CHAPTER 5
THEORETICAL INVESTIGATION

5.1 INTRODUCTION
Recently researchers attempted to simulate the behaviour and failure mechanisms of reinforced concrete members strengthened with ferrocement composites using finite element method modeled using two dimensional plate elements. The geometry and the material properties as adopted during experimentation were used for this study. In this section a static non-linear analysis was carried out for perfect, undamaged laminated, predamaged rehabilitated beams and the results were compared with experimental results. In addition theoretical load deflection relationship was derived using the section analysis procedure for the perfect beams and undamaged ferrocement laminated beams and the results were validated with experimental results.

5.2 FINITE ELEMENT ANALYSIS
The finite element method in comparison with the most common analytical methods of concrete behaviour is a powerful tool to study the behaviour of the reinforced concrete and composite beams. A general purpose finite element code, ANSYS was utilized in this study to analysis the behaviour of ferrocement reinforced concrete composite beams. ANSYS, including a variety of its routines, allows for the implementation of specific material models (concrete and steel), boundary conditions, and bond behaviour. Interaction between reinforcing steel and concrete could also be considered. The model was a smeared crack model, in the sense that it does not track in individual macro cracks, but applies the constitutive equation independently at each integration point in the model to determine failure in concrete. The failure criterion was defined by providing the uniaxial tensile and compressive failure stresses of concrete as well as the ratio of biaxial stresses to each uniaxial state. Using this defined stress limits for concrete material, the crack detection surface was developed allowing detection of crack in each element of the model.

In nonlinear analysis, the total load applied to a finite element model was divided into a series of load increments called load steps. The analysis was performed by applying an
incremental load, with iterations in each increment. The modified time to algorithm with assumed proportional loading history was used. This approach determines the static equilibrium solutions for unstable response in concrete, due to cracking in tension, yielding of reinforcement, and concrete softening in compression. It neglect any permanent strains associated with cracking. In this study, the analysis was carried out using the following element: SOLID 65 –Concrete element, LINK 8 –Rebars, PIPE 16-Dummy elements, and SHELL 99 –for composite section (ferrocement with concrete beam).

5.2.1 Assumptions and Restrictions

- Cracking was permitted in three orthogonal directions at each integration point
- If cracking occurs at an integration point, the cracking was model through an adjustment of material properties which effectively treats the cracking as a smeared band of cracks, rather than discrete cracks
- The concrete materials were assumed to be initially isotropic
- Whenever the reinforcement capability of element was used, the reinforcement was assumed to be “smeared” throughout the element
- In addition to cracking and crushing, the concrete may also undergo plasticity with the Drucker – Prager failure surface being most commonly used.

5.2.2 Finite Element Modeling Description

SOLID 65 was used for the three-dimensional modeling of solids with or without reinforcing bars (Rebars). The solid is capable of cracking in tension and crushing in compression. In concrete application, for example, the solid capability of the element may be used to model the concrete, while the rebar capability was available for modeling reinforcement behaviour. Other case for which the element was also applicable would be reinforce composites (such as fibre glass), and geological materials (such as rocks). The element was defined by eighth nodes having three degree of freedom at each node translation in the nodal x, y and z directions as shown in Figure 5.1.

The element had plasticity, creep, swelling, stress stiffening, large deflection and large strain, in addition with special cracking crushing capabilities. The most important aspect of this element was treatment of nonlinear material properties. The concrete is capable of
cracking (in three diagonal directions), crushing, plastic deformation and creep. The rebar was capable of tension and compression, but not shear. They are also capable of plastic deformation and creep. Additional concrete material data, such as shear transfer coefficient, tensile and compressive stress could be the input in the data table. Typical shear co-efficient ranges from 0.0 to 1.0, with 0.0 representing a smooth crack (complete loss of shear transfer) and 1.0 representing a rough crack (no loss of shear transfer). This specification may be made for both closed and open crack.

![Diagram of SOLID 65 3-D Reinforced Concrete Solid](image)

**Figure 5.1 SOLID 65 3-D Reinforced Concrete Solid**

5.2.3 Modeling of Perfect and Composite Beams

The two layers (concrete and ferrocement) were assumed to be perfectly bonding along their interface by constraining the same number of nodes in each of freedom i.e displacement and rotation. The non-linear analysis program was based on the assumption of full bond between both layers. The longitudinal layering details of perfect beam and the layered configuration of perfect beam and the undamaged laminated beam by plate bonding technique are shown in Figure 5.2 and Figure 5.3. A three dimensional finite element mesh for a perfect beam and the predamaged rehabilitated beams by plate bonding are shown in Figure 5.4 and Figure 5.5.
Figure 5.2 Longitudinal layering details of perfect beam

Figure 5.3 Layered configurations of perfect and the laminated beam
5.2.4 Response of Finite Element Analysis

A static non-linear analysis was carried out for the twenty six beams, perfect beams, undamaged laminated beams and predamaged rehabilitated beams as given in series 1, series 2, series 3, series 4, and series 6 by adopting plate bonding technique. The beams
strengthened by cast insitu bonding are not considered because the input data given for the material properties resembles the same for the two techniques. The main obstacle found in this analysis was the difficulty in characterizing the material properties. A realistic model was predicted for the behaviour of these beams. All the beams were analysed under four bending regime using the ANSYS finite element software. The results were compared in terms of load-deflection profile at the initial cracking stage, yielding stage and ultimate stage for all beams with respect to perfect beams. During the analysis the deformed shape of the beams at every 10 kN loading intervals where obtained. Some typical deformed shapes of perfect beam, undamaged laminated and predamaged rehabilitated beams by using plate bonding technique are shown in Figure 5.6 – Figure 5.21.

Figure 5.6 Deformed shape of perfect beam (BP1) at initial cracking stage
Figure 5.7 Deformed shape of perfect beam (BP1) at ultimate stage

Figure 5.8 Deformed shape of undamaged laminated beam (BRP1) at initial cracking stage
Figure 5.9 Deformed shape of undamaged laminated beam (BRP1) at ultimate stage

Figure 5.10 Deformed shape of undamaged laminated beam (BRP3) at yielding stage
Figure 5.11 Deformed shape of undamaged laminated beam (BRP3) at ultimate stage

Figure 5.12 Deformed shape of undamaged laminated beam (BRP5) at ultimate stage
Figure 5.13 Deformed shape of predamaged rehabilitated beam (BOP1) at initial stage

Figure 5.14 Deformed shape of predamaged rehabilitated beam (BOP3) at yielding stage
Figure 5.15 Deformed shape of predamaged rehabilitated beam (BOP5) at ultimate stage

Figure 5.16 Deformed shape of predamaged rehabilitated beam (BOP7) at yield stage
Figure 5.17 Deformed shape of predamaged rehabilitated beam (BOP 9) at ultimate stage

Figure 5.18 Deformed shape of predamaged rehabilitated beam (BOP 11) at ultimate stage
Figure 5.19 Deformed shape of predamaged rehabilitated beam (BOP 13) at initial stage

Figure 5.20 Deformed shape of predamaged rehabilitated beam (BOP 15) at yield stage
5.2.5 Comparison of Analytical and Experimental load-deflection behaviour of beams

From the analytical data obtained, load-deflection behaviour of perfect beams, undamaged laminated beams and predamaged rehabilitated beams by using plate bonding technique were drawn, and the results were compared with the experimental results of the perfect beams, undamaged laminated beams and predamaged rehabilitated beams by using plate bonding technique. The analytical load-deflection curves clearly indicate that the bonding of ferrocement laminates to the tension face of the beams has given an improvement in load carrying capacity and reduction in deflection.

The load-deflection behaviour of perfect, undamaged laminated beams and predamaged rehabilitated beams as obtained analytically were compared with the experimental results to authenticate the study, and are shown in Figure 5.22 to Figure 5.26. In the analytical study, the general behaviour of the finite element models represented by the load-deflection graphs showed good agreement with the experimental results. The effect of bond slip in between the concrete and steel reinforcement and also the micro cracks occurring in the actual beams were excluded in the finite element models as they...
contribute to higher stiffness represents that the undamaged laminated beams and predamaged rehabilitated beams exhibit a similar pattern with each other.

Figure 5.22 Comparison of load-deflection response of perfect beam (series 1)

Figure 5.23 Comparison of load-deflection response of undamaged laminated beams (series 2)
Figure 5.24 Comparison of load-deflection response of predamaged rehabilitated beams (series 3)

Figure 5.25 Comparison of load-deflection response of predamaged rehabilitated beams (series 4)

Figure 5.26 Comparison of load-deflection response of predamaged rehabilitated beams (series 5)
5.3 THEORETICAL FORMULATIONS BY SECTION ANALYSIS

In this section the theoretical load-deflection relationship were derived using the section analysis procedure for the perfect beams and undamaged laminated beams and the results were compared with the experimental results.

5.3.1 Theoretical Load-Deflection for Perfect Beams

For a simply supported beam subjected to two point loads (magnitude of each load is $P/2$), at one third spans, the maximum bending moment occurs at middle third zone. The maximum bending moment

$$M = \frac{PL}{6} \quad 5.1$$

The displacements corresponding to the loads could be found out using conjugate beam method of analysis. The detailed calculations of theoretical load and deflection for perfect beams and undamaged laminated beams are presented in the subsequent headings.

5.3.1.1 Initial Cracking Stage

$$Cracking\ load\ \ P_{cr} = 2 \times M_{cr}$$

$$= 2 \times 4.92$$

$$= 9.84\ kN$$

The curvature distribution over the conjugate beam is shown in Figure 5.27.

![Figure 5.27 Conjugate beam for cracking stage](image-url)
Mid span deflection at the cracking stage,

\[
\delta_{cr} = \frac{1}{2} \times L / 3 \times \phi_{cr} \times \frac{2}{3} \times L / 3 + \frac{L / 6 \times \phi_{cr}}{L / 3 + L / 12}
\]

\[
= L^2 / 27 \times \phi_{cr} + 5L^2 / 72 \times \phi_{cr}
\]

\[
= \left(\frac{23}{216}\right) \times L^2 \times \phi_{cr}
\]

\[
\delta_{cr} = \left(\frac{23}{216}\right) \times 3000^2 \times 1.12 \times 10^{-6}
\]

\[
= 1.073 \text{ mm}
\]

5.3.1.2 Yielding Stage

Load corresponding to yielding stage \( P_{y2} = 2 \times M_y \)

\[
= 2 \times 18.460
\]

\[
= 36.92 \text{ mm}
\]

The curvature distribution over the conjugate beam for yield stage is shown in Figure 5.28.

Figure 5.28 Conjugate beam for yielding stage
\[ L_1 = \frac{L}{3} \times \frac{M_{cr}}{M_y} \]
\[ = \frac{3000}{3} \times \frac{4.92}{18.64} \]
\[ = 266.52 \text{ mm} \]

\[ L_2 = \left( \frac{L}{3} \times \frac{M_{cr}}{M_y} \right) - L_1 \]
\[ = \frac{3000}{3} \times \frac{14.811}{18.460} - 266.52 \]
\[ = 535.81 \text{ mm} \]

\[ L_3 = L_1 + \frac{\phi_{cr} + L_2^2 + \frac{L_2}{2} \times (\phi_{y2} - \phi_{y1}) \times \frac{2}{3} \times L_2}{\phi_{cr} \times L_2 + \frac{L_2}{2} \times (\phi_{y2} - \phi_{y1})} \]
\[ = 226.52 + \frac{1.12 \times 10^{-6} \times (535.81^2 / 2) + (535.81 / 2) \times (1.01 \times 10^{-5} - 1.12 \times 10^{-6}) \times 2 / 3 \times 535.81}{1.12 \times 10^{-6} \times 535.81 + (535.81 / 2) \times (1.01 \times 10^{-5} \times 1.12 \times 10^{-6})} \]
\[ = 339.38 \text{ mm} \]

\[ L_4 = \frac{L}{3} - L_1 - L_2 \]
\[ = \frac{3000}{3} - 266.52 - 535.81 \]
\[ = 197.67 \text{ mm} \]

\[ L_5 = L_1 + L_2 + \frac{\phi_{y1} \times L_1^2 / 2 + L_2 / 2 \times (\phi_{y2} - \phi_{y1}) \times \frac{2}{3} \times L_4}{\phi_{y1} \times L_4 + L_2 / 4 \times (\phi_{y2} - \phi_{y1})} \]
\[ = 266.52 + 535.81 + \frac{0.197321 + 0.19937868}{1.996467 \times 10^{-3} + 1.51296618 \times 10^{-3}} \]
\[ = 915.37 \text{ mm} \]

\[ L_6 = 500 \text{ mm} \]
\[ L_7 = \frac{L}{3} + \frac{L}{12} \]
\[ = \frac{3000}{3} + \frac{3000}{12} \]
\[ = 1250 \text{ mm} \]

Central Deflection \( \delta_y = \{ \frac{L}{2} \times \phi_x \times 2L_1/3 \text{ + } 1/2(\phi_x - \phi_y) \times L_2 \times L_3 \text{ + } 1/2(\phi_y + \phi_z) \times L_4 \times L_5 \text{ + } \phi_y \times L_6 \times L_7 \} \]

\[ \delta_{y2} = \left( \frac{266.52}{2} \times 1.12 \times 10^{-6} \times (2 \times 266.52) / 3 \right) + \\
\frac{1}{2}(1.12 \times 10^{-6} + 1.01 \times 10^{-6}) \times 535.81 \times 339.38 + \\
\frac{1}{2}(1.01 \times 10^{-5} + 2.5408 \times 10^{-5}) \times 197.67 \times 915.37 + \\
1.01 \times 10^{-5} \times 500 \times 1250 \]

\[ \delta_{y2} = 0.02651895322 + 1.02014034 + 3.21242985 + 6.3125 \]

\[ = 10.57 \text{ mm} \]

5.3.1.3 Ultimate Stage

\[ P_{ul} = 2 \times M_{ul} \]
\[ = 2 \times 19.54 \]
\[ = 39.08 kN \]

Ultimate deflection \( \delta_u = (\phi_{ul} - \phi_{yz}) \times L_p \times L_e + \delta_{y2} \)

\[ \delta_u = \left( 241.94 \times 10^{-6} - 2.5408 \times 10^{-5} \right) \times 250 \times 500 + 10.57 \]

\[ = 37.637 \text{ mm} \]
5.3.2 Theoretical Load-Deflections for Undamaged Laminated Beam ($V_r=2.192\%$)

5.3.2.1 Initial cracking stage

For 2.192\% volume fraction of ferrocement laminates,

\[ P_{cr} = 2 \times M_{cr} \]

\[ = 2 \times 8.65 \]

\[ = 17.3 \text{kN} \]

\[ \delta_{cr} = \left( \frac{23}{216} \right) \times L^2 \times \phi_{cr} \]

\[ \delta_{cr} = \left( \frac{23}{216} \right) \times 3000^2 \times 1.034 \times 10^{-6} \]

\[ = 0.991 \text{mm} \]

5.3.2.2 Yielding stage

*Load yield stage* $P_{y2} = 2 \times M_{y2}$

\[ = 2 \times 14.46 \]

\[ = 28.92 \text{kN} \]

\[ \phi_{yz} = 2.54 \times 10^{-5} \text{ radians/mm} \]

\[ L_1 = \frac{L}{3} \times \frac{M_{cr}}{M_{yz}} \]

\[ = \left( \frac{3000}{3} \right) \times \frac{8.65}{14.46} \]

\[ = 598.20 \text{mm} \]
\[ L_2 = \left( \frac{L}{3} \times \frac{M_{yz}}{M_{y2}} \right) - L_1 \]

\[ = \left( \frac{3000}{3} \times \frac{13.85}{14.46} \right) - 598.20 \]

\[ = 359.61 \text{ mm} \]

\[ L_3 = L_1 + \frac{\phi_{cr} + L_2^2 + L_2^2/2 \times (\phi_{sl} - \phi_{cr}) \times 2/3 \times L_2}{\phi_{cr} \times L_2 + L_2^2/2 \times (\phi_{sl} - \phi_{cr})} \]

\[ L_3 = 598.20 + \frac{1.034 \times 10^{-6} \times \frac{359.61^2}{2} + \frac{359.61^2}{2} \times (1.06 \times 10^{-6} - 1.034 \times 10^{-6}) \times \frac{2}{3} \times 359.61}{1.034 \times 10^{-6} \times 359.61 + \frac{359.61^2}{2} \times (1.06 \times 10^{-6} - 1.034 \times 10^{-6})} \]

\[ L_3 = 598.20 + \frac{0.066858105 + 0.4123563074}{3.7183674 \times 10^{-4} + 1.72001463 \times 10^{-3}} \]

\[ = 229.08 \text{ mm} \]

\[ L_4 = \frac{L}{3} - L_1 - L_2 \]

\[ = \frac{3000}{3} - 598.20 - 359.61 \]

\[ = 42.19 \text{ mm} \]

\[ L_5 = L_1 + \frac{\phi_{sl} \times L_1^2 / 2 + L_2^2 / 2 \times (\phi_{sl} - \phi_{sl}) \times 2/3 \times L_4}{\phi_{sl} \times L_4 + L_2^2/4 \times (\phi_{sl} - \phi_{sl})} \]

\[ = 598.20 + 359.61 + \frac{9.43397933 \times 10^{-3} + 8.781314093 \times 10^{-3}}{4.47214 \times 10^{-3} + 3.12206 \times 10^{-4}} \]

\[ = 598.20 + 359.61 + \frac{0.01821529342}{7.5942 \times 10^{-4}} \]

\[ = 981.80 \text{ mm} \]
\[ L_6 = \frac{L}{6} \]

\[ = \frac{3000}{6} \]

\[ = 500 \text{ mm} \]

\[ L_7 = \left( \frac{L}{3} + \frac{L}{12} \right) \]

\[ = \left( \frac{3000}{3} + \frac{3000}{12} \right) \]

\[ = 1250 \text{ mm} \]

**Central Deflection**

\[ \delta_{y2} = \{ L_6 / 2 \times \phi_{y1} \times 2L_7 / 3 + 1 / 2(\phi_{y4} - \phi_{y6}) \times L_2 \times L_3 + 1 / 2(\phi_{y4} + \phi_{y6}) \times L_4 \times L_5 + \phi_{y4} \times L_6 \times L_7 \} \]

\[ \delta_{y2} = \frac{598.20}{2} \times 1.034 \times 10^{-6} \times \frac{2}{3} \times 598.20 + \frac{1}{2} \left( \frac{1.034 \times 10^{-6} + 1.06 \times 10^{-6}}{2} \right) \times 359.61 \times 229.68 + \frac{1}{2} \left( 1.06 \times 10^{-5} + 2.54 \times 10^{-5} \right) \times 42.19 \times 981.8 + 1.06 \times 10^{-5} \times 500 \times 1250 \]

\[ \delta_{y2} = 0.1233366367 + 0.4792013118 + 6.625 \]

\[ = 7.97 \text{ mm} \]

**5.3.2.3 Ultimate stage**

\[ P_{ul} = 2 \times M_{ul} \]

\[ = 2 \times 25.18 \]

\[ = 50.36 \text{ kN} \]
Similarly the theoretical load and deflections calculation for undamaged laminated beams bonded with 4.384% and 6.576% volume fractions of ferrocement laminates have been evaluated at various stages are shown in Table 5.1.

Table 5.1 Theoretical value of load and deflection of beams

<table>
<thead>
<tr>
<th>Stage</th>
<th>Perfect beam BP</th>
<th>Undamaged Laminated beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load kN</td>
<td>Deflection Mm</td>
</tr>
<tr>
<td>V_r=2.192% BRP1</td>
<td>9.84</td>
<td>1.07</td>
</tr>
<tr>
<td>V_r=4.384% BRP3</td>
<td>36.92</td>
<td>10.57</td>
</tr>
<tr>
<td>V_r=6.576% BRP5</td>
<td>39.08</td>
<td>37.64</td>
</tr>
</tbody>
</table>

The comparison of experimental, theoretical by section analysis and non linear analysis by ANSYS for perfect beam BP, undamaged laminated beams BRP1, BRP3 and BRP5 are shown in Figure 5.29 to Figure 5.32. From the test results it is understood that the experimental and the analytical values are in good agreement with each other and also with the values of theoretical analysis.

\[
\delta_{ul} = (\phi_{ul} - \phi_{r}) \times L_p \times L_c + \delta_{r}
\]

\[
\delta_{ul} = (3.47 \times 10^{-4} - 2.54 \times 10^{-5}) \times 250 \times 500 + 7.97 = 48.17 \text{ mm}
\]
Figure 5.29 Comparison of load-deflection response of perfect beam (BP 1)

Figure 5.30 Comparison of load-deflection response of undamaged laminated beam (BRP1)
Figure 5.31 Comparison of load-deflection response of undamaged laminated beams (BRP3)

Figure 5.32 Comparison of load-deflection response of undamaged laminated beams (BRP5)

5.4 SUMMARY

A static non-linear analysis using ANSYS has been carried out to simulate the behaviour of the perfect beam, undamaged laminated beam, and predamaged rehabilitated beams strengthened in flexure with ferrocement laminates by adopting plate bonding technique.
Analytical study was carried out in comparison with the beams that were experimentally tested. The load-deflection plot obtained from these studies made use of to authenticate the model. Theoretical load-deflection curves were also formulated for perfect beams, undamaged ferrocement laminated beams by using section analysis procedure and are compared with that of experimental and analytical curves. The results obtained with the theoretical studies showed good agreement with experimental data.