CHAPTER 1

INTRODUCTION
1.1 Introduction:

In almost every endeavor, it is difficult to determine what constitutes ‘best practice’. Businesses around the world spend millions of dollars searching for ‘best practice’, believing that there is a silver bullet solution that will cure their inventory problems but often don’t know whether they are ‘best practice’ or just somewhere in the pack

"Achieving a best practice starts with a clear understanding of what it is!"

Inventory is often a company's largest current asset and the single largest contributor of working capital requirements. If inventory is properly managed, working capital requirements are reduced and cash-flow is increased, enhancing the organization's chances to prosper and grow. On the other hand, mismanagement of inventory often leads to declining cash-flow and a loss in market share to competitors

Putting it in simple terms, Inventory can be defined as stock of physical assets having some economic value, which can be either in the form of material, money or labor. Inventory is also known as an idle resource as long as it is not utilized. Inventory may be regarded as those goods, which are procured, stored and used for day to day functioning of the organization. Thus, inventory is the common thread that ties all the functions and departments of the organization together. The objective of inventory management has been to maintain inventory at desired level and also be cost effective.

Even a small businessman focuses on the fact that the 'management system' is optimized not just the inventory. Ensuring the availability of materials and goods is one of the major responsibilities and goals of inventory management to earn goodwill. The more important task is to put the right processes, policies and measures in order to limit inventory purchases to those
items that are most likely to be used / sold and in the right quantity, than clearing out the old stock.

If the retailer orders less than the demand, it would result into the situation of stock-out. For any retailer, stock-out may result into (i) the loss of potential profit which he could gain by selling the product; (ii) more importantly, the loss of goodwill that can result in a permanent loss of customers and future sales. Hence stock-out has to be avoided and therefore, inventories should be maintained at proper levels.

In contrast, if retailer orders more than demand; it would be a case of the over-stocking. Excessive inventory levels usually lead to lower profits and cash drain. When inventory just sits there, the money that was used to pay for it is tied-up, the cash that is spent to store and handle it is wasted, and the opportunity to sell a more profitable product is missed. Sitting on the shelf, inventory declines in value due to obsolescence, changing fashions, seasonality, wear and tear, etc. When its value declines, management actually has less incentive to sell it.

Therefore, the stock level of various items in inventory must reach at fine balance in view of the cost minimization / profit maximization while obtaining this delicate balance. To ensure this, following two fundamental questions need to be answered:

1. How much to order?
2. When to order?

Following are the various types of inventories classified on the basis of their intended use.

1. **Anticipation Inventory:** It is the stock of items kept to meet predictable changes in demand or in availability of raw materials.
2. **Decoupling Inventory**: It provides maximum efficiency of operations within a single facility. Decoupling is done by breaking operations apart so that one operation's supply is independent of another's supply. This decoupling function serves two purposes. First, inventories are needed to reduce the dependencies among successive stages of operations so that breakdowns, material shortages, or other production fluctuations at one stage do not cause later stages to shut down.

   A second purpose of decoupling is to let one organization unit schedule its operations independently of another. For example, in an automobile organization, engine built up can be scheduled separately from seat assembly, and each can be decoupled from final automobile assembly operations through in-process inventories.

3. **Seasonal Inventory**: It is the inventory of items retained that is relevant in a particular season, stocked by the firms before the start of that season.

4. **Safety / Buffer Stock**: These are the items needed to ensure that there is no risk of complete breakdown of production. Buffer inventories are considered necessary at different stages of supply chain to provide independence between stages and to avoid work stoppages or delays. Safety stock works as a hedge against the possibility of the stock out.

5. **Pipeline / Transit Stock**: It is the stock that is moved between suppliers, manufacturers, distributors and the end users.

1.2 **Basic Concepts and Terminology**:

Following are the basic concepts and the terminology associated with an inventory system.
• **Demand:** The number of units required per time unit is called demand. The demand pattern of a commodity may be either deterministic or probabilistic. In the deterministic case, it is assumed that the quantities needed over subsequent periods of time are known with certainty. This may be expressed over equal periods of times in terms of known constant demands (referred as static demand) or in terms of known variable demands (referred as dynamic demand).

Probabilistic demand occurs when requirements over a certain period of time are not known with certainty but their pattern can be described by a known probability distribution. The demand for a given period may be satisfied instantaneously at the beginning of the period or uniformly during that period. The effect of instantaneous and uniform demand reflects directly on the total cost of holding inventory.

• **Lead–Time:** The time gap between the placement of the order and actual realization of the inventory is known as lead–time.

Lead–Time has two components viz. administrative lead–time from initiation of procurement action until the placing of an order, and the delivery lead–time from placing of an order until the delivery of the ordered material.

• **Planning Horizon:** The time period over which the inventory level will be controlled is called the planning horizon. It may be finite or infinite depending upon the nature of the inventory system of the commodity.

• **Reorder Level:** The reorder level is the level of inventory at which a new order should be placed.

• **Economic Order Quantity (EOQ):** EOQ is the optimal size of an order that minimizes total inventory costs.
• **Order Cycle:** The time period between two successive placements of orders is referred to as an order cycle. The orders may be placed on the basis of the following two types of inventory review systems.

  (A) **Continuous Review:** In this review system, a pre-determined reorder level is set for each item and continual record of the inventory level for every item is maintained. Whenever the inventory on hand reaches at the reorder level, a new order is placed to replenish the stock of inventory. The replenishment quantity is invariably the EOQ. This is also known as the two-bin system. This divides the inventory into two parts and places it physically in two bins. Items are drawn from only one bin and when it is empty, a new order is placed. Demand is then satisfied from the second bin until the order is received. Upon receipt of the order, specified items are placed in the second bin and the remaining items are placed in the first bin. Every time this process is repeated.

  (B) **Periodic Review:** In this system, the inventory on hand is counted at specific time intervals and orders are placed at such intervals. The quantity ordered each time depends on the available inventory level at the time of review and not necessarily the EOQ. In Constant Review Systems, fixed quantities are ordered at variable time intervals where as in Periodic Review Systems variable quantities are ordered at fixed time intervals.

• **Deterioration:** Deterioration is defined as decay, evaporation, obsolescence, and loss of utility or marginal value of the commodity that
results into the reduction of usefulness from the original condition. It may happen due to the following reasons.

1. The items may have a fixed life span and may be of no use if stored after the life period, e.g., medicines, photo-films, bulbs, etc.
2. The items deteriorate when storage conditions are inadequate, unsatisfactory or poor, e.g., dairy products
3. Deterioration can also result from poor handling in the store, e.g., fruits and vegetables, glass items, etc.

The rate of deterioration is measured by the fraction deteriorated per quantity per time unit and it may remain constant or vary with time.

- **Ordering Cost:** This is the cost associated with replenishing the stock of inventory being held. This is independent of the order size and varies with the number of orders made. Costs incurred, when an order is made, can include requisition and purchase orders, transportation and shipping, advertisement, inspection, and accounting and auditing costs.

- **Purchase Cost:** The cost of purchasing a unit of an item is known as purchase cost. The purchase price is of special interest when quantity discounts are considered for purchases above a certain quantity or when large production runs may result in a decrease in the production cost.

- **Inventory Carrying (Holding) Cost:** The carrying cost is associated with the handling of inventory. This cost varies with the level of inventory and occasionally with the length of time an item is held. This cost includes the cost of loosing the use of funds tied-up in inventory; direct storage costs such as rent, heating, cooling, lighting, security, refrigeration, record keeping, and transportation; interest on loans used to purchase inventory; depreciation; obsolescence as markets for products in
inventory diminish; product deterioration and spoilage, breakage, taxes; and pilferage.

- **Shortage Cost:** This cost occurs when customer demand cannot be met because of insufficient inventory. The avoidance of these costs is the basic reason why stocks are held in the first instance. These costs include the loss of potential profit through sales of items and loss of customer good will, in terms of permanent loss of customers and loss of future sales. In some instances, the inability to meet customer demand or lateness in meeting demand results in penalties in the form of price discounts or rebates. When demand is internal, a shortage can cause work stoppages in the production process and create delays, resulting in down-time costs and the cost of lost production. Evidently, many of these costs are difficult to quantify, but they are often significant.

- **Opportunity Cost:** The value or benefit from an alternative proposal, e.g., investment decision that is forgone in favour of another.

- **Inflation:** The phenomenon of sustained increase in the prices of goods and services in general is known as inflation. It may arise due to a scarcity of supplies in relation to demand; this is known as 'demand–pull inflation.' It may also result from an increase in the cost of some critical input, such as steel or petroleum, which then triggers off a gradual rise in prices, in general; this is known as 'cost–push inflation.' When inflation increases, it results into the decline in the purchasing power of money.

- **Present Value:** The discounted value of future cash-flow is known as present value of that particular future cash-flow. For example, present value of $1000 expected two years from today is $826.45, if the discounting rate is 10%, i.e. $1000/(1+0.10)^2$
• **Discounting Rate:** The interest rate used in calculating the present value of future cash-flows is known as discounting rate. This interest rate is often considered to be the inflation rate. The procedure for calculating the present value of future cash-flows by applying an appropriate rate of discounting rate is known as **Discounting**. This approach is also known as **Discounted-cash-flow (DCF) approach**. This is based on the concept of **time value of money**, which differentiates between a dollar offered today and one expected at some point of time in future.

• **Net Present Value:** It is a discounted cash-flow measure to evaluate the viability of an investment proposal. It serves to determine whether the present value of future cash-flow exceeds the investment on a project.

1.3 Literature Survey:

In 1915, Harris first developed the analysis of an inventory system and derived the classical lot size formula, though Erlenkotter (1989) reported that the earliest model was developed by Harris in 1913. After a few years, Wilson (1934) developed independently the same formula obtained by Harris. This formula has been named as Harris–Wilson formula or Wilson’s formula that was formulated under certain simple assumptions, such as known and uniform demand, no shortages, infinite replenishment time, negligible lead-time etc. This model was developed under the concept of the average cost approach. The Wilson’s (1934) EOQ model is derived with the assumption that the retailer must pay for the items as soon as it is received by the system.

However, the most prevailing practice is that the supplier may offer a delay period to the retailer to settle his account. This delay period, termed as **trade credit**, is an important economic phenomenon. It can be used as a many faceted marketing / relationship management tool and / or as a means of
directing information to the market or to specific buyers about the firm, its products and its future prospects / commitments. Most of the extensions are customer focused; encouraging frequent purchasers, which accommodate customer's demand for credit to help financially for their production period. The requirements / bargaining power of large customers can influence a firm to extend more credit. Firms will vary terms in anticipation of capturing new business, to attract specific customers and in order to achieve specific marketing goals. Thus, the delay in the payment offered by the supplier is a kind of price discount since paying later indirectly reduces the purchase cost and it encourages the retailers to increase their order quantity.

Ferris (1981) derived a transaction theory of trade credit to economize on the joint costs of exchange. Trade credit can be viewed as a tool that separates the exchange of money between two trading parties. Trade credit permits a reduction in precautionary money holdings and the more effective management of net money accumulation.

For supplier who offers trade credit, it is an effective means of price discrimination as well as efficient tool to stimulate the demand of his products. The length of credit period is considered as supplier's dominant strategy against the competitive suppliers. The factors viz. credit risk, the size of the account, customer type and market competition are the most prominent to determine the length of the credit period. For the sake of better production and inventory control, manufacturers prefer less frequent orders with larger order sizes to frequent orders with smaller order sizes, if the annual ordering quantities are equal. In such situation, they offer a longer credit period for larger amount of purchase. Their policies attract the retailer to make order size large enough to avail for a credit period.
Mehta (1968) applied the statistical technique of sequential decision process to the problems of trade credit management. He tried to examine two problems viz. credit extension policy on a specific request or account and construction of indices measuring the effectiveness of such a policy. The aim was to establish a decision system which has analytical solution. He studied indices in terms of bad debt level, receivable level etc. and measure the impact of credit extension procedures on the subsequent phases of credit policy. The logical relationship between the operating decision rules and the control indices were suggested to the management in framing the optimal credit policy.

Haley and Higgins (1973) studied the relationship between inventory policy and credit policy in the context of the classical lot-size model. It is observed that in general, optimality of the total cost of an inventory system requires order quantity and payment time decisions simultaneously. They derived the conditions under which the standard solution reduces to optimal solution.

Chapman et al. (1984) derived an economic order quantity model which considers possible credit periods allowable by suppliers. This model is shown to be very sensitive to the length of the permissible credit period and to the relationship between the credit periods and inventory level. It is also shown to be more sensitive to estimates of the demand for inventory items and less sensitive to ordering cost than the classical economic order quantity model. They gave numerical example to show how inventory costs may be reduced considerably by taking the advantage of a credit period into account.

Goyal (1985) pioneered to develop mathematical model when supplier announces credit period in settling the account, so that no interest charges are payable from the outstanding amount if the account is settled within the
allowable delay period. The supplier will obviously charge higher interest if the account is not settled by the end of the permissible delay period. In fact, this brings some economic advantage to the system, as retailer would try to earn some interest from the revenue realized during the period of permissible delay. Shah et al (1988) extended above model by allowing shortages. Mandal and Phaujdar (1989–a, 1989–b) have studied Goyal’s model by including interest earned from the sales revenue on the stock remaining beyond the settlement period. Chung and Huang (2003) extended Goyal’s model when replenishment rate is finite.

Davis and Gaither (1985) developed optimal order quantities for firms which are offered a one-time opportunity to delay payment for an order of a commodity. Such delayed payments result in a reduction of the effective purchase cost, which is a function of the return available on alternative investments, the number of units of commodity ordered and the length of the extended period. Optimal order quantities are developed for extended payment privileges that occur at a reorder point of between reorder points. Six suppliers’ extended payment scenarios are studied. The sensitivity of derived models to the changes in the various input parameter is carried out. The simulation with realistic parametric values revealed that the additional discounted order quantity is insensitive to large changes in the ordering cost and unit price, sensitive to changes in the inventory carrying cost and return rate of funds, but without significantly affecting the total cost; and extremely sensitive to the annual demand. They gave simple analytic decision rules to guide firms which are offered such extended payment privileges. Using principles of financial analysis, Dallenbach (1986, 1988), Ward and Chapman (1987), Chapman and Ward (1988) argued that if trade credit has the character of a renewable source of
capital, the usual assumptions as to the incidence and the value of the inventory investment opportunity cost made by the traditional inventory theory are correct, contrary to research articles on this subject. They also established that if trade credit surplus is taken into account, the optimal ordering quantities decreases, rather than increase, as argued in some papers.

Chung (1998) established the convexity of the total annual variable cost function for optimal economic order quantity under conditions of permissible delay in payments. Analytically he showed that the economic order quantity under conditions of permissible delay in payments is generally higher than the economic order quantity given by the classical economic order quantity model.

Abad and Jaggi (2003) considered the seller – buyer channel in which the end demand is price sensitive and the supplier offers trade credit to the buyer. The unit price charged by the seller and the length of the credit period offered by the seller to the buyer both influence the final demand for the product. They considered both to be the policy variables for the seller. The case of no credit was used as a benchmark in the analysis. The article provides algorithm for determining the seller’s and the buyer’s policies under non-cooperative as well as cooperative relationships. In the non-cooperative relationship, they determined the seller’s optimal unit price and the length of the credit period. For the cooperative scenario, they provided a procedure for characterizing Pareto efficient solutions.

Chung et al. (2005) determined the economic order quantity under conditions of permissible delay in payments where the delay in payments depends on the quantity ordered when the order quantity is less than the quantity at which the delay in payments is permitted; the payment for the item must be made immediately. Otherwise, the fixed credit period is allowed.
Shinn and Hwang (2003) dealt with the problem of determining the retailer's optimal price and order size simultaneously under the condition of order size dependent delay in payments. It is assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price.

Most of the researchers while developing EOQ models for a retailer when the supplier offers a permissible delay in payments assumed that the selling price is same as the purchase cost. Teng (2002) expanded Goyal's model by considering the difference between unit selling price and unit cost to derive closed-form solution to the offer of the trade credit. Teng et al. (2005) developed the model by considering the difference between the selling price and the purchase cost. They gave algorithm for a retailer to determine optimal selling price and lot-size simultaneously when the supplier offers a permissible delay in payments. They concluded that the economic replenishment interval and order quantity increases marginally under the permissible credit period.

Carlson and Roussean (1989) examined EOQ under date terms supplier credit by partitioning carrying cost into financial cost and variable holding costs. When a distinction between these types of holding costs is made, Wilson's EOQ may no longer hold good. They gave search procedure to find optimal order quantity over a well-defined range of order quantities which encompasses the classical EOQ. They contradicted the fact that the optimal order quantity under date terms is always given by an integral multiple of monthly demands. The unique feature of date terms credit is the possible existence of multiple EOQs is established.

Shah et al. (1997) derived (T, S)–policy with increasing demand when delay in payment is permissible. Shinn (1997) dealt with the problem of
optimizing the retailer's selling price and lot-size simultaneously under the condition of permissible delay in payments when the retailer's demand rate is represented by a constant price elasticity function which is a decreasing function of selling price. The effect of credit period on retailer's decision variables is examined.

The most of the inventory replenishment policies under trade credit are developed under the assumption of instantaneous receipt of the goods in an inventory system and the supplier also offer a cash discount to encourage retailer to pay for his purchases at the earliest. Ouyang (2005) developed an inventory model with non-instantaneous receipt under trade credit, in which the supplier provides not only a permissible delay in payments but also a cash discount to the retailer.

Arcelus and Srinivasan (1993) formulated decision procedure for a vendor who wants to dispose of the extra stock $Q$ due to large levels of inventory. In such situation, vendor offers either a discount in the unit price or a credit period, $M$, within which no payment is required, in exchange for the purchase of an additional $x \leq Q$ units over and above the regular order. They evaluated a bargaining range within which negotiations may take place as to purchase lot-size and its corresponding credit period, a range of indifference as to whether the transaction should take place at all, the set of $(x, M)$ values within the bargaining range which gives the desired benefits to both parties and leads to the largest combined benefits.

The aforesaid models are derived under the assumption that items in the warehouse remain intact for its use for infinite planning horizon. However, physical changes occurring in the items hinder it to use for its purpose. Refer to review articles by Raafat (1991), Shah and Shah (2000) and Goyal and Giri...

Sarker et al (2001) obtained optimal payment time under permissible delay in payments when units in an inventory are subject to constant rate of deterioration. Jamal et al. (2000) discussed the problem in which the retailer can pay the supplier either at the end of credit period or later incurring interest charges on the unpaid balance for the overdue period. They developed a retailer’s model for optimal cycle and payment time for a retailer when units in inventory are subject to a constant rate of deterioration where a wholesaler allowed a specified credit period to the retailer for payment without penalty. The objective was taken to be a cost minimization to determine the optimal payment time under various system parameters. They concluded that the retailer has always an option to pay after the permissible credit period depending on interest rates, unit purchase and selling price and the deterioration rate of the units.

Chang and Dye (2000) derived mathematical model of an inventory system for deteriorating items with partial back-logging when supplier offers fixed credit period to settle the account.

Liao (2007) derived a production model for the lot-size inventory system with finite production rate, taking into consideration the effect of deterioration under permissible trade credit. He made restrictive assumption of a relaxed
permissible delay at the end of the credit period. It is assumed that the retailer will make a partial payment on total purchasing cost to the supplier and pay off the remaining balance by loan from the bank. The existence of a unique optimal cycle time to minimize the total variable cost per unit time is established. The bounds for the optimal cycle time are obtained.

Chung and Liao (2004) extended Hwang and Shinn's (1997) and Khouja and Mehrez's (1996) model for exponentially deteriorating items under the conditions of permissible delay in payments. They assumed that the delay in payments depends on the quantity ordered.

Lokhandwala et al. (2005) extended Davis and Gaither's (1985) model to determine optimal order quantities for firms where units in an inventory are subject to deterioration at a constant rate, which are offered a one-time opportunity to delay payment for an order of a commodity. Such delayed payment reduces purchase cost which is a function of the return available on alternative investments, the number of units ordered and the length of the extended period. For the following four scenarios, the optimal order quantities are developed for extended payment privileges that occur at a reorder point viz.

1. Extended payment privilege is allowed on all units, when \((Q + x) - n\) units are ordered, if \(x > 0\) at a reorder point.

2. Extended payment privilege is allowed on all units, when \((Q + x) - n\) units are ordered if \(x > x_{mn}\) where \(x_{mn}\) is pre-stated quantity at a reorder point.

3. Extended payment privilege is allowed on all additional \(x\)–units only, when \((Q + x) - n\) units are ordered, if \(x > 0\) at a reorder point.

4. Extended payment privilege is allowed on all additional \(x\)–units ordered if \(x > x_{mn}\) where \(x_{mn}\) is pre-stated quantity at a reorder point. (Here reorder
quantity Q is specified by the supplier to qualify for the offer of extended payment privilege).

Chung et al (2001) considered linear demand to derive inventory model for deteriorating items under permissible trade credit. Chang et al (2003) derived an EOQ model for deteriorating items in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a pre-determined quantity.

In general, the rate of deterioration increases with the age of the longer the items remain unused, the higher the rate at which it looses its usability. Gor and Shah (2003, 2005–a, 2005–b) developed a mathematical model by allowing / not allowing shortages in the inventory system in which units are subject to time dependent deterioration and supplier allows credit period to settle the account.

These aforesaid models were developed under the concept of the average cost approach. The average cost approach used by Wilson (1934) has following two main drawbacks:

1. The time value of money is not explicitly taken into account.

2. There is no distinction between out-of-pocket holding costs and opportunity costs due to inventory investments.

To overcome these drawbacks of the average cost approach, number of researchers suggested present value (PV)) approach (discounted – cash – flow (DCF)). The DCF approach allows proper recognition of the financial implication of the opportunity cost and out – of – pocket costs in the inventory analysis. It also permits an explicit recognition of the exact timing of cash – flow associated with inventory system and considers the time value of money as well.
The effect of inflation on the inventory system plays an important role. Buzacott (1975), Bierman and Thomas (1977), Misra (1979–a) investigated the inventory decisions under inflationary conditions for the EOQ model. Misra (1979–b) derived an inflation model for the EOQ, in which the time value of money and different inflation rates were considered. Brahmbhatt (1982) derived an EOQ model under a variable inflation rate and mark–up prices. Gurnani (1983) gave the economic analysis of inventory systems and claimed that the discounting effects on EOQ were substantial. Chandra and Bahner (1985) studied the effects of inflation and time value of money on optimal order policies.

Kim et al. (1986) developed inventory model by taking into account the effect of constant rate of deterioration on the optimal order quantity using the DCF – approach. Chung and Lin (2000) derived mathematical model considering time value of money over a fixed planning horizon. They established convexity of total variable cost. Chung and Tsai (2001) also extended above concept for trended demand.

Chung (1989) presented the DCF – approach for the analysis of the optimal inventory policy under the effect of trade credit. He studied effect of the delayed payment in determining the optimal order size. Rachamadugu (1989) established that the best order quantity is an increasing function of allowable delay period.

Carlson et al. (1996) obtained economic order quantity under both all – units and incremental – quantity discounts when purchase cost, ordering cost and inventory holding cost are all incurred on date – terms supplier credit. Payment dates for the three cost components need not be the same. Differences in the characteristics of day – terms and date – terms solutions to the quantity discount case are studied.
Chung and Liao (2006) extended Jaggi and Aggarwal’s (1994) article under the assumption that the trade credit is linked to ordering quantity using DCF - approach. Hwang and Shinn (1997) developed mathematical model to determine the retailer’s optimal price and lot-size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is a function of constant price elasticity and units in inventory are subject to constant rate of deterioration.

Sarker et al. (2000), Chung et al. (2001) and Shah (2006) discussed a model to determine an optimal ordering policy for deteriorating items under inflation, permissible trade credit and allowable shortage. The optimal order quantity and maximum allowable shortages are obtained by optimizing the present value of the total cost incurred. The effect of inflation rate and time discount is derived on the optimal order quantity and maximum allowable shortages.

Yang and Wee (2006) developed a collaborative inventory system of a single vendor and single buyer to maximize the total profit of the whole system. However, the optimal solution for the whole system is not always acceptable to both parties. A negotiating factor was introduced to share profit between two players according to their contributions. The permissible delay in payment is a win–win strategy for sharing profit in the collaborative system. A deteriorating inventory model with finite replenishment rate and price sensitive demand is assumed to occur in a high-tech, short life cycle and perishable electronic product. It is established that the percentage of extra total profit is significant when both the collaboration strategy and the deterioration rate are considered.

The demand of product may not be known or constant, though it can be forecasted using past experience by distributions. Shah and Shah (1992)
derived the probabilistic order level system in which the scheduling period $T$ is fixed and the supplier offers a fixed credit period (say) $M$-time units. The optimal order level is obtained by minimizing the total average expected cost of the system. Later on Shah (2004) extended above model when units in inventory are subject to constant rate of deterioration.

Shah (1993-b, 1993-c) derived a probabilistic time – scheduling model for an exponential decaying inventory when supplier offers a credit period to settle the account. The expressions are derived for the total average expected cost of the system, the optimum cycle time and the time for obtaining optimum order level. Shah and Shah (1998) derived above model by treating time to be discrete variable.

Salameh et al. (2003) examined the continuous review inventory model under permissible delays in payments, i.e. a retailer can pay for the goods immediately upon the receipt of the order or delay the payment till the next replenishment order, where supplier will charge interest over the delayed period. They assumed demand to be constant over the time and the lead – time to be random variable. The optimal ordering quantity and reorder level are obtained by maximizing the vendor’s total expected profit per time unit when trade credit is offered.

Harris and Wilson’s Economic order quantity (EOQ) model is based on the stringent assumption, that, the items received by the retailer exactly matches with order requisitioned. However, due to a variety of reasons, viz, machine’s breakdown, worker’s strike, electricity failure, shortage of raw materials etc., it is found that the quantity received does not necessarily match with the quantity ordered but may be a random variable depending on the quantity ordered. Silver (1976) developed an EOQ model when the quantity
received is uncertain. Kalro and Gohil (1982) extended Silver's result to allow shortages. Shah and Shah (1992–a, 1992–b) developed optimal ordering policy when units received are random in nature and subject to constant rate of deterioration for infinite / finite production rate. Gor and Shah (1994) extended above said model by allowing shortages. Yano and Lee (1995) reviewed the literature on quantitatively oriented approach for determining lot-sizes when production or procurement yields are random. They discussed the issues relating to the modeling cost yield uncertainty and performance of the system with random yield. Shah and Trivedi (2005) analyzed an EOQ model for deteriorating items when the supply is random and supplier allows a certain fixed credit period for settling the accounts.

The observations of supermarkets reveal that the demand rate is usually influenced by the amount in the stock. Levin et al. (1972) quoted at times; the presence of inventory has motivational effect on people around it. It is common belief that large piles of goods, displayed in a supermarket, will attract customers to buy more. Silver and Peterson (1982) observed that a sale at the retail level is directly proportional to the amount of inventory displayed. This fact attracted a number of researchers to drive EOQ models focused on stock-dependent demand rate patterns. Gupta and Vrat (1986) considered demand rate to be a function of initial stock level. Mandal and Phaujdar (1989–b) derived production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Dixit and Shah (2001) extended the above model for more general demand. Baker and Urban (1988) formulated a model in which sales were directly affected by the allocation of shelf-space. Datta and Pal (1990) extended Mandal and Phaujdar (1989–b)'s work for deteriorating items with the assumption that the demand rate is a linear
function of the on-hand inventory by allowing shortages, which are completely backlogged for both finite and infinite time horizons. Some of the related works in this area are by Goh (1994), Padmanabhan and Vrat (1995), Wee (1995), Ray and Chaudhuri (1997), Sarker et al. (1997), Giri and Chaudhari (1998), Yan and Chen (1998), Mandal and Maiti (1999), etc. Liao et al. (2000) developed an inventory model for stock dependent demand rate when delay in payments is permissible. Shah et al. (2004) extended Gor and Shah’s (2003) model by taking demand to be stock dependent and units in inventory are subject to deterioration with respect to time.

1.4 Outline of the Thesis:

The proposed thesis has been divided into seven chapters on the basis of the structure of the different models which are as under.

- **Chapter 1**: Introduction
- **Chapter 2**: Optimal Ordering Policies for exponentially deteriorating items under scenario of progressive credit period.
- **Chapter 3**: EOQ model for progressive trade credit scheme under DCF – approach.
- **Chapter 4**: Optimal Ordering and Pricing policies for two stage credit policies
- **Chapter 5**: Inventory models for Stock – dependent demand under progressive payment scheme
- **Chapter 6**: Lot – size model for progressive trade credit scheme under random supply.
- **Chapter 7**: EOQ model for two level credit policies under random supply.
The following is a chapter-wise description of inventory models dealt with in the proposed thesis.

**Chapter 1** contains an introduction giving an overview of the development of various inventory models under different assumptions.

The concept of **progressive credit period** given by the supplier to the retailer for settling the account is as follows: If the retailer settles outstanding amount by \( M \) - time units, then the supplier does not charge any interest. If the retailer pays after \( M \) but before \( N \) (\( N > M \)), then the supplier charges the retailer on un-paid balance at the rate \( I_c \). If the retailer settles the account after \( N \), then he will have to pay an interest rate of \( I_c^2 \) (\( I_c^2 > I_c \)).

**Chapter 2** discusses optimal ordering policies for exponentially deteriorating items when supplier offers two progressive credit periods to the retailer for settling his account.

**Chapter 3** is about present value formulation of two EOQ models for progressive trade credit scheme. **Section 3.1** deals with the present value formulation of EOQ model when supplier facilitates his retailers with two credit periods for settling the account. The theoretical results are illustrated with hypothetical numerical example. The effect of allowable credit period on present value of all future cash-out-flows and optimum cycle time are studied. In **Section 3.2**, a present value formulation of lot-size inventory model with constant rate of deterioration under progressive payment scheme is formulated. The effects of deterioration of units, inflation rate and ordering cost on optimal procurement quantity and the present value of all future cash-out-flows studied numerically.

In **Section 4.1** of **Chapter 4**, an optimal pricing and ordering policies for retailers when the supplier offers two progressive credit periods to settle the
accounts is derived. The objective function is to optimize the net profit which is difference of gross revenue and all cash-out-flows. The extension of section 4.1, when units in inventory are subject to constant rate of deterioration is carried out in Section 4.2. The effects of deterioration of units in inventory and credit periods on objective function and decision variables are studied using hypothetical numerical example.

In Chapter 5, we study two inventory models. Section 5.1 deals with formulation of optimal ordering policies for retailer; when demand is stock-dependent and the supplier offers progressive credit periods to settle the account. The effects of permissible credit periods and stock dependent factor on optimum purchase quantity, demand and total cost of an inventory system are observed. In Section 5.2, joint pricing and replenishment policy for stock dependent demand under progressive payment scheme

In Chapter 6, an attempt is made to formulate ordering policy for retailer when supplier offers progressive trade credit to settle the account and the quantity received by the retailer does not match with that of the ordered. The optimal replenishment policy is derived by minimizing total expected cost of an inventory system. Analytic results are discussed to observe the effect of various parameters on the objective function.

In Chapter 7, a mathematical model is developed to study the retailer's optimal replenishment decisions under two levels trade credit, i.e. retailer passes some credit offered by supplier to his customer to stimulate his own sale, when the production rate of the supplier is finite and units received by the retailer are uncertain. The optimal replenishment policy is established by minimizing the total expected cost of an inventory system. The numerical examples are given to validate the developed model.
The easy-to-use flowchart is given to search for the optimal replenishment/pricing policy for each formulated model.

List of papers *published / accepted for publication / presented / communicated* follows Chapter 7

The thesis concludes with Bibliography
ASSUMPTIONS – A.1

The following are the assumptions used in the thesis:

1. The inventory system deals with single item.

2. The demand of \( R \) – units for an item is constant during the cycle time.

3. Shortages are not allowed and lead-time is zero.

4. Replenishment is instantaneous. Replenishment rate is infinite/finite.

5. The units in inventory deteriorate at a constant rate (say) \( \theta \), \( 0 \leq \theta < 1 \), during the cycle time.

6. The deteriorated units can neither be repaired nor replaced during the period under review.

7. If the retailer pays by \( M \), then supplier does not charge to the retailer If the retailer pays after \( M \) and before \( N \) (\( M < N \)), he can keep difference in unit sale price and unit cost in an interest bearing account at the rate of \( Ic_1 \) per unit/year. During \([M, N]\), the supplier charges the retailer an interest rate of \( Ic_1 \) per unit/year. If the retailer pays after \( N \), then supplier charges the retailer an interest rate of \( Ic_2 \) per unit per annum with \( Ic_2 > Ic_1 \).

Additional assumptions, if any, will be mentioned in the relevant chapters.
NOTATIONS – N.1

The following are the notations used in the thesis:

\( A \) : Ordering cost of inventory, $/per order

\( C \) : The purchase cost per unit.

\( h \) : The inventory holding cost per unit per year excluding interest charges; $/unit/time unit.

\( P \) : The selling price per unit

\( Q \) : The procurement quantity.

\( T \) : The replenishment cycle time.

\( Q(t) \) : The on-hand inventory level at time \( t \) \((0 \leq t \leq T)\).

\( M \) : The first permissible credit period in settling the account without any extra charges.

\( N \) : The second permissible delay period in settling the account with higher interest charge and \( N > M \).

\( Ie \) : The interest earned, $/year.

\( Ic_1 \) : The interest charged per $ in stock per year by the supplier when the retailer pays after \( M \) but before \( N \). \( (Ic_1 > Ie) \)

\( Ic_2 \) : The interest charged per $ in stock per year by the supplier when the retailer pays after \( N \). \( (Ic_2 > Ic_1) \)

Additional notations, if any, will be mentioned in the relevant chapters.