$$

- Gross revenue, $GR = (P-C)R(P)$ \tag{4.1.2.6}

Regarding interest charged and earned, based on the length of the cycle time $T$, three cases arise:

Case 1: $T \leq M$

Case 2: $M < T < N$
Case 3: $T \geq N$

We discuss each case in detail.

Case 1: $T \leq M$ (Fig. 4.1.2.1)

Here, the retailer sells $Q$-units during $[0, T]$ and is paying for $CR(P)T$-units in full to the supplier at time $M \geq T$. Therefore, interest charges are zero, i.e.

$$IC_1 = 0 \quad (4.1.2.7)$$

The retailer sells products during $[0, T]$ and deposits the revenue in an interest bearing account at the rate of $le$/year. In the period $[T, M]$, the retailer deposits revenue into the account that earns interest at the rate $le$/year. Therefore interest earned per time unit is

$$IE_1 = \frac{Pie}{T} \left[ \int_0^T R(P) \, dt + R(P)T(M - T) \right] = Pie(a - bP) \left( M - \frac{T}{2} \right) \quad (4.1.2.8)$$

The net profit, $NP_1$, is given by

$$NP_1(P, T) = GR - OC - IHC - IC_1 + IE_1 \quad (4.1.2.9)$$
$P$ and $T$ are continuous decision variables. Hence optimal values of $P$ and $T$ can be obtained by setting

$$\frac{\partial NP_1(P, T)}{\partial P} = a - 2bP + bC + \frac{hbT}{2} + Ie\left(M - \frac{T}{2}\right) (a - 2bP) = 0 \quad (4.1.2.10)$$

$$\frac{\partial NP_1(P, T)}{\partial T} = \frac{A}{T^2} - \frac{h}{2} (a - bP) = 0 \quad (4.1.2.11)$$

The obtained $P = P_1$ and $T = T_1$, maximizes net profit, provided

$$XY - Z^2 < 0 \quad (4.1.2.12)$$

where

$$X = \frac{\partial^2 NP_1(P, T)}{\partial P^2} = -2b \left[1 + Ie\left(M - \frac{T}{2}\right)\right]$$

$$Y = \frac{\partial^2 NP_1(P, T)}{\partial T^2} = - \frac{2A}{T^3}$$

$$Z = \frac{\partial^2 NP_1(P, T)}{\partial T \partial P} = \frac{hb - Ie(a - 2bP)}{2}$$

**Case 2: $M < T < N$ (Fig. 4.1.2.2)**

![Inventory Level Diagram](image-url)
The retailer sells units and deposits the revenue into an interest bearing account at an interest rate \( r \text{/unit/year} \) during \([0, M]\). Therefore, the interest earned during \([0, M]\) is

\[
IE_2 = P \int_0^M R(P) t \, dt = \frac{1}{2} P le(a-bP)M^2
\]  

(4.1.2.13)

Buyer has to pay for \( Q = R(P)T \)–units purchased initially at the rate of \( C \) \$/unit to the supplier during \([0, M]\). The retailer sells \( R(P)M \)–units at sale price \( SP/unit \). So he has generated revenue of \( P R(P) M \) plus the interest earned, \( IE_2 \), during \([0, M]\). Two sub–cases may arise.

Sub–case 2.1: Let \( P R(P) M + IE_{2,1} \geq CQ \) i.e. the retailer has enough money to pay for all \( Q \)–units procured. Then, interest charges, \( IC_{2,1} = 0 \) (4.1.2.14) and the interest earned, \( IE_{2,1} \), per time unit is

\[
IE_{2,1} = \frac{IE_2}{T}
\]  

(4.1.2.15)

Using equations (4.1.2.4) to (4.1.2.6), (4.1.2.14) and (4.1.2.15), the net profit; \( NP_{2,1} (P, T) \) is given by

\[
NP_{2,1} (P, T) = GR - OC - IHC - IC_{2,1} + IE_{2,1}
\]  

(4.1.2.16)

The optimum values of \( P = P_{2,1} \) and \( T = T_{2,1} \) are the solutions of

\[
\frac{\partial NP_{2,1}(P,T)}{\partial P} = a - 2bP + bC + \frac{hbT}{2} + le\left(M - \frac{T}{2}\right)(a - 2bP) = 0
\]  

(4.1.2.17)

\[
\frac{\partial NP_{2,1}(P,T)}{\partial T} = \frac{A}{T^2} - \frac{h (a - bP)}{2} - \frac{P le(a - bP)}{2} = 0
\]  

(4.1.2.18)

The obtained \( P = P_{2,1} \) and \( T = T_{2,1} \) maximizes the net profit provided

\[
XY - Z^2 < 0
\]  

(4.1.2.19)

where \( X, Y \) and \( Z \) are same as defined in (4.1.2.12) \( NP_1(P,T) \) is replaced by \( NP_{2,1}(P,T) \).
**Sub-case 2.2:** \( P R(P)M + IE_2 < CQ \)

Here, retailer will have to pay interest on the unpaid balance \( U_1 = CR(P) - [PR(P)M + IE_2] \) at rate of \( Ic_1 \) at time \( M \) to the supplier. The interest to be paid, \( IC_{2.2} \), per time unit is:

\[
IC_{2.2} = \frac{U_1^2}{PR(P)T} - \frac{Ic_1}{M} \int_Q^T Q(t)dt = \frac{IC_1 U_1^2 \left[ M \left( \frac{M}{2} - T \right) - \frac{T^2}{2} \right]}{2PT} \tag{4.1.2.20}
\]

and interest earned per time unit is:

\[
IE_{2.2} = \frac{IE_2}{T} \tag{4.1.2.21}
\]

Using equations (4.1.2.4) to (4.1.2.6), (4.1.2.20) and (4.1.2.21), the net profit, \( NP_{2.2}(P, T) \) is given by

\[
NP_{2.2}(P, T) = GR - OC - IHC - IC_{2.2} + IE_{2.2} \tag{4.1.2.22}
\]

The optimum values of \( P = P_{2.2} \) and \( T = T_{2.2} \) are the solutions of

\[
\frac{\partial NP_{2.2}(P, T)}{\partial P} = a - bP - (P - C)b + \frac{hbT}{2} - \frac{IC_1 U_1[M(\frac{M}{2} - T) - \frac{T^2}{2}]}{PT} \\
\times CbT - (a - bP)M + PMb - \frac{IE_2 M^2}{2(a - 2bP)} + \frac{IC_{2.2}}{P} \\
+ \frac{IE_2 M^2(a - 2bP)}{2T} = 0 \tag{4.1.2.23}
\]

\[
\frac{\partial NP_{2.2}(P, T)}{\partial T} = A \frac{h(a - bP)}{T^2} - \frac{IC_1 U_1[M(\frac{M}{2} - T) - \frac{T^2}{2}]}{PT} C(a - bP) + \frac{IC_{2.2}}{2PT} \left( M + T \right) \\
+ \frac{IC_{2.2}}{T} - \frac{IE_{2.2}}{T} = 0 \tag{4.1.2.24}
\]

The obtained \( P = P_{2.2} \) and \( T = T_{2.2} \) maximizes the net profit \( NP_{2.2}(P, T) \) provided

\[
EF - G^2 < 0 \tag{4.1.2.25}
\]
\[
E = \frac{\partial^2 N P_{2,2}(P, T)}{\partial P^2} = -2b \frac{Ic_1 \%1^2 \%2}{PT} + \frac{2Ic_1 U_1 \%1 \%2}{P^2T} - \frac{Ic_1 U_1 \%2}{PT} \frac{2bM + IebM^2}{P^2} \\
- \frac{2IC_{2,2}}{P^2} \frac{IebM^2}{T}
\]

\[
F = \frac{\partial^2 N P_{2,2}(P, T)}{\partial T^2} = \frac{-2A}{T^3} \frac{IC_1 C^2(a-bP)^2 \%2}{PT} + \frac{2IC_1 C(a-bP)U_1(M+T)}{PT} \\
+ \frac{2IC_1 (a-bP)U_1 \%2}{PT^2} + \frac{IC_1 U_1^2 (M+T)}{PT^2} - \frac{2IC_{2,2}}{T} + \frac{2IE_{2,2}}{T^2}
\]

\[
G = \frac{\partial^2 N P_{2,2}(P, T)}{\partial P \partial T} = \frac{hb}{2} \frac{IC_1 (a-bP)\%1 \%2}{PT} + \frac{IC_1 U_1 (M+T) \%1}{PT} + \frac{IC_1 U_1 \%2}{P^2T} \\
+ \frac{IC_1 U_1 \%2 Cb}{PT} + \frac{IC_1 U_1 \%2 C(a-bP)}{P^2T} - \frac{IC_1 U_1^2 (M+T)}{2P^2T} \\
- \frac{IC_1 U_1^2 \%2}{2P^2T^2} + \frac{IeM^2}{2T^2} (a-2bP)
\]

where \( \%1 = [-CbT - (a-bP)M + PMb - \frac{IeM^2}{2}(a-2bP)] \)

\( \%2 = [M \left( \frac{M}{2} - T \right) - \frac{T^2}{2}] \)

**Case 3: \( T \geq N \) (Fig. 4.1.2.3)**

Inventory Level

![Figure 4.1.2.3](image-url)
Based on the total purchase cost, $C_Q$, total money $PR(P)M + IE_2$ in account at $M$ and total money in account at $N$ is $PR(P)N + PIR(P)\frac{N^2}{2}$, three sub-cases may arise.

**Sub-case 3.1:** Let $PR(P)M + IE_2 \geq C_Q$ Then this sub-case is same as sub-case 21 (Note: Decision variables and objective function are designated by subscript 3.1)

**Sub-case 3.2:** Let $PR(P)M + IE_2 < C_Q$ but

$$PR(P)(N-M) + \frac{PIR(P)(N-M)^2}{2} \geq C_Q - (PR(P)M + IE_2)$$

This sub-case coincides with sub-case 3.2. (Note: Decision variables and objective function are designated by subscript 3.2)

**Sub-case 3.3:** Let $PRMe^{-rM} + IE_2 < C_Q$ but

$$PR(P)(N-M) + \frac{PIR(P)(N-M)^2}{2} < C_Q - (PR(P)M + IE_2)$$

Here, the retailer does not have enough money in his account to pay off for total purchase cost at $N$. He will do payment of $\left[PR(P)M + IE_2\right]$ at $M$ and $PR(P)(N-M) + \frac{PIR(P)(N-M)^2}{2}$ at $N$. So, he has to pay interest charges on un-paid balance $U_1 = C_Q - \left[PR(P)M + IE_2\right]$ with interest rate $Ic_1$ during $[M, N]$ and $U_2 = U_1 - \left[PR(P)(N-M) + \frac{PIR(P)(N-M)^2}{2}\right]$ with interest rate $Ic_2$ during $[N, T]$.

Therefore, total interest charges, $IC_{33}$, per time unit is given by
\[ IC_{3.3} = \frac{U_1 Ic_1 (N - M)}{T} + \frac{U_2^2}{PR(P)T} Ic_2 \left[ Q(t)dt \right]_N^T \]
\[ = \frac{U_1 Ic_1 (N - M)}{T} + \frac{U_2^2}{PT} Ic_2 \left\{ N \left( \frac{N}{2} - T \right) - \frac{T^2}{2} \right\} \] (4.1.2.26)

and interest earned per time unit is;
\[ IE_{3.3} = \frac{IE_2}{T} \] (4.1.2.27)

Using equations (4.1.2.4) to (4.1.2.6), (4.1.2.26) and (4.1.2.27), the net profit;

\[ NP_{3.3} (P, T) \]

The optimum values of \( P = P_{3.3} \) and \( T = T_{3.3} \) are the solutions of

\[ \frac{\partial NP_{3.3}(P,T)}{\partial P} = a - bP - (P - C)b + \frac{hbT}{2} - \frac{\%1 Ic_1 (N - M)}{T} - \frac{2U_2 \%2 \%3}{PT} \]
\[ + \frac{Ic_2 U_2^2 \%2}{P^2 T} + \frac{IeM^2}{2T} (a - 2bP) = 0 \] (4.1.2.29)

\[ \frac{\partial NP_{3.3}(P,T)}{\partial T} = \frac{A}{T^2} - \frac{h(a - bP)}{2} - \frac{C(a - bP)Ic_1 (N - M)}{T} + \frac{Ic_1 U_1 (N - M)}{T^2} \]
\[ - \frac{2Ic_2 U_2^2 \%2 (a - bP)}{PT} + \frac{Ic_2 U_2^2 (N + T)}{PT} + \frac{Ic_2 U_2^2 \%2}{PT^2} - \frac{Ple(a - bP)M^2}{2T^2} \]
\[ = 0 \] (4.1.2.30)

where \( \%1 = [-CbT - (a - bP)M + PMb - \frac{IeM^2}{2}(a - 2bP)] \)
\[ \%2 = N \left( \frac{N}{2} - T \right) - \frac{T^2}{2} \]
\[ \%3 = [\%1 - (N - M)(a - 2bP) - \frac{Ie(N - M)^2}{2}(a - 2bP)] \]
The obtained $P = P_{3,3}$ and $T = T_{3,3}$ maximizes the net profit $NP_{3,3}$ provided

$$BK - J^2 < 0 \quad (4.1.231)$$

\[
B = \frac{\partial^2 NP_{3,3}(P, T)}{\partial P^2} = -2b \frac{Ic_1(N - M)(2bM + IeM^2)}{T} - \frac{2Ic_2 \%2 \%3^2}{PT} \]
\[
+ \frac{4Ic_2 U_2 \%1 \%2 \%3}{P^2T} - \frac{2bIc_2 U_2 \%2}{PT} \left[2N + IeM^2 + Ie(N - M)^2 \right] \]
\[
- \frac{2Ic_2 U_2 \%2}{P^3T} - \frac{Ie b M^2}{T} \]

\[
K = \frac{\partial^2 NP_{3,3}(P, T)}{\partial T^2} = -2A \frac{2C(a - bP)Ic_1(N - M)}{T^3} + \frac{2Ic_1 U_1(N - M)}{T^3} \]
\[
- \frac{2Ic_2 C^2(a - bP)^2}{PT} - \frac{4Ic_2 U_2(N + T)}{PT} + \frac{4Ic_2 U_2 \%2 C(a - bP)}{PT^2} \]
\[
+ \frac{Ic_2 U_2}{PT} - \frac{2Ic_2 U_2 \%2(N + T)}{PT^3} - \frac{2Ie(a - bP)M^2}{T^3} \]

\[
J = \frac{\partial^2 NP_{3,3}(P, T)}{\partial P \partial T} = \frac{h b}{2} \frac{Cbi_1(N - M)}{T} + \frac{U_1 Ic_1(N - M)}{T^2} + \frac{2Ic_2 U_2 \%2 C(a - bP)}{P^2T} \]
\[
+ \frac{2Ic_2 U_2 \%2 Cb}{PT} + \frac{2Ic_2 U_2 \%3(N + T)}{PT} - \frac{2Ic_2 U_2 \%2(N + T)}{P^2T} \]
\[
- \frac{2Ic_2 U_2 \%2}{PT^2} - \frac{2Ic_2 U_2 \%2}{P^2T^2} - \frac{IeM^2}{2T^2}(a - 2bP) \]

where

\[
\%1 = [-CbT - (a - bP)M + PMb - \frac{IeM^2}{2}(a - 2bP)] \]
\[
\%2 = N \left(\frac{N}{2} - T\right) - \frac{T^2}{2} \]
\[
\%3 = [\%1 - (N - M)(a - 2bP) - \frac{Ie(N - M)^2}{2}(a - 2bP)] \]

In the next section, we present computational flowchart to search for optimal solution.
4.1.3 Flowchart:

Compute $T = T_i$ and $P = P_i$ from Case - 1

Is $T < M$?

Yes

No

Is $M < T_i < N$?

Yes

No

Is $PR(P)M + IE_i \geq CQ$?

Yes

No

Compute $T = T_{i2}$ and $P = P_{i2}$ from Sub-case 2.2 OR $T = T_{i1}$ and $P = P_{i1}$ from Sub-case 3.1

Compute $NP_i(P, T) = \max\{NP_i(P, T_i)\}$, $i = 1, 2.1, 2.2, 3.1, 3.2, 3.3$

Stop

4.1.4 Theoretical Results:

**Proposition 4.1.4.1** : $NP_i(P, T)$ is maximum for $i = 1, 2, 1, 2, 2, 3, 1, 3.1, 3.2$, and 3.3.

**Proof** : It follows from the equations (4.1.2.12), (4.1.2.19), (4.1.2.25), (4.1.2.31).

**Proposition 4.1.4.2** : For $T > N$, $NP_{33}(P, T)$ is increasing function of $M$ and $N$.

**Proof** :
\[
\frac{\partial NP_3^3(P,T)}{\partial M} = \frac{PR(P) (1+M)l(N-M)}{T} + \frac{U_1 l c_1}{T}
\]

\[
+ \frac{2Ic_2 U_2 \%2}{PT} \left[ P \varepsilon R(P) (N-2M) \right] + \frac{P \varepsilon R(P) M}{T}
\]

> 0

\[
\frac{\partial NP_3^3(P,T)}{\partial N} = \frac{-U_1 l c_1}{T} + \frac{2Ic_2 U_2 \%2}{PT} \left[ P \varepsilon R(P) (1+ \varepsilon e(N-M)) \right]
\]

\[
- \frac{Ic_2 U_2^2 (N-T)}{PT}
\]

> 0

where \( \%2 = N(N-T) - \frac{T^2}{2} \)

4.1.5 Numerical Example and observations:

Consider following parametric values in appropriate units.

\[
[h, A, C, \varepsilon e, a, b] = [0.2, 100, 20, 0.12, 1000, 10]
\]
Table 4.1.5.1

Variation in $M$ and $N$

<table>
<thead>
<tr>
<th>$N$</th>
<th>$M$</th>
<th>$15/365$</th>
<th>$20/365$</th>
<th>$25/365$</th>
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<tbody>
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<td></td>
<td></td>
<td>$Ic_1 = 15%$</td>
<td>$Ic_1 = 16%$</td>
<td>$Ic_1 = 17%$</td>
</tr>
<tr>
<td>30/365</td>
<td>$Ic_2 = 18%$</td>
<td>$T = 0.3065$</td>
<td>$T = 0.3061$</td>
<td>$T = 0.3060$</td>
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<tr>
<td></td>
<td></td>
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<tr>
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<td>$R = 394.63$</td>
</tr>
<tr>
<td></td>
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<td>$NP = 15261.98$</td>
<td>$NP = 15281.28$</td>
<td>$NP = 15299.44$</td>
</tr>
<tr>
<td>35/365</td>
<td>$Ic_2 = 20%$</td>
<td>$T = 0.3338$</td>
<td>$T = 0.3334$</td>
<td>$T = 0.3331$</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
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<td>$Q = 131.60$</td>
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<tr>
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<td>$NP = 15339.84$</td>
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</tr>
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<td>$NP = 15408.71$</td>
</tr>
</tbody>
</table>

Observations:
- Increase in first allowable credit period decreasing the order quantity and increases net profit whereas selling price is inactive.
- Increase in extended permissible credit period increases order quantity and net profit increases significantly while selling price leisurely goes down.

4.2 Optimal Pricing and Ordering Policies for deteriorating items under progressive trade credit scheme:

In this section, a mathematical model is developed to formulate optimal pricing and ordering policies when the units in inventory are subject to constant rate of deterioration and the supplier offers progressive credit periods to settle the account.
4.2.1 Assumptions and Notations:

An EOQ model for deteriorating items with progressive payment scheme is developed under the same assumptions and notations as given in A.1, N.1 and section 4.1.1.

4.2.2 Mathematical Formulation:

The on-hand inventory \(Q(t), 0 \leq t \leq T\) depletes due to constant demand \(R(P)\) and deterioration of units. The instantaneous state of inventory \(Q(t)\) at any instant of time \(t, 0 \leq t \leq T\) is governed by the differential equation

\[
\frac{dQ(t)}{dt} + \theta Q(t) = -R(P), \quad 0 \leq t \leq T
\]

(4.2.2.1)

with the initial condition \(Q(0) = Q\) and boundary condition \(Q(T) = 0\).

The solution of (4.2.2.1) is given by

\[
Q(t) = \frac{R(P)(e^{\theta(T-t)} - 1)}{\theta}, \quad 0 \leq t \leq T
\]

(4.2.2.2)

and the order quantity is

\[
Q = \frac{R(P)(e^{\theta T} - 1)}{\theta}
\]

The cost components per unit time are as follows:

- Ordering cost; \(OC = \frac{A}{T}\)  

(4.2.2.3)

- Inventory holding cost;

\[
IHC = \frac{h}{T} \int_0^T Q(t) dt = \frac{h(a-bP)(e^{\theta T} - 1 - \theta T)}{\theta^2 T}
\]

(4.2.2.4)

- Cost due to deterioration;

\[
DC = \frac{C(a-bP)(e^{\theta T} - 1 - \theta T)}{\theta T}
\]

(4.2.2.5)

- Gross revenue; \(GR = (P-C)R(P)\)

(4.2.2.6)
Regarding interest charged and earned, based on the length of the cycle time $T$, three cases arise:

Case 1: $T \leq M$

Case 2: $M < T < N$

Case 3: $T \geq N$

We discuss each case in detail.

Case 1: $T \leq M$ (Fig. 4.2.2.1)

Here, the retailer sells $Q$-units during $[0, T]$ and is paying for $CQ$-units in full to the supplier at time $M > T$. Therefore, interest charges are zero. i.e.

$$IC_1 = 0 \quad (4.2.2.7)$$

The retailer sells products during $[0, T]$ and deposits the revenue in an interest bearing account at the rate of $lel$/year. In the period $[T, M]$, the retailer deposits revenue into the account that earns interest at the rate $lel$/year. Therefore, interest earned per time unit is

$$IE_1 = \frac{Ple}{T} \left[ \int_0^T R(P)t \, dt + R(P)T(M - T) \right] = \frac{Ple(a - bP)(2M - T)}{2} \quad (4.2.2.8)$$

The net profit $NP_1$ is given by

$$NP_1(P, T) = GR - OC - IHC - DC - IC_1 + IE_1 \quad (4.2.2.9)$$
P and T are continuous decision variables. Hence optimal values of P and T can be obtained by setting

\[
\frac{\partial NP_1(P,T)}{\partial P} = a - 2Pb + bC + \frac{(e^{\theta T} - 1 - \theta T)b}{\theta T}(h + C) + \frac{Ie(2M - T)}{2}(a-2Pb) = 0
\]

(4.2.10)

\[
\frac{\partial NP_1(P,T)}{\partial T} = \frac{A}{T^2} \left( \frac{h + C}{\theta} (a - bP) \right) \left\{ \frac{e^{\theta T} - 1}{T} - \frac{e^{\theta T} - 1 - \theta T}{\theta T} \right\} - \frac{Ple(a-bP)}{2} = 0
\]

(4.2.11)

The obtained \( P = P_1 \) and \( T = T_1 \), maximizes the net profit; provided

\[ XY - Z^2 < 0 \]

(4.2.12)

where

\[
X = \frac{\partial^2 NP_1(P,T)}{\partial P^2} = -2b - le b(2M - T)
\]

\[
Y = \frac{\partial^2 NP_1(P,T)}{\partial T^2} = \frac{-2A}{T^3} \left( \frac{h + C}{\theta} (a - bP) \right) \left\{ \theta e^{\theta T} - \frac{2(e^{\theta T} - 1)}{T^2} + \frac{2(e^{\theta T} - 1 - \theta T)}{\theta T^3} \right\}
\]

\[
Z = \frac{\partial^2 NP_1(P,T)}{\partial T \partial P} = \frac{(e^{\theta T} - 1)b(h + C)}{T} - \frac{b(e^{\theta T} - 1 - \theta T)}{\theta T^2} \left( \frac{h + C}{\theta} \right) - \frac{le(a-2bP)}{2}
\]

**Case 2: \( M < T < N \) (Fig. 4.2.2.2)**

![Inventory Level](image)
The retailer sells units and deposits the revenue into an interest bearing account at an interest rate \( i \) unit/year during \([0, M]\). Therefore the interest earned during \([0, M]\) is given by

\[
IE_2 = Pte \int_0^M R(P)t \, dt = \frac{1}{2} Pte(a - bP)M^2
\]  

(4.2.2.13)

Buyer has to pay for \( Q = R(P)T \) units purchased initially at the rate of \( C \) $/unit to the supplier during \([0, M]\). The retailer sells \( R(P)M \) units at sale price \( P \) $/unit. So he has generated revenue of \( P R(P)M \) plus the interest earned, \( IE_2 \), during \([0, M]\). Two sub-cases may arise.

**Sub-case 2.1:** Let \( P R(P)M + IE_2 \geq CQ \), i.e. the retailer has enough money to pay for all \( Q \) units procured. Then, interest charges, \( IC_{2.1} = 0 \)  

(4.2.2.14)

and the interest earned, \( IE_{2.1} \), per time unit is

\[
IE_{2.1} = \frac{IE_2}{T}
\]  

(4.2.2.15)

Using equations (4.2.2.3) to (4.2.2.6), (4.2.2.14) and (4.2.2.15), the net profit, \( NP_{2.1}(P, T) \) is given by

\[
NP_{2.1}(P, T) = GR - OC - IHC - DC - IC_{2.1} + IE_{2.1}
\]  

(4.2.2.16)

The optimum values of \( P = P_{2.1} \) and \( T = T_{2.1} \) are solutions of

\[
\frac{\partial NP_{2.1}(P, T)}{\partial P} = a - 2Pb + bC + \frac{(e^{\theta T} - 1 - \theta T)b(h + C)}{\theta T} + \frac{Ple M^2(a - 2Pb)}{2T} = 0
\]  

(4.2.2.17)

\[
\frac{\partial NP_{2.1}(P, T)}{\partial T} = \frac{A}{T^2} \left( \frac{h + C}{\theta} (a - bP) \right) \left\{ \frac{(e^{\theta T} - 1)}{T} - \frac{(e^{\theta T} - 1 - \theta T)}{\theta T^2} \right\}
\]

\[
- \frac{Ple(a - bP)M^2}{2T^2} = 0
\]  

(4.2.2.18)

The obtained \( P = P_{2.1} \) and \( T = T_{2.1} \) maximizes the net profit provided
where $X$, $Y$ and $Z$ are same as defined earlier. [Note $NP_1(P,T)$ is replaced by $NP_21(P,T)$]

**Sub-case 2.2:** $PR(P)M + IE_2 < CQ$

Here, retailer will have to pay interest on the un-paid balance $U_1 = CR(P) - [PR(P)M + IE_2]$ at rate of $Ic_1$ at time $M$ to the supplier. The interest to be paid, $IC_{2.2}$, per time unit is:

$$IC_{2.2} = \frac{U_1^2}{PR(P)T} Ic_1 \int_{M}^{T} Q(t) dt = \frac{Ic_1 U_1^2 (\theta(T - M) + M\theta - \theta T)}{2\theta^2 P T}$$

and interest earned per time unit is:

$$IE_{2.2} = \frac{IE_2}{T}$$

Using equations (4.2.2.3) to (4.2.2.6), (4.2.2.20) and (4.1.2.21), the net profit; $NP_{2.2}(P, T)$ is given by

$$NP_{2.2}(P, T) = GR - OC - IHC - DC - IC_{2.2} + IE_{2.2}$$

The optimum values of $P = P_{2.2}$ and $T = T_{2.2}$ are the solutions of

$$\frac{\partial NP_{2.2}(P,T)}{\partial P} = a - 2Pb + bC + \frac{(\theta T - 1 - \theta T)b}{\theta T} \left( h + C \right)$$

$$- \frac{IC_1 U_1^2 Cb - (a - 2bP)M - \frac{Ie M^2(a - 2bP)}{2}}{\theta^2 P T}$$

$$+ \frac{IC_1 U_1^2 Cb}{2\theta^2 P^2 T} + \frac{Ie M^2(a - 2bP)}{2T} = 0$$

(4.2.2.23)
\[ \frac{\partial NP_{2,2}(P,T)}{\partial T} = \frac{A}{T^2} - \left( \frac{h}{\theta} + C \right)(a-bP) \left\{ \frac{(e^{\theta T} - 1)}{T} - \frac{(e^{\theta T} - 1-\theta T)}{\theta T^2} \right\} \]

\[ - \frac{Ic_1 U_1^2 C(a-bP)}{\theta^2 P T} - \frac{Ic_1 U_1^2 (e^{\theta(T-M)} - 1)}{2\theta PT} + \frac{Ic_1 U_1^2 %2}{\theta^2 P T^2} \]

\[ - \frac{Ple(a-bP)M^2}{2T^2} = 0 \]  

(4.2.2.24)

where \( %2 = (e^{\theta(T-M)} + M\theta - 1-\theta T) \)

The obtained \( P = P_{2,2} \) and \( T = T_{2,2} \) maximizes the net profit \( NP_{2,2}(P, T) \) provided

\[ EF - G^2 < 0 \]  

(4.2.2.25)

\[ E = \frac{\partial^2 NP_{2,2}(P,T)}{\partial P^2} = -2b - \frac{Ic_1 (e^{\theta(T-M)} + M\theta - 1-\theta T) %2}{\theta^2 P T} + \frac{2Ic_1 U_1 %1 %2}{\theta^2 P^2 T} \]

\[ \frac{Ic_1 U_1 %2 (2bM + Ie b M^2)}{\theta^2 P T} - \frac{Ic_1 U_1^2 %2}{\theta^2 P^2 T} - \frac{Ie b M^2}{T} \]

\[ F = \frac{\partial^2 NP_{2,2}(P,T)}{\partial T^2} = -2A \left( \frac{h}{\theta} + C \right)(a-bP) \left\{ \frac{\theta e^{\theta T}}{T} - \frac{2(e^{\theta T} - 1)}{T^2} + \frac{2(e^{\theta T} - 1-\theta T)}{\theta T^3} \right\} \]

\[ - \frac{Ic_1 C^2 (a-bP)^2 {e^{\theta(T-M)}} + M\theta - 1-\theta T}{\theta^2 P T} \]

\[ - \frac{2Ic_1 U_1 (e^{\theta(T-M)} - 1)C(a-bP)}{\theta P T} + \frac{2Ic_1 U_1 %2 C(a-bP)}{\theta^2 P T^2} \]

\[ - \frac{Ic_1 U_1^2 e^{\theta(T-M)}}{2 P T} + \frac{Ic_1 U_1^2 (e^{\theta(T-M)} - 1)}{\theta P T^2} \frac{Ic_1 U_1^2 %2}{\theta^2 P T^3} \]

\[ + \frac{Ple(a-bP)M^2}{2T^3} \]
\[ G = \frac{\partial^2 N P_{22}(P,T)}{\partial T \partial P} = \frac{(e^{\theta T} - 1)b(h + C)}{T} - \frac{b(e^{\theta T} - 1 - \theta T)}{\theta T^2} \left( \frac{h}{\theta} + C \right) - \frac{Ic_1 \%1 C(a - bp) \%2}{\theta^2 P T} \]
\[ + \frac{Ic_1 U_1 \%2 C(a - bp)}{\theta^2 P^2 T} + \frac{Ic_1 U_1 \%2 Cb}{\theta^2 P T} - \frac{Ic_1 U_1 (e^{\theta(T-M)} - 1) \%1}{\theta P T} \]
\[ + \frac{Ic_1 U_1^2 \%2 (e^{\theta(T-M)} - 1)}{2\theta P^2 T} + \frac{Ic_1 U_1 \%1 C(a - bp) \%2}{\theta^2 P T^2} \]
\[ - \frac{Ic_1 U_1^2 \%2}{2\theta^2 P^2 T^2} - \frac{Ic_1 M^2 (a - 2bP)}{2T^2} \]

where
\[ \%1 = (-CbT - (a - 2bP)M - \frac{Ic M^2 (a - 2bP)}{2}) \]
\[ \%2 = (e^{\theta(T-M)} + M\theta - 1 - \theta T) \]

**Case 3:** \( T \geq N \) (Fig. 4.2.2.3)

![Figure 4.2.2.3](image)

Based on the total purchase cost, \( CQ \), total money \( PR(P)M + IE_2 \) in account at \( M \) and total money in account at \( N \) is \( PR(P)N + PR(P) \frac{N^2}{2} \), three sub-cases may arise:
Sub-case 3.1: Let \( PR(P)M + IE_2 \geq CQ \) Then this sub-case is same as sub-case 2.1. (Note: Decision variables and objective function are designated by subscript 3.1)

Sub-case 3.2: Let \( PR(P)M + IE_2 < CQ \) but

\[
PR(P)(N-M) + \frac{PleR(P)(N-M)^2}{2} \geq CQ - (PR(P)M + IE_2)
\]

This sub-case coincides with sub-case 3.2. (Note: Decision variables and objective function are designated by subscript 3.2)

Sub-case 3.3: Let \( PRMe^{-rM} + IE_2 < CQ \) but

\[
PR(P)(N-M) + \frac{PleR(P)(N-M)^2}{2} < CQ - (PR(P)M + IE_2)
\]

Here, the retailer does not have enough money in his account to pay off for total purchase cost at \( N \). He will do payment of \( [PR(P)M + IE_2] \) at \( M \) and \( PR(P)(N-M) + \frac{PleR(P)(N-M)^2}{2} \) at \( N \). So, he has to pay interest charges on un-paid balance \( U_1 = CQ - [PR(P)M + IE_2] \) with interest rate \( Ic_1 \) during \([M, N]\]

and \( U_2 = U_1 - [PR(P)(N-M) + \frac{PleR(P)(N-M)^2}{2}] \) with interest rate \( Ic_2 \) during \([N, T]\).

Therefore, total interest charges, \( IC_{3.3} \), per time unit is given by

\[
IC_{3.3} = \frac{U_1Ic_1(N-M)}{T} + \frac{U_2^2}{PR(P)T} \int_N^T Q(t)dt
\]

\[
= \frac{U_1Ic_1(N-M)}{T} + \frac{Ic_2U_2^2(e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2 P T}
\]

(4.2.2.26)

and interest earned per time unit is;
Using equations (4.2.2.3) to (4.2.2.6), (4.1.2.26) and (4.1.2.27), the net profit;

\[ NP_{3,3} (P, T) \] is given by

\[ NP_{3,3} (P, T) = GR - OC - IHC - DC - IC_{3,3} + IE_{3,3} \quad (4.2.2.28) \]

The optimum values of \( P = P_{3,3} \) and \( T = T_{3,3} \) are the solutions of

\[ \frac{\partial NP_{3,3}(P, T)}{\partial P} = a - 2Pb + bC + \left( \frac{e^{\theta T}}{\theta T} \right) \left( -1 - \theta T \right) \frac{h}{\theta} + C \left( \frac{h}{\theta} + C \right) - \frac{Ic_1(N-M)1/3}{T} \]

\[ - \frac{2Ic_2U_2}{\theta^2 P T} - \frac{Ic_2U_2^2}{\theta^2 P^2 T} + \frac{Ic_2U_2^2}{2T} = 0 \quad (4.2.2.29) \]

\[ \frac{\partial NP_{3,3}(P, T)}{\partial T} = A \left( \frac{h}{\theta} + C \right) (a-bP) \left\{ \frac{\left( e^{\theta T} - 1 \right)}{T} - \frac{\left( e^{\theta T} - 1 - \theta T \right)}{\theta T^2} \right\} \]

\[ - \frac{Ic_1(a-bP)C(N-M)}{T} + \frac{U_1Ic_1(N-M)}{T^2} - \frac{2Ic_2U_2^22C(a-bP)}{\theta^2 P T} + \left( \frac{Ic_2U_2^2}{\theta P T} - \frac{Ic_2U_2^2}{\theta^2 P^2 T} + \frac{Ple(a-bP)M^2}{2T^2} \right) = 0 \quad (4.2.2.30) \]

where

\( \%1 = (-CbT - (a - 2bP)M - \frac{le M^2(a-2bP)}{2}) \)

\( \%2 = (e^{\theta(T-N)} + M \theta - 1 - \theta T) \)

\( \%3 = \left\{ -CbT - (a - 2bP)M - \frac{le M^2(a-2bP)}{2} - (a - 2bP)(N-M) - \frac{le (N-M)^2(a-2bP)}{2} \right\} \)

The obtained \( P = P_{3,3} \) and \( T = T_{3,3} \) maximizes the net profit \( NP_{3,3} \) provided

\[ BK - J^2 < 0 \quad (4.2.2.31) \]
\[ B = \frac{\partial^2 N_{P,3}(P,T)}{\partial P^2} = -2b - \frac{(2bM + IebM^2)lc_1(N - M)}{T} - \frac{2lc_2%2%3^2}{\partial^2 P T} \]
\[ + \frac{4lc_2U_2%2%3}{\partial^2 P^2 T} - \frac{2lc_2U_2%22(2bN + le b M^2 + le b (N - M)^2)}{\partial^2 P T} \]
\[ - \frac{2lc_2U_2%2%2}{\partial^2 P^3 T} - \frac{le b M^2}{T} \]
\[ K = \frac{\partial^2 N_{P,3}(P,T)}{\partial T^2} = -\frac{2A}{T^3} \left( \frac{h + C}{\theta} (a - bP) \left( \frac{\theta e^{\theta T}}{T} - \frac{2(e^{\theta T} - 1)}{T^2} + \frac{2(e^{\theta T} - 1 - \theta T)}{\theta T^3} \right) \right) \]
\[ + \frac{2C(a - bP)lc_1(N - M)}{T^2} - \frac{2U_1lc_1(N - M)}{T^3} - \frac{2lc_2C^2(a - bP)^2%2}{\partial^2 P T} \]
\[ - \frac{4lc_2U_2(e^{\theta(T - N)} - 1)c(a - bP)}{\partial P T} + \frac{4lc_2U_2%22C(a - bP)}{\partial^2 P T^2} \]
\[ - \frac{lc_2U_2%22\theta(T - N)}{PT} + \frac{2lc_2U_2%22(e^{\theta(T - N)} - 1)}{\partial P T^2} - \frac{lc_2U_2%2%2}{\partial^2 P T^3} \]
\[ + \frac{Plc(a - bP)M^2}{2T^3} \]
\[ J = \frac{\partial^2 N_{P,3}(P,T)}{\partial T \partial P} = \frac{(e^{\theta T} - 1)b(h + C)}{T} - \frac{b(e^{\theta T} - 1 - \theta T)}{\theta T^2} \left( \frac{h + C}{\theta} \right) + \frac{Cblc_1(N - M)}{T} \]
\[ + \frac{%1lc_1(N - M)}{T^2} - \frac{2lc_2%22C(a - bP)%3}{\partial^2 P T} + \frac{2lc_2U_2%22C(a - bP)}{\partial^2 P^2 T} \]
\[ + \frac{2lc_2U_2%2C b}{\partial^2 P T} - \frac{2lc_2U_2%22(e^{\theta(T - N)} - 1)%3}{\partial P T} + \frac{lc_2U_2%22(e^{\theta(T - N)} - 1)}{\partial P^2 T} \]
\[ + \frac{lc_2U_2%2%3}{\partial^2 P T^2} - \frac{lc_2U_2%2%2}{\partial^2 P^2 T^2} - \frac{le(a - 2bP)M^2}{2T^2} \]

where
\[ %1 = (-CbT - (a - 2bP)M - \frac{le M^2(a - 2bP)}{2}) \]
\[ %2 = (e^{\theta(T - N)} + M\theta - 1 - \theta T) \]
%3 = \{-CbT - (a - 2bP)M - \frac{IeM^2(a - 2bP)}{2} - (a - 2bP)(N - M) - \frac{Ie(N - M)^2(a - 2bP)}{2}\}

4.2.3 Flowchart:

Computational flow is same as mentioned in section 4.1.3

4.2.4 Theoretical Results:

**Proposition 4.2.4.1** : $NP, (P, T)_{i}$ is maximum for $i = 1, 2.1, 2.2, 3.1, 3.2,$ and $3.3.$

**Proof** : It follows from the equations (4.2.2.12), (4.2.2.19), (4.2.2.25), (4.2.2.31).

**Proposition 4.2.4.2** : For $T > N, NP_{3.3} (P, T)$ is increasing function of $M$ and $N.$

**Proof**

\[
\frac{\partial NP_{3.3}(P,T)}{\partial M} = \frac{P(a - bP)(1 + M)Ic_i(N - M)}{T} + \frac{U_1 Ic_i}{T} \\
- \frac{2Ic_iU_2 %2}{\theta^2 PT} [Ple R(P)(N - 2M)] + \frac{PleR(P)M}{T} > 0
\]

and

\[
\frac{\partial NP_{3.3}(P,T)}{\partial N} = -\frac{U_1 Ic_i}{T} + \frac{2Ic_iU_2 %2}{\theta^2 PT} [PR(P)(1 + Ie(N - M))] \\
+ \frac{Ic_iU_2(e^{\theta(T - N)} - 1)}{\theta PT} > 0
\]

where $%2 = (e^{\theta(T - N)} + N\theta - 1 - \theta T)$

**Proposition 4.2.4.3** : $NP, (P, T)_{i}$ is decreasing function of $\theta$ for $i = 1, 2.1,$ $2.2, 3.1, 3.2,$ and $3.3.$
Proof:

\[
\frac{\partial NP_{2,1}(P,T)}{\partial \theta} = -\frac{R(P)(e^{\theta T} - 1)}{\theta} \left( \frac{h}{\theta} + C \right) + \frac{R(P)(e^{\theta T} - 1 - \theta T)}{\theta^2 T} \left( \frac{2h}{\theta} + C \right) < 0
\]

\[
\frac{\partial NP_{2,2}(P,T)}{\partial \theta} = -\frac{R(P)(e^{\theta T} - 1)}{\theta} \left( \frac{h}{\theta} + C \right) + \frac{R(P)(e^{\theta T} - 1 - \theta T)}{\theta^2 T} \left( \frac{2h}{\theta} + C \right) - \frac{Ic_2 U_2^2 (T - M) e^{\theta (T - M)} + M - T}{2 \theta^2 P T} + \frac{2Ic_2 U_2^2 \%2}{\theta^3 P T} < 0
\]

where \( \%2 = (e^\theta (T - M) + N\theta - 1 - \theta T) \)

\[
\frac{\partial NP_{3,3}(P,T)}{\partial \theta} = -\frac{R(P)(e^{\theta T} - 1)}{\theta} \left( \frac{h}{\theta} + C \right) + \frac{R(P)(e^{\theta T} - 1 - \theta T)}{\theta^2 T} \left( \frac{2h}{\theta} + C \right) - \frac{Ic_2 U_2^2 (T - N) e^{\theta (T - N)} + N - T}{2 \theta^2 P T} + \frac{2Ic_2 U_2^2 \%2}{\theta^3 P T} < 0
\]

where \( \%2 = (e^\theta (T - N) + N\theta - 1 - \theta T) \)

In the following section, numerical example is given to carry out the sensitivity analysis of parameters for sub case 3.3 of section 4 2.
4.2.5 Numerical Example and observations:

Consider following parametric values in appropriate units

\[ [h, A, C, le, a, b] = [0.2, 100, 20, 0.12, 1000, 10] \]

### Table 4.2.5.1: Optimal decision variables when \( \theta = 0.01 \)

<table>
<thead>
<tr>
<th>N</th>
<th>M</th>
<th>15/365</th>
<th>20/365</th>
<th>25/365</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( Ic_1 = 15% )</td>
<td>( Ic_1 = 16% )</td>
<td>( Ic_1 = 17% )</td>
</tr>
<tr>
<td>30/365</td>
<td>( Ic_2 = 18% )</td>
<td>T = 0.3058</td>
<td>T = 0.3055</td>
<td>T = 0.3054</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P = 60.59</td>
<td>P = 60.57</td>
<td>P = 60.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q = 120.70</td>
<td>Q = 120.65</td>
<td>Q = 120.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R = 394.09</td>
<td>R = 394.34</td>
<td>R = 394.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NP = 15250.85</td>
<td>NP = 15269.16</td>
<td>NP = 15287.32</td>
</tr>
<tr>
<td>35/365</td>
<td>( Ic_2 = 20% )</td>
<td>T = 0.3332</td>
<td>T = 0.3327</td>
<td>T = 0.3325</td>
</tr>
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<td></td>
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<td>Q = 131.54</td>
<td>Q = 131.52</td>
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<td></td>
<td>R = 394.44</td>
<td>R = 394.69</td>
<td>R = 394.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NP = 15306.35</td>
<td>NP = 15327.64</td>
<td>NP = 15346.31</td>
</tr>
<tr>
<td>40/365</td>
<td>( Ic_2 = 22% )</td>
<td>T = 0.3637</td>
<td>T = 0.3630</td>
<td>T = 0.3626</td>
</tr>
<tr>
<td></td>
<td></td>
<td>P = 60.53</td>
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<td>P = 60.48</td>
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<td>R = 394.71</td>
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<td>R = 395.22</td>
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<td>NP = 15353.73</td>
<td>NP = 15374.53</td>
<td>NP = 15394.32</td>
</tr>
</tbody>
</table>

### Table 4.2.5.2: Optimal decision variables when \( \theta = 0.02 \)

<table>
<thead>
<tr>
<th>N</th>
<th>M</th>
<th>15/365</th>
<th>20/365</th>
<th>25/365</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( Ic_1 = 15% )</td>
<td>( Ic_1 = 16% )</td>
<td>( Ic_1 = 17% )</td>
</tr>
<tr>
<td>30/365</td>
<td>( Ic_2 = 18% )</td>
<td>T = 0.3052</td>
<td>T = 0.3049</td>
<td>T = 0.3048</td>
</tr>
<tr>
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<td>P = 60.60</td>
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</tr>
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<td>Q = 120.55</td>
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<td>R = 394.04</td>
<td>R = 394.28</td>
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<td></td>
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<td>NP = 15257.04</td>
<td>NP = 15275.19</td>
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<tr>
<td>35/365</td>
<td>( Ic_2 = 20% )</td>
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<td>T = 0.3321</td>
<td>T = 0.3318</td>
</tr>
<tr>
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<td>P = 60.54</td>
<td>P = 60.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q = 131.59</td>
<td>Q = 131.48</td>
<td>Q = 131.46</td>
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<tr>
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<td></td>
<td>R = 394.38</td>
<td>R = 394.63</td>
<td>R = 394.87</td>
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<td></td>
<td>NP = 15293.16</td>
<td>NP = 15313.44</td>
<td>NP = 15333.11</td>
</tr>
<tr>
<td>40/365</td>
<td>( Ic_2 = 22% )</td>
<td>T = 0.3630</td>
<td>T = 0.3624</td>
<td>T = 0.3620</td>
</tr>
<tr>
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<td>P = 60.54</td>
<td>P = 60.51</td>
<td>P = 60.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q = 143.78</td>
<td>Q = 143.63</td>
<td>Q = 143.57</td>
</tr>
<tr>
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<td>R = 394.64</td>
<td>R = 394.90</td>
<td>R = 395.16</td>
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<td>NP = 15338.30</td>
<td>NP = 15359.17</td>
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Table 4.2.5.3: Optimal decision variables when $\theta = 0.03$

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<th>25/365</th>
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</thead>
<tbody>
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<td>$I_c = 15%$</td>
<td>$I_c = 16%$</td>
<td>$I_c = 17%$</td>
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<tr>
<td>30/365</td>
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<td>$R = 393.98$</td>
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<td>$R = 394.45$</td>
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<td>$NP = 15226.61$</td>
<td>$NP = 15245.93$</td>
<td>$NP = 15263.08$</td>
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<tr>
<td>35/365</td>
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<td>$T = 0.3314$</td>
<td>$T = 0.3312$</td>
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<tr>
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<td>$P = 60.57$</td>
<td>$P = 60.54$</td>
<td>$P = 60.52$</td>
</tr>
<tr>
<td></td>
<td>$Q = 131.53$</td>
<td>$Q = 131.42$</td>
<td>$Q = 131.40$</td>
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<tr>
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<td>$R = 394.31$</td>
<td>$R = 394.57$</td>
<td>$R = 394.81$</td>
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<td>$T = 0.3618$</td>
<td>$T = 0.3614$</td>
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<td>$P = 60.54$</td>
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</tr>
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<td>$R = 394.57$</td>
<td>$R = 394.83$</td>
<td>$R = 395.04$</td>
</tr>
<tr>
<td></td>
<td>$NP = 15324.87$</td>
<td>$NP = 15345.76$</td>
<td>$NP = 15366.50$</td>
</tr>
</tbody>
</table>

Observations:

- Increase in first allowable credit period decreasing the order quantity and increases net profit whereas selling price is insensitive.
- Increase in extended permissible credit period lowers cycle time and selling price Net profit decreases significantly.
- Increase in deterioration rate reduces cycle time and net profit whereas selling price leisurely goes up.

4.3 Conclusion:

In this chapter, two models are discussed. In section 4.1, optimal pricing and ordering policy is derived for the retailer when the supplier offers two progressive credit periods. The demand is assumed to be decreasing function of selling price of unit Net profit increases for increase in the first allowable credit period and extended credit period. In the section 4.2, constant deterioration of units in inventory is assumed. The increase in deterioration of units decreases net profit significantly. The model developed in section 4.2 reduces to that in section 4.1 if $\theta \to 0$ i.e. there is no deterioration of units in an inventory system.