CHAPTER 3

EOQ MODEL FOR PROGRESSIVE TRADE CREDIT

SCHEME UNDER DCF – APPROACH
3.0 Introduction:

In this chapter, we develop present value formulation of two EOQ models for progressive trade credit scheme i.e. when supplier facilitates his retailers with two credit periods for settling the account. The theoretical results are illustrated with hypothetical numerical example. The effect of allowable credit periods on present value of all future cash-out-flows and optimum cycle time are studied. In the second section of the chapter, a present value formulation of lot-size inventory model with constant rate of deterioration under progressive payment scheme is formulated. The effects of deterioration of units, inflation rate and ordering cost on optimal procurement quantity and the present value of all future cash-out-flows are studied numerically.

The two models are formulated under following sections (viz)

Section 3.1 – An EOQ model for progressive payment scheme under DCF approach.

Section 3.2 – An EOQ Model for Deteriorating Items with Progressive Payment Scheme under DCF Approach

3.1 An EOQ model for progressive payment scheme under DCF approach:

In this section, an EOQ model is developed for progressive payment scheme under DCF approach. This study deals with mathematical derivation when supplier offers two progressive credit periods to the retailer to settle the accounts. The objective function to be optimized is considered as present value of all future cash-out-flows. The effect of various parameters on objective function is studied analytically. A flow chart is given to explore the computational flow.
3.1.1 Assumptions and Notations:

The following additional assumptions and notations other than those given in A.1 and N.1 are used to formulate the proposed model:

The inflation rate, \( r \) (\( 0 < r < 1 \)) is constant during the period under review.

\( r = \) the discounting rate/time unit (\( 0 < r < 1 \))

\( PV(T) = \) Present value of cash–out–flows/cycle.

\( PV_\infty(T) = \) Present value of all future cash–out–flows.

\( T \) is a decision variable.

3.1.2 Mathematical Formulation:

The on–hand inventory \( Q(t), 0 < t < T \) depletes due to constant demand \( R \). Hence, the instantaneous state of inventory \( Q(t) \), at any instant of time \( t, 0 < t < T \) is governed by the differential equation

\[
\frac{dQ(t)}{dt} = -R, \; 0 \leq t \leq T
\]

(3.1.2.1)

with the initial condition \( Q(0) = Q \) and boundary condition \( Q(T) = 0 \).

Consequently, the solution of (3.1.2.1) is given by

\[
Q(t) = R(T - t), \; 0 \leq t \leq T
\]

(3.1.2.2)

and the order quantity is \( Q = RT \).

Let us compute various cost components:

• Ordering cost; \( OC = A \)  
  (3.1.2.3)

• Purchase cost; \( PC = CRT \)  
  (3.1.2.4)

• Inventory holding cost:

\[
IHC = h \int_0^T Q(t) e^{-rt} dt = \frac{hR}{r^2} \left( e^{-rT} + rT - 1 \right)
\]

(3.1.2.5)

Regarding interest charged and earned, based on the length of the cycle time \( T \), three cases arise:
Case 1. $T \leq M$

Case 2. $M < T < N$

Case 3. $T \geq N$

We discuss each case in detail.

**Case 1: $T \leq M$ (Fig. 3.1.2.1)**

Here, the retailer sells $Q$-units during $[0, T]$ and is paying for $CRT$-units (in full) to the supplier at time $M \geq T$. Therefore, interest charges are zero. i.e.

$$ IC_1 = 0 $$ \hfill (3.1.2.6)

The retailer sells products during $[0, T]$ and deposits the revenue in an interest bearing account at the rate of $\i$ $\$/year. In the period $[T, M]$, the retailer deposits revenue into the account that earns interest at the rate $\i$ $\$/year. Therefore, interest earned per year is

$$ IE_1 = Pte \left( \int_0^T Rte^{-rt} dt + RT(M - T)e^{-r(M-T)} \right) $$

$$ = Pte \left[ \frac{1}{r^2} \left( 1 - (1 + rT)e^{-rT} \right) + T(M - T)e^{-r(M-T)} \right] $$ \hfill (3.1.2.7)
Using (3.1.2.3) – (3.1.2.7), the present value of cash–out–flow for one cycle is

\[ PV_1(T) = OC + PC + IHC + IC_1 - IE_1 \]

The present value of all future cash–out–flows is given by

\[ PV_\infty(T) = \sum_{n=0}^{\infty} PV_1(T) e^{-nrT} = \frac{PV_1(T)}{1 - e^{-rT}} \]

Since \( r < 1 \), \( rT < 1 \), using series expansion of exponential series (ignoring higher powers of \( rT \)), we get

\[ \frac{1}{1 - e^{-rT}} = \frac{1}{rT} \left(1 - \frac{rT}{2} \right)^{-1} = \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \]

Then \( PV_\infty(T) = \left(1 + \frac{1}{2} + \frac{rT}{4}\right)PV_1(T) \) \hspace{1cm} (3.1.2.8)

The optimal value of \( T \) (say \( T_1 \)) can be obtained by solving equation

\[ \frac{\partial PV_\infty(T)}{\partial T} = \left\{ CR + \frac{hR}{r} \left(1 - e^{-rT}\right) - \left(Tr e^{-rT} + (M - 2T + T(M - T)r)\right) \right\} \times PeRe^{-r(M - T)} \left(\frac{1}{rT} + \frac{1}{2} + \frac{rT}{4}\right) + \left(\frac{r}{4} - \frac{1}{rT^2}\right) PV_1(T) = 0 \] \hspace{1cm} (3.1.2.9)

The obtained \( T = T_1 \) minimizes present value of future cash–out flows because

\[ \frac{\partial^2 PV_\infty(T)}{\partial T^2} = \left\{ hRe^{-rT} - PeR((1-rT) - e^{-r(M - T)}) \times(2M - 4rT + r^2T(M - T) - 2))\right\} + 2\left\{ CR + \frac{hR(1-e^{-rT})}{r} \right\} \]

\[ - Pe R(e^{-rT}T + e^{-r(M - T)}((M - T) - T + T(M - T)r)) \]

\[ \times \left(\frac{r}{4} - \frac{1}{rT^2}\right) + \frac{2PV_1(T)}{rT^3} \]

\[ > 0, \forall T \] \hspace{1cm} (3.1.2.10)
Case 2: \( M < T < N \) (Fig 3.1.2.2)

The retailer sells units and deposits the revenue into an interest bearing account at an interest rate \( i \) per unit/year during \([0, M]\). Hence the interest earned, \( IE_{2,1} \) during \([0, M]\) is

\[
I E_{2,1} = P e \int_{0}^{M} R e^{-r t} dt = P e R \left( 1 - (1 + r M) e^{-r M} \right) \tag{3.1.2.11}
\]

Buyer has to pay for \( Q \)-units purchased at time \( t = 0 \) at the rate of \( C \$ \)/unit to the supplier during \([0, M]\), the retailer sells \( RM \)-units at sale price \( P \$/unit. So he has generated revenue of \( PRMe^{-r M} \) plus the interest earned, \( IE_{2,1} \), during \([0, M]\). Two sub-cases may arise:

Sub-case 2.1: Let \( PRMe^{-r M} + IE_{2,1} \geq CQ \) i.e. the retailer has enough money to pay for all \( Q \)-units procured. Then, interest charges, \( IC_{2,1} = 0 \) \( \tag{3.1.2.12} \)

and the interest earned is given by \( IE_{2,1} \).

Using equations (3.1.2.3) to (3.1.2.5), (3.1.2.11) and (3.1.2.12), the present value of all cash-out-flows per cycle is given by
\[ PV_{2,1}(T) = OC + PC + IHC + IC_{2,1} - IE_{2,1} \]

Arguing as in case 1, the present value of all future cash-out-flows is given by

\[ PV_{2,1}\infty(T) = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) PV_{2,1}(T) \tag{3.1.2.13} \]

The optimum value of \( T = T_{2,1} \) is the solution of non-linear equation

\[
\frac{\partial PV_{2,1}\infty(T)}{\partial T} = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) \left\{ CR + \frac{hR}{r} \left( 1 - e^{-rT} \right) \right\} \\
+ \left( \frac{r}{4} - \frac{1}{rT^2} \right) PV_{2,1}(T) = 0 \tag{3.1.2.14} \]

The obtained \( T = T_{2,1} \) minimizes present value of all future cash-out-flows because

\[
\frac{\partial^2 PV_{2,1}\infty(T)}{\partial T^2} = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) + 2 \left[ CR + \frac{hR(1 - e^{-rT})}{r^2} \right] \left( \frac{r}{4} - \frac{1}{rT^2} \right) \\
+ \frac{2PV_{2,1}(T)}{rT^3} > 0, \forall T \tag{3.1.2.15} \]

**Sub-case 2.2:** \( PR Me^{-rM} + IE_2 < CQ \)

Here, retailer will have to pay interest on the un-paid balance

\[ U_1 = CQ - \left[ PR Me^{-rM} + IE_2 \right] \] at rate of \( Ic_1 \) at time \( M \) to the supplier. The interest to be paid, \( IC_{2,2} \), is:

\[
IC_{2,2} = \frac{U_1^2}{PR} Ic_1 \int_M^T Q(t)e^{-rt} \, dt \\
= \frac{U_1^2}{PR} Ic_1 \frac{1}{r^2} \left\{ e^{-rT} + e^{-rM} \left( rT - rM - 1 \right) \right\} \tag{3.1.2.16} \]
Equations (3.1.2.3) to (3.1.2.5), (3.1.2.11) and (3.1.2.16) give the present value of all cash–out–flows per cycle as

\[ PV_{2.2}(T) = OC + PC + IHC + IC_{2.2} - IE_{2.1} \]

The present value of all future cash–out–flows is given by

\[ PV_{2.2\infty}(T) = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) PV_{2.2}(T) \tag{3.1.2.17} \]

The optimum value of \( T = T_{2.2} \) is the solution of non–linear equation

\[
\frac{\partial PV_{2.2\infty}(T)}{\partial T} = \left\{ CR + \frac{hR}{r} \left( 1 - e^{-rT} \right) - \frac{2U_1 Ic_1}{p r^2} \left( e^{-rT} + e^{-M(rT-rM-1)} \right) \right.
\]

\[
+ \frac{U_1 Ic_1}{PRr} \left( e^{-rM} - e^{-T} \right) \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) \]

\[
\left. + \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) PV_{2.2}(T) = 0 \right\} \tag{3.1.2.18}
\]

can be solved by suitable numerical method for \( T = T_{2.2} \) minimizes present value of all future cash–out–flows as

\[
\frac{\partial^2 PV_{2.2\infty}(T)}{\partial T^2} = \left\{ hRe^{-rT} - 2C^2 R%1 - 4U_1 Ic_1 %2 + \frac{U_1^2 Ic_1 e^{-rT}}{PR} \right\}
\]

\[
\times \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) + 2 \left\{ CR + \frac{hR(1-e^{-rT})}{r} - 2U_1 C%1 - U_1^2 %2 \right\}
\]

\[
\times \left( \frac{r}{4} - \frac{1}{rT^2} \right) + \frac{2PV_{2.2}(T)}{rT^3} > 0, \ \forall \ T \tag{3.1.2.19}
\]

where

\[
%1 = \frac{Ic_1 (-rTe^{-rM} + rMe^{-rM} + e^{-rM} - e^{-rT})}{r^2 p}
\]

\[
%2 = \frac{Ic_1 (-e^{-rM} + e^{-rT})}{rp}
\]
Case 3: $T \geq N$ (Fig. 3.1 2.3)

Based on the total purchase cost, $CQ$, total money $PRMe^{-rM} + IE_{2,1}$ in account at $M$ and total money, $PRNe^{-rN} + IE_{2,1}$ in account at $N$, three sub-cases may arise:

Sub-case 3.1: Let $PRMe^{-rM} + IE_{2,1} \geq CQ$ Then this sub-case is same as sub-case 2.1 (Note. Decision variables and objective function are designated by 3.1)

Sub-case 3.2: Let $PRMe^{-rM} + IE_{2,1} < CQ$ but

$$
PR(N-M)e^{-r(N-M)} + Pte^{\int_{M}^{N}Rt^{-r}dt} \geq CQ - \left(PRMe^{-rM} + IE_{2,1}\right)
$$

This sub-case coincides with sub-case 3.2. (Note Decision variables and objective function are designated by 3.2)
Sub-case 3.3: Let \( PRM e^{-rM} + IE_2 < CQ \) but

\[
P R(N - M) e^{-r(N - M)} + P l e \int_N^M R t e^{-r t} d t < C Q - \left( P R M e^{-rM} + IE_2 \right)
\]

Here, the retailer does not have enough money in his account to pay off for total purchase cost at \( N \). He will do payment of \( P R M e^{-rM} + IE_2 \) at \( M \) and \( PR(N - M) e^{-r(N - M)} + P l e \int_M^N R t e^{-r t} d t \) at \( N \). So, he has to pay interest charges on un-paid balance \( U_1 = C Q - \left( P R M e^{-rM} + IE_2 \right) \) with interest rate \( Ic_1 \) during \( [M, N] \) and \( U_2 = U_1 - \left[ PR(N - M) e^{-r(N - M)} + P l e \int_M^N R t e^{-r t} d t \right] \) with interest rate \( Ic_2 \) during \([N, T]\).

Therefore, total interest charges, \( IC_{3,3} \), is

\[
IC_{3,3} = U_1 Ic_1(N - M) + \frac{U_2^2}{PR} Ic_2 \int_N^T Q(t) e^{-r t} d t
\]

\[
= U_1 Ic_1(N - M) + \frac{U_2^2}{PR} Ic_2 \frac{1}{r^2} \left( e^{-r T} + e^{-r N} (r T - r N - 1) \right)
\]

(3.1.2.20)

Using equation (3.1.2.3) to (3.1.2.5), (3.1.2.11) and (3.1.2.20), the present value of all cash-out-flows per cycle is

\[
PV_{3,3} (T) = OC + PC + IHC + IC_{3,3} - IE_{21}
\]

The present value of all future cash-out-flows is given by

\[
PV_{3,3\infty} (T) = \left( \frac{1}{r T} + \frac{1}{2} \frac{r T}{4} \right) PV_{3,3} (T)
\]

(3.1.2.21)

The optimal value of \( T = T_{3,3} \) is the solution of non-linear equation
\[
\frac{\partial PV_{3,3\infty}(T)}{\partial T} = \left\{CR + \frac{hR}{r} \left(1 - e^{-rT}\right) + CRl_1(N-M) + \frac{2U_2C_2l_2}{P r^2} \right.
\]
\[
\times \left(e^{-rT} + e^{-rN(rT-rN-1)} + \frac{U_2^2l_2}{P R} \left(e^{-rM} - e^{-rT}\right)\right)
\]
\[
+ \left(\frac{1}{rT} + \frac{1}{2} + \frac{rT}{4}\right)PV_{3,3}(T) = 0
\]  

(3.1.2.22)

The sufficiency condition for \( PV_{3,3\infty}(T) \) is

\[
\frac{\partial^2 PV_{3,3\infty}(T)}{\partial T^2} = \left\{hR e^{-rT} - 2C^2R%1 - \frac{4U_2l_2C \left(e^{-rN} - e^{-rT}\right)}{rP} \right.
\]
\[
+ \frac{U_2^2l_2e^{-rT}}{PR} \times \left(\frac{1}{rT} + \frac{1}{2} + \frac{rT}{4}\right)
\]
\[
+ 2\left\{CR + \frac{hR(l-1-e^{-rT})}{r} + CRl_1(N-M) + 2U_2C_2l_2%1 \right.
\]
\[
- \frac{U_2l_2\left(e^{-rN} - e^{-rT}\right)}{rPR} \right\} \left(\frac{r}{4} - \frac{1}{rT^2}\right) + \frac{2PV_{3,3}(T)}{rT^3}
\]
\[
> 0, \ \forall T
\]  

(3.1.2.23)

where \( %1 = \frac{-rTe^{-rN} + rNe^{-rN} + e^{-rN} - e^{-rT}}{r^2P} \)

In the next section, we present computational flowchart to search for optimal solution.

3.1.3 Flowchart:
3.1.4 Theoretical Results:

Proposition 3.1.4.1: $PV_{i\infty}$ is minimum for $i = 1, 2.1, 2.2, 3.1, 3.2$, and $3.3$.

Proof: \[
\frac{\partial^2 PV_{i\infty}(T)}{\partial T^2}
\]
given by equation (3.1.2.10), (3.1.2.15), (3.1.2.19), (3.1.2.23) is non-negative for obtained $T$. 
Proposition 3.1.4.2: For $T > N$, $PV_{3,3\infty}(T)$ is decreasing function of $M$ and increasing function of $N$.

Proof:

\[
\frac{\partial PV_{3,3\infty}(T)}{\partial M} = -\left\{ PRe^{-r} M (1 - rM - le R M) Ic_1 \times (N - M) + U_1 Ic_1
\right.
\]
\[
+ 2 U_2 \frac{1}{rT} \left( \frac{1}{2} + \frac{rT}{4} \right)
\]
\[
\times \left[ PRe^{-r} M (1-rM) + PRe^{-r} (N-M)(1-r(N-M)) \right]
\]
\[
< 0, \forall T
\]

and

\[
\frac{\partial PV_{3,3\infty}(T)}{\partial N} = \left\{ U_1 Ic_1 + 2 U_2 \frac{1}{rT} \left[ PRe^{-r} (N-M) \right]
\right.
\]
\[
\times (r(N-M)-1) - P le R e^{-r} N I N
\]
\[
+ U_2 Ic_2 \frac{e^{-r} N (N-1)}{PR} \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right)
\]
\[
> 0, \forall T
\]

where $%1 = \frac{Ic_2(e^{-r} N (rT-rN-1) + e^{-r} T)}{r^2 PR}$
Proposition 3.1.4.3: For $T > N$, $P_{V_{3,3\infty}}(T)$ is decreasing function of inflation rate $r$.

Proof:

$$\frac{\partial P_{V_{3,3\infty}}(T)}{\partial r} = \left\{ \begin{array}{l}
hR\left(T-e^{-rT}T\right) - \frac{2hR(rT-1+e^{-rT})}{r^2} + \%3Ic_1(N-M) \\
+ 2U_2\%2\times[\%3 - PR(M-N)(N-M)e^{-r(N-M)}] - \frac{(rM-2)}{r^2}(P_{IeRM}e^{-rM}) + \frac{(rN-2)}{r^2}(P_{IeRNe}^{-rN})
\end{array} \right\}$$

$$+ 2P_{IeR}(e^{-rM}e^{-rN}) \left\{ \begin{array}{l}
\frac{U_2^2}{PR^2} - \frac{2U_2^2}{PR^3} \\
x(e^{-rN}(-rNT + T + rN^2) - e^{-rT}) - \%4
\end{array} \right\}$$

$$\times \left( \frac{1}{rT^2} + \frac{1}{2} + \frac{rT}{4} \right)$$

$$+ \left\{ A + CRT + \frac{hR(rT-1+e^{-rT})}{r^2} + U_1Ic_1(N-M) \\
+ \%2^2\%3 + P_{IeR}(-1 + e^{-rM}rM + e^{-rM}) \right\}$$

$$\times \left( \frac{T}{4} - \frac{1}{r^2T} \right)$$

$< 0, \ \forall \ T > N$

where $\%2 = \frac{Ic_2(e^{-rN}(Tr - rN - 1) + e^{-rT})}{PRr^2}$

$\%3 = PRM^2e^{-rM} - \%4$

$\%4 = \frac{P_{IeRM}e^{-rM}}{r} + \frac{2P_{IeR}(-1 + e^{-rM}rM + e^{-rM})}{r^3}$
3.1.5 Numerical Example and observations:

Consider following parametric values in appropriate units.

\[ [R, h, A, P, C, Ie, r] = [1000, 0.2, 100, 30, 20, 0.08, 0.03] \]

Table 3.1.5.1

Variations in \( M \) and \( N \)

<table>
<thead>
<tr>
<th>( M )</th>
<th>( N )</th>
<th>( Ic_2 )</th>
<th>( Ic_1 = 15% )</th>
<th>( Ic_1 = 16% )</th>
<th>( Ic_1 = 17% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15/365</td>
<td>18%</td>
<td>( T_{32} = 0.06158 )</td>
<td>( T_{33} = 0.08224 )</td>
<td>( T_{33} = 0.10281 )</td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td>( Q = 61.58 )</td>
<td>( Q = 82.24 )</td>
<td>( Q = 102.81 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td>( PV_{3,\infty}(T_{3,2}) = 720575 )</td>
<td>( PV_{3,\infty}(T_{3,3}) = 706887 )</td>
<td>( PV_{3,\infty}(T_{3,3}) = 698687 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20/365</td>
<td>20%</td>
<td>( T_{32} = 0.06134 )</td>
<td>( T_{33} = 0.08218 )</td>
<td>( T_{33} = 0.10279 )</td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td>( Q = 61.34 )</td>
<td>( Q = 82.18 )</td>
<td>( Q = 102.79 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td>( PV_{3,\infty}(T_{3,2}) = 720780 )</td>
<td>( PV_{3,\infty}(T_{3,3}) = 706910.38 )</td>
<td>( PV_{3,\infty}(T_{3,3}) = 698689 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25/365</td>
<td>22%</td>
<td>( T_{32} = 0.06083 )</td>
<td>( T_{33} = 0.08198 )</td>
<td>( T_{33} = 0.10278 )</td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td>( Q = 60.83 )</td>
<td>( Q = 81.98 )</td>
<td>( Q = 102.78 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp;</td>
<td>( PV_{3,\infty}(T_{3,2}) = 727217 )</td>
<td>( PV_{3,\infty}(T_{3,3}) = 707004.00 )</td>
<td>( PV_{3,\infty}(T_{3,3}) = 698693 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations:

- Increase in first allowable credit period increases the order quantity to be procured and the replenishment cycle. It decreases the present value of future costs.
- Increase in extended permissible trade credit increase the present value of future costs.

Consider following parametric values in appropriate units

\[ [R, h, A, P, C, Ic_1, Ic_2, Ie, M, N] = [1000, 0.2, 100, 30, 20, 16\%, 18\%, 8\%, 20/365, 30/365] \]
Table 3.1.5.2
Variation in $r$

<table>
<thead>
<tr>
<th>$r$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$PV_{3,3\infty}(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.030</td>
<td>0.08240</td>
<td>82.24</td>
<td>706887.00</td>
</tr>
<tr>
<td>0.035</td>
<td>0.08221</td>
<td>82.21</td>
<td>606038.70</td>
</tr>
<tr>
<td>0.040</td>
<td>0.08219</td>
<td>82.19</td>
<td>530400.40</td>
</tr>
</tbody>
</table>

**Observation:**

- As inflation rate increases, optimum order quantity, replenishment cycle and present value of future cost decreases.

**3.2 An EOQ Model for Deteriorating Items with Progressive Payment Scheme under DCF Approach:**

In this section, a mathematical model is developed when units in inventory are subject to constant rate of deterioration when supplier offers his retailers two progressive credit periods to settle the account of the procured goods.

**3.2.1 Assumptions and Notations:**

An EOQ Model for deteriorating items with progressive payment scheme under DCF approach is developed under the same assumptions and notations given in A.1, N.1 and section 3.1.1.

**3.2.2 Mathematical Formulation:**

The depletion of inventory $Q(t)$, $0 \leq t \leq T$ occurs due to combined effect of the demand and deterioration in the time interval $[0, T]$. The differential equation governing instantaneous state of $Q(t)$ at some instant of time $t$, $0 \leq t \leq T$ is given by

$$\frac{dQ(t)}{dt} + \theta Q(t) = -R, \quad 0 \leq t \leq T \quad (3.2.2.1)$$

with the initial condition $Q(0) = Q$ and boundary condition $Q(T) = 0$. 
The solution of (3.2.2) is given by

\[ Q(t) = \frac{R}{\theta} \left( e^{\theta(T-t)} - 1 \right) \]  

(3.2.2.2)

and the order quantity is

\[ Q = \frac{R}{\theta} \left( e^{\theta T} - 1 \right) \]

The components of total inventory cost of the system per cycle time are as follows:

- Ordering cost; \( OC = A \)  

(3.2.2.3)

- Procurement Cost; \( PC = CQ = \frac{C R}{\theta} \left( e^{\theta T} - 1 \right) \)  

(3.2.2.4)

- Inventory holding cost;

\[
IHC = h \int_{0}^{T} Q(t) e^{-r t} dt = \frac{h R}{r(r+\theta)} \left\{ e^{-r t} - \frac{1}{\theta} \left( \theta + r \left( 1 - e^{\theta T} \right) \right) \right\} 
\]

(3.2.2.5)

Computation of interest charged and interest earned depends on the length of the cycle time \( T \). There are three possibilities, which are discussed in detail.

**Case 1:** \( T \leq M \)

**Case 2:** \( M < T < N \)

**Case 3:** \( T \geq N \)
**Case 1: \( T \leq M \) (Fig. 3 2 2.1)**

Here, retailer sells \( Q \) units in cycle time \( T \) and will have to pay \( CQ \) to the supplier in full at time \( M \) but \( M \geq T \). So interest charges are zero, i.e.

\[
IC_1 = 0
\]

(3.2.2.6)

During \([0, T]\), the retailer sells products at selling price \( P \)/unit and deposits the revenue into interest earning account at the rate of \( \ell e$/year. In the period \([T, M]\), the retailer deposits only the total revenue into an account that earns interest at the rate \( \ell e$/year. Hence, interest earned for one cycle is

\[
IE_1 = P \int_0^T e^{-rt} dt + RT(M-T) e^{-r(M-T)}
\]

\[
= P \ell eR \left( \frac{1}{r^2} \left( 1 - e^{-rT} (rT +1) \right) + T(M-T) e^{-r(M-T)} \right)
\]

(3.2.2.7)

Using (3.2.2.2) – (3.2.2.7), the present value of cash–out–flow for one cycle is

\[
PV_1(T) = OC + PC + IHC + IC_1 - IE_1
\]

The present value of all future cash–out–flows is given by

\[
PV_{\infty}(T) = \sum_{n=0}^{\infty} PV_1(T) e^{-nrT} = \frac{PV_1(T)}{1 - e^{-rT}}
\]
\[ PV_{1\infty} (T) = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) PV_1 (T) \quad (3.2.2.8) \]

\[ \frac{\partial PV_{1\infty} (T)}{\partial T} = \left\{ \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right\} \left[ CRe^{\theta T} + \frac{hR}{r+\theta} \left( e^{\theta T} - e^{-rT} \right) \right. \]

\[ \left. \qquad \quad - \left[ \frac{1}{r} \left( rTe^{-rT} \right) - (M-T)e^{-r(M-T)} \right] e^{-r(M-T)} \right\] \]

\[ + rT(M-T)e^{-r(M-T)} \right] \]

\[ \times PeR + \left( \frac{r}{4} - \frac{1}{rT^2} \right) PV_1 (T) = 0 \quad (3.2.2.9) \]

provided that it satisfies the sufficiency condition at \( T = T_1 \)

\[ \frac{\partial^2 PV_{1\infty} (T)}{\partial T^2} = (h-PeR) \left\{ \left[ CRe^{\theta T} - \%1 + \%2 - \%3 \right] \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) \right. \]

\[ \left. + 2 \left[ CRe^{\theta T} - \%2 \right] \left( \frac{r}{4} - \frac{1}{rT^2} \right) \right\} \}

\[ + PeRe^{-r(M-T)} \left( \frac{rT}{4} - \frac{1}{rT^2} \right) \]

\[ - PeRe^{-r(M-T)}(M-2T+rT) \left( \frac{1}{T} + \frac{3r}{4} + \frac{r^2T}{4} + \frac{1}{rT^2} \right) \]

\[ + \frac{2}{rT^2} PV_1 (T) \]

\[ > 0 \quad \forall T \quad (3.2.2.10) \]
Case 2: $M < T < N$ (Fig 3.2.2.2)

Here, interest earned, $IE_2$, during $[0, M]$ is

$$IE_2 = P \int_0^M R t e^{-rt} \, dt = \frac{P \int e R}{r^2} \left[ 1 - (1 + rM) e^{-rM} \right]$$

Buyer has to pay the procurement cost $CQ$ of $Q$-units at time $t = M$ to the supplier. Up to time $M$, the retailer sells $RM$-units and has the revenue of $PRMe^{-rM}$ plus interest earned $IE_2$ to pay the supplier. Depending on the difference between the total procurement cost $CQ$ and the total revenue $PRMe^{-rM} + IE_2$, two sub-cases are possible.

Sub-case 2.1: $PRMe^{-rM} + IE_2 \geq CQ$
Sub-case 2.2: \( P R Me^{-rM} + IE_2 < CQ \)

Let us discuss both the sub-cases in details.

Sub-case 2.1: \( P R Me^{-rM} + IE_2 \geq CQ \)

Here, the retailer has sufficient amount in his account to pay off the total purchase cost at \( M \).

Therefore, Interest charges, \( IC_{2.1} = 0 \) \hspace{1cm} (3.2.2.11)

The present value of cash-out-flow for one cycle for this sub-case is

\[
PV_{2.1}(T) = OC + PC + IHIC + IC_{2.1} - IE_2
\]

The present value of all future cash-out-flows is given by

\[
PV_{2.1}(T) = \left( \frac{1}{r} \right)^r \left( e^{\theta T} - e^{-r T} \right) \]

The optimum value of \( T = T_{2.1} \) is the solution of non-linear equation

\[
\frac{\partial PV_{2.1}(T)}{\partial T} = 0
\]

provided that it satisfies the sufficiency condition \( \frac{\partial^2 PV_{2.1}(T)}{\partial T^2} > 0 \) at \( T = T_{2.1} \).

Sub-case 2.2: \( P R Me^{-rM} + IE_2 < CQ \)

In this sub-case, the retailer does not have sufficient money in his account to make the payment at permissible credit time \( M \). Then, supplier charges retailer on the un-paid balance \( U_1 = CQ - \left[ P R Me^{-rM} + IE_2 \right] \) at the interest rate \( Ic_1 \) after time \( M \). Therefore, Interest charges payable; \( IC_{2.2} \) is
The present value of cash-out-flow for one cycle for this sub-case is

\[ PV_{22} (T) = OC + PC + IHC + IC_{22} - IE_2 \]

The present value of all future cash-out-flows is given by

\[ \therefore PV_{22 \infty} (T) = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) PV_{22} (T) \]

The optimum value of \( T = T_{22} \) is the solution of non-linear equation

\[
\frac{\partial PV_{22 \infty} (T)}{\partial T} = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) \left\{ CR e^{\theta T} + \frac{hR}{r + \theta} \left( e^{\theta T} - e^{-rT} \right) \right. \\
+ \frac{2U_1 Ic_1}{\theta P r (\theta + r)} \left[ -e^{-rM (r + \theta - e^{\theta (T - M)})} + e^{-rT} \right] \\
\left. + \frac{U_1^2 Ic_1}{P (\theta + r)} \left[ -e^{-rT} + e^{-rM + \theta (T - M)} \right] \right\} \\
+ \left( \frac{r}{4} - \frac{1}{rT^2} \right) PV_{22} (T) = 0
\]

provided that it satisfies the sufficiency condition at \( T = T_{22} \)
\[
\frac{\partial^2 PV_{2,2\infty}(T)}{\partial T^2} = \left\{ CR e^{\theta T} - \%1 + \%2 - \%3 + \frac{1}{Pr(r + \theta)\theta} [2CR^2 R^2 Ic_1 \%4 \right.
\]

\[
+ 4 U_1 Ic_1 \%5 CR - 4 U_1 Ic_1 \%4 CR + U_1^2 Ic_1 \%6
\]

\[
- \frac{1}{Pr(r + \theta)\theta} [2U_1^2 Ic_1 \%5 - U_1^2 Ic_1 \%4]) \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right)
\]

\[
+ 2 \left\{ CR e^{\theta T} - \%2 + \%3 + \frac{1}{Pr(r + \theta)\theta} [2 U_1 Ic_1 \%4 CR
\]

\[
+ U_1^2 Ic_1 \%5] - \frac{1}{Pr(r + \theta)\theta} U_1^2 Ic_1 \%4 \right) \left( \frac{r}{4} - \frac{1}{rT^2} \right) + \frac{2PV_{2\infty}(T)}{rT^3}
\]

\[
> 0, \ \forall T \quad (3.2.2.17)
\]

where

\[
\%1 = \frac{hR \left[ -\left( -\theta - \theta \right)^2 e \left[ (r + \theta)T_r + e^{rT} r^2 + e^{rT} r_\theta \right] e^{-rT} \right]}{(r + \theta) \theta r}
\]

\[
\%2 = \frac{2hR \left[ -\left( -\theta \right) e \left[ (r + \theta)T_r + e^{rT} r^2 + e^{rT} r_\theta \right] e^{-rT} \right]}{(r + \theta) \theta}
\]

\[
\%3 = \frac{hR \left[ -e \left( r + \theta \right) T_r + e^{rT} r_\theta - e^{rT} r \right] e^{-rT}}{(r + \theta) \theta}
\]

\[
\%4 = \left( -\theta e^{rM} - e \left( rT + \theta T - \theta M \right) r + e^{rT} r + e^{rT} \theta \right) e^{-\left( rT - \theta M \right)}
\]

\[
\%5 = \left( -\left( r + \theta \right) e \left[ (r + \theta T - \theta M) r + e^{rT} r^2 + e^{rT} r_\theta \right] e^{-\left( rT - \theta M \right)} \right)
\]

\[
\%6 = \left( -\left( -\theta - \theta \right)^2 e \left[ (r + \theta T - \theta M) r + e^{rT} r^2 + e^{rT} r_\theta \right] e^{-\left( T + M \right)} \right)
\]
Case 3: \( T \geq N \) (Fig. 3.2.2.3)

![Inventory Level vs Time Graph](image)

\( \text{Figure 3.2.2.3} \)

Depending upon the procurement cost \( CQ \), total amount available to retailer at \( M \) is \( PRM e^{-rM} + IE_2 \) and total amount available to the retailer at \( N \) is \( PRN e^{-rN} + IE_2 \); following three sub-cases are possible

**Sub-case 3.1:** \( PRM e^{-rM} + IE_2 \geq CQ \)

This sub-case is same as sub-case 2.1.

The present value of all future cash-out-flows is given by
\[
\therefore PV_{3,1\infty}(T) = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) PV_{2,1}(T) \tag{3.2.2.18}
\]

The optimum value of \( T = T_{3,1} \) is the solution of non-linear equation
\[
\frac{\partial PV_{3,1\infty}(T)}{\partial T} = \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) \left\{ C R e^{\theta T} + \frac{hR}{r+\theta} \left( e^{\theta T} - e^{-rT} \right) \right\} + \left( \frac{r}{4} - \frac{1}{rT^2} \right) PV_{2,1}(T) = 0 \tag{3.2.2.19}
\]

provided that it satisfies the sufficiency condition \( \frac{\partial^2 PV_{3,1\infty}(T)}{\partial T^2} > 0 \) at \( T = T_{3,1} \).
Sub-case 3.2: Let $PRM e^{-rM} + IE_2 < CQ$ but

$$PR(N-M) e^{-r(N-M)} + Pl e \int_M^N Rte^{-r t} dt \geq CQ \left( PRM e^{-rM} + IE_2 \right)$$

In this case, the retailer does not have enough money at $M$ to settle the account of supplier completely, but he can pay the balance on or before $N$.

Hence, the retailer makes the payment $PRM e^{-rM} + IE_2$ at $M$ and will pay the interest charges at the rate of $Ic_1$ on the unpaid balance

$$U_1 = CQ - \left[ PRM e^{-rM} + IE_2 \right]$$

This case is similar to sub-case 2.2.

The present value of all future cash-out-flows is given by

$$\therefore PV_{3.2\infty}(T) = \left( \frac{1}{rT} + \frac{1}{2} \right) PV_{2.2}(T) \quad (3.2.2\; 20)$$

The optimum value of $T = T_{3.2}$ is the solution of non-linear equation

$$\frac{\partial PV_{3.2\infty}(T)}{\partial T} = 0 \quad (3.2.2\; 21)$$

provided that it satisfies the sufficiency condition at $\frac{\partial^2 PV_{3.2\infty}(T)}{\partial T^2} > 0$.

$T = T_{3.2}$.

Sub-case 3.3: Let $PRM e^{-rM} + IE_2 < CQ$ but

$$PR(N-M) e^{-r(N-M)} + Pl e \int_M^N Rte^{-r t} dt < CQ - \left( PRM e^{-rM} + IE_2 \right)$$

In this sub-case, the retailer does not have enough money to pay off the total purchase cost at time $N$ Therefore, he makes the payment

of $PRM e^{-rM} + IE_2$ at $M$ and $\left[ PR(N-M) e^{-r(N-M)} + Pl e \int_M^N Rte^{-r t} dt \right]$ at $N$. 
Hence, the retailer will pay the interest charges on the un-paid balance $U_1 = CQ - \left[ P R M e^{-r M} + IE_2 \right]$ at the interest rate $Ic_1$ during $[M, N]$ and on

$U_2 = U_1 - \left[ P R (N - M) e^{-r (N - M)} + \frac{P r e^{-r t}}{M} \right]$ at interest rate $Ic_2$ at time $N$. Therefore, interest charges payable $IC_{3,3}$ is

$$IC_{3,3} = U_1 Ic_1 (N - M) + \frac{U_2^2 T}{P R} \int_{N}^{T} Q(t) e^{-r t} dt$$

$$= U_1 Ic_1 (N - M) + \frac{U_2^2 Ic_2}{P r (\theta + r)} \left[ e^{-r T} + \frac{1}{\theta} e^{-r N (r e^{\theta (T - N)} - \theta - r)} \right]$$

(3.2.22)

and interest earned, $IE_{3,3} = \frac{P I e R}{r^2} \left[ 1 - (1 + r M) e^{-r M} \right]$.

The present value of cash-out-flow for one cycle for this sub-case is

$$PV_{3,3} (T) = OC + PC + IH C + IC_{3,3} - IE_{3,3}$$

The present value of all future cash-out-flows is given by

$$PV_{3,3\infty} (T) = \left( \frac{1}{r T} + \frac{1}{2} + \frac{r T}{4} \right) PV_{3,3} (T)$$

(3.2.23)

The optimum value of $T = T_{3,3}$ is the solution of non-linear equation

$$\frac{\partial PV_{3,3\infty} (T)}{\partial T} = \left( \frac{1}{r T} + \frac{1}{2} + \frac{r T}{4} \right) \left[ C R e^{\theta T} + \frac{h R}{r + \theta} \left( e^{\theta T} - e^{-r T} \right) \right]$$

$$+ C R e^{\theta T} Ic_1 (N - M) + \left[ e^{-r T} + \frac{1}{\theta} e^{-r N (r e^{\theta (T - M)} - \theta - r)} \right]$$

$$\times \frac{2 U_2 C e^{\theta T}}{P r (\theta + r)} + \frac{U_2^2 Ic_2}{P (\theta + r)} \left[ -e^{-r T} + e^{-r N + \theta (T - N)} \right]$$

$$+ \left( \frac{r}{4} - \frac{1}{r T^2} \right) PV_{3,3} (T) = 0$$

(3.2.24)
provided that it satisfies the sufficiency condition at $T = T_3$

\[
\frac{\partial^2 PV_{3,3\infty} \left( T \right)}{\partial T^2} = \left\{ CR e^{\theta T} \left[ - \%1 + \%2 + \%3 + \frac{1}{Pr(r + \theta)\theta} \left[ 2C^2 R^2 l c_2 \%4 + 4 U_2 l c_2 \%5 + U_2^2 l c_2 \%6 \right] - \frac{1}{P(r + \theta)\theta}[4 U_2 l c_2 \%5 CR + U_2^2 l c_2 \%6(2 \%7 - r \%4)] \right\}\left( \frac{1}{r T} + \frac{1}{2} \frac{r T}{4} \right) + \left( \frac{r}{4} - \frac{1}{r T^2} \right) \]

\[ \times 2 \left\{ CR e^{\theta T} \left[ - \%2 + \%3 + C R l c_1(N - M) + \frac{1}{Pr(r + \theta)\theta} \right] \left[ 2 U_2 l c_2 \%4 CR + U_2^2 l c_2 \%7 \right] - \frac{1}{P(r + \theta)\theta} U_2^2 l c_2 \%4 \right\} \]

\[ + \frac{2PV_{3,3}(T)}{r T^3} \]

> 0, \ \forall T \quad (3.2.2.25)

where,

\[
\%1 = \frac{hR \left( -(r - \theta) e^{(r + \theta) T} r + e^{r T} r + e^{r T} r^2 \theta \right) e^{-r T}}{(r + \theta) \theta r} \]

\[
%2 = \frac{2hR \left( -(r - \theta) e^{(r + \theta) T} r + e^{r T} r + e^{r T} r^2 \theta \right) e^{-r T}}{(r + \theta) \theta} \]

\[
%3 = \frac{hR \left( -e^{(r + \theta) T} r + e^{r T} r + e^{r T} \theta - \theta \right) r e^{-r T}}{(r + \theta) \theta} \]

\[
%4 = \left( e^{(r T + \theta T - \theta N)} r - e^{r T} r - \theta (e^{r T} e^{r N}) \right) e^{-r T - \theta N} \]

\[
%5 = \left( -(r + \theta) e^{(r T + \theta T - \theta N)} r + e^{r T} r^2 + e^{r T} r^2 \theta \right) e^{-r T - \theta N} \]

\[
%6 = \left( (r + \theta)^2 e^{(r T + \theta T - \theta N)} r - e^{r T} r^3 - e^{r T} r^2 \theta \right) e^{-r (T + N)} \]

\[
%7 = \left( (r + \theta) e^{(r T + \theta T - \theta N)} r - e^{r T} r^2 - e^{r T} r \theta \right) e^{-r (T + N)} \]
In the next section, we present computational flowchart to search for optimal solution.

3.2.3 Flowchart:

The flowchart is same as defined in section 3.1.3

3.2.4 Theoretical Results:

**Proposition 3.2.4.1**: $P_{V_{i\infty}}(T)$ is minimum for $i = 1, 2.1, 2.2, 3.1, 3.2,$ and 3.3.

**Proof**: given by equation (3.1.2.10), (3.1.2.17), (3.1.2.25) is non-negative for obtained $T$.

**Proposition 3.2.4.2**: For $T > N$, $P_{V_{3.3\infty}}(T)$ is decreasing function of $M$ and increasing function of $N$.

**Proof**:

$$
\frac{\partial^2 P_{V_{3.3\infty}}(T)}{\partial N^2} = \left\{ U_1 I c_1 - \frac{1}{P(r + \theta)\theta} \left[ 2 U_2 I c_2 \%1 e^{-(r T - r N)} \times (-P R e^{-(r(N - M))} + P R (N - M) r e^{-(r(N - M))}) 
\right.

\left. -P I e N e^{-r N_j} \right\} \frac{1}{P(r + \theta)\theta} (U_2^2 I c_2 \%1) \right\}

\times \left( \frac{1}{r T} + \frac{1}{2} + \frac{r T}{4} \right)

< 0, \ \forall \ T

\text{where} \ \%1 = (-e^{(r T + \theta T - \theta N)} r + e^{r T} r + \theta(e^{r T} - e^{r N}))

\text{and}
where $\%_1 = (e^{(rT + \theta T - \theta N)} - e^{rT} - \theta(e^{rT} - e^{rN}))$

**Proposition 3.2.4.3**: For $T > N$, $PV_{3,3\omega}(T)$ is decreasing function of inflation rate $r$.

**Proof**:

\[
\frac{\partial PV_{3,3\omega}(T)}{\partial r} = \left\{ \frac{hRe^{-rT}}{\theta r (r + \theta)}(T e^{(rT + \theta T)} + e^{(rT + \theta T)} - e^{rT}r_T) - e^{rT}e^{(rT + \theta T)} - \%_1 \right\} \\
- \frac{1}{rP(r + \theta)\theta} \left[ 2U_2lc_2%3%2 + PR(N - M)^2e^{-r(N - M)} \right] \\
\times (rMe^{-rM} + e^{-rM} - rNe^{-rN}) \\
- \frac{1}{Pr(r + \theta)\theta} \left( U^2_2lc_2%3(-N - T) + \frac{1}{Pr^2(r + \theta)\theta} U^2_2lc_2%3 \right) \\
+ \frac{1}{Pr(r + \theta)^2\theta} U^2_2lc_2%3 - \frac{PlMe^2e^{-rM}}{r} \right\}
\]
\[-\frac{2PJe(-1 + rMe^{-rM} + e^{-rM})}{r^3} \left( \frac{1}{rT} + \frac{1}{2} + \frac{rT}{4} \right) + PV_{3.3}(T) \left( \frac{T}{4} - \frac{1}{r^2T} \right) \]

\[< 0, \ \forall \ T > N \]

where

\[\%1 = \frac{hR \left( e^{(r + \theta)T} - e^{-rT} + e^{rT} \theta - \theta \right) e^{-rT}}{(r + \theta) r \theta} e^{-rT}\]

\[\%2 = PRM^2 e^{-rM} - \frac{PJeM^2 e^{-rM}}{r} - \frac{2PJe}{r^3} (-1 + rMe^{-rM} + e^{-rM})\]

\[\%3 = \left( -e^{(rT + \theta T - \theta N)} r + e^{rT} - e^{rN} \right) e^{-rT - \theta N} \]

### 3.2.5 Numerical Example and Observations:

Consider following parametric values in appropriate units.

\[ [R, h, A, P, C, Ic_1, Ic_2, Ie, M, N] = [1000, 4, 100, 35, 20, 10\%, 12\%, 8\%, 15/365, 30/365] \]

#### Table 3.2.5.1

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<th>( \downarrow \theta )</th>
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<th>0.04</th>
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<td>0.217</td>
<td>0.204</td>
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</table>
Observation:

- For fixed rate of deterioration, as discounting rate increases, optimum cycle time, optimum procurement quantity and PV decreases but it is evident that the developed model is more sensitive to the changes in discounting rate.

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<th>Ordering Cost A</th>
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<th>$Q$</th>
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<td>$T_1 = 0.0391$</td>
<td>39</td>
<td>74678.89</td>
</tr>
<tr>
<td>55</td>
<td>$T_2 = 0.0504$</td>
<td>51</td>
<td>72549.62</td>
</tr>
<tr>
<td>60</td>
<td>$T_2 = 0.0704$</td>
<td>70</td>
<td>70132.47</td>
</tr>
<tr>
<td>70</td>
<td>$T_3 = 0.125$</td>
<td>125</td>
<td>67909.32</td>
</tr>
<tr>
<td>85</td>
<td>$T_3 = 0.187$</td>
<td>187</td>
<td>65489.10</td>
</tr>
<tr>
<td>100</td>
<td>$T_3 = 0.221$</td>
<td>221</td>
<td>63125.58</td>
</tr>
</tbody>
</table>

Observation:

- From Table 3.2.5.2, it is observed that for fixed value of discounting rate and deterioration rate, as ordering cost increases, optimum procurement quantity increases significantly but PV decreases

3.3 Conclusion:

In this chapter, two mathematical models are formulated to take account of present values of all future cash–out–flows. In the second section, model is developed when units in inventory deteriorate at a constant rate and when supplier offers retailer two progressive credit periods to settle the account for the procured items. The present value of all future costs decreases with increase in first allowable credit period while it increase with increase in extended credit period. Also, the present value of all future costs decreases with increase in inflation rate. The deterioration of units increases the present value of all future costs.