CHAPTER 3

INVENTORY MODELS WITH TIME DEPENDENT
DETERIORATION OF UNITS UNDER CONDITIONS OF
PERMISSIBLE DELAY IN PAYMENTS
3.0 Introduction.

In this chapter, three inventory models have been proposed. The mathematical models are developed to determine the optimal ordering policy for variable deterioration of units in an inventory system under permissible delay in payments. In the models in Section 3.1 and 3.2 the following two scenarios are discussed:

Scenario-1: When permissible delay period in payments is less than the cycle time
Scenario-2: When permissible delay period in payments is greater than the cycle time

Section 3.3 deals with an inventory model in which the supplier provides a cash discount as well as a credit period to the customer. It deals with the following four cases.

Case 1. The payment is paid at \( M_1 \) to get a cash discount and cycle time \( T \geq M_1 \)
Case 2. The customer pays in full at \( M_1 \) to get a cash discount but cycle time \( T < M_1 \)
Case 3. The payment is paid at time \( M \) to the permissible credit and cycle time \( T \geq M \)
Case 4. The customer pays in full at \( M \) and cycle time \( T < M \)

where \( M_1 \): the period of cash discount
and \( M \): the period of permissible delay

3.1 A LOT-SIZE MODEL WITH VARIABLE DETERIORATION RATE UNDER SUPPLIER CREDITS

This section deals with a lot-size model with variable deterioration rate under supplier credits. The model is developed under assumption of instantaneous and infinite replenishments and no shortages. The deterioration of items in the inventory follows the Weibull density function. The Newton-Raphson method has been used to find the optimum solutions. An easy-to-use algorithm to find the solution is given. Sensitivity
analysis of the optimal solution with respect to the parameters of the system is carried out.

3.1.1 Assumptions and Notations

The following additional notations and assumptions other than those given in A.1 and N.1 are used to derive the proposed model

Assumptions:

- Shortages are not allowed.
- The distribution of the time for deterioration of units is as given in 1.3.1 earlier.

$Q$ is a decision variable.

$M$ is the permissible delay payment time.

3.1.2 Mathematical Formulation:

The model has two scenarios:

Scenario I: when permissible delay period 'M' in payments is less than the cycle time $T$;

Scenario II: when permissible delay period 'M' in payments is greater than the cycle time $T$.

In the first scenario, if the customer does not pay the supplier by time $M$, then he can incur an interest for the outstanding balance. In the second case, the customer would not only be able to use all the product he bought and get the revenue for that but also he would be able to earn the interest until the time he has to settle the account.

Let $Q(t)$, $0 \leq t \leq T$ be on-hand inventory of units at time $t$. The instantaneous state of $Q(t)$ for any instant of time follows the differential equation.

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \quad 0 \leq t \leq T$$

(3.1.2.1)

With initial condition $Q(0) = Q$ and $Q(T) = 0$
The study of optimal ordering policies for time varying decay rate of inventory.

Equation (3.1.2.1) with given condition has solution.

\[ Q(t) = Re^{-\alpha t^\beta} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n^\beta + 1)} (t^{n^\beta + 1} - t^{n^\beta + 1}) \quad 0 \leq t \leq T \]

\[ Q(0) = Q = R \sum_{n=0}^{\infty} \frac{\alpha^n t^{n^\beta + 1}}{n!(n^\beta + 1)} \quad (3.1.2.2) \]

The total demand during cycle time \( T \) is \( RT \). Hence, the number of units deteriorated during \([0, T]\) is given by

\[ D(T) = Q - RT \]

\[ = R[ \sum_{n=0}^{\infty} \frac{\alpha^n t^{n^\beta + 1}}{n!(n^\beta + 1)} - T] \]

The cost due to deterioration / time unit is

\[ CD(T) = \frac{CD(T)}{T} = \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n t^{n^\beta + 1}}{n!(n^\beta + 1)} - T \right] \quad (3.1.2.3) \]

The ordering cost per time unit

\[ OC = \frac{A}{T} \quad (3.1.2.4) \]

The inventory holding cost per time unit is

\[ IHC = \frac{CI}{T} \int_{0}^{T} Q(t) dt \]

\[ = \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n^\beta + 1)^2} T^{2(n^\beta + 1)} \quad (3.1.2.5) \]
Scenario I: \( M \leq T \) (permissible delay period is less than the cycle time \( T \))

![Diagram of inventory level over time](image)

Fig. 3.1.1

Here the permissible payment period ends on or before the inventory depletes to zero.

As a result, the variable cost consists of the sum of the ordering cost, inventory holding cost, cost due to deterioration of units and the interest charged minus the interest earned.

The interest payable per time unit is:

\[
IC = \frac{CI}{T} \int_{M}^{T} Q(t) dt
\]

\[
= \frac{CI}{T} \sum_{n=0}^{\infty} \frac{(-1)^{n} \alpha^{2n}}{(n!)^{2} (n\beta+1)^{2}} \left[ T^{2(n\beta+1)} - \frac{M^{n\beta+1}(T^{n\beta+1} - \frac{M^{n\beta+1}}{2})}{2} \right] \quad (3.1.2.6)
\]

The interest earned per time unit is

\[
IE = \frac{CI}{T} \int_{0}^{M} R dt = \frac{CI R M^{2}}{2T} \quad (3.1.2.7)
\]
Using equations (3.1.2.3) - (3.1.2.7), the total variable cost per time unit $K_i(T)$ is

$$K_i(T) = OC + CD + IHC + IC - IE$$

$$= \frac{A}{T} + CR \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T \right] + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n} T^{2(n\beta+1)}}{2(n\beta+1)^2}$$

$$+ \frac{CIR}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{2(n\beta+1)^2} \left[ \frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1} (T^{n\beta+1} - \frac{M^{n\beta+1}}{2}) \right]$$

$$- \frac{CIR}{T} \left( M - \frac{T}{2} \right)$$  \hspace{1cm} (3.1.2.8)

The optimal value of $T = T_1^*$ can be obtained by solving $\frac{dK_i(T)}{dT} = 0$

$$\frac{dK_i(T)}{dT} = -\frac{A}{T^2} + CR \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T \right] - \frac{CIR}{T^2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n} T^{2(n\beta+1)}}{2(n\beta+1)^2} \right]$$

$$+ \frac{CIR}{T^2} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{2(n\beta+1)^2} \left[ \frac{T^{2(n\beta+1)}}{2} - M^{n\beta+1} (T^{n\beta+1} - \frac{M^{n\beta+1}}{2}) \right]$$

$$- \frac{CIR}{T^2} \left( M - \frac{T}{2} \right) = 0$$  \hspace{1cm} (3.1.2.9)

by suitable numerical method. The cycle time $T = T_1^*$ obtained by solving the equation (3.1.2.9), minimizes the total cost because $\frac{d^2K_i(T)}{dT^2} > 0$
\[
\frac{\partial^2 K_1(T)}{\partial T^2} = \frac{2A}{T^3} + \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n n \beta \beta^{-1}}{n!} \right] - \frac{2CR}{T^2} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta}{n!} \right] - 1 \\
+ \frac{2CR}{T^3} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta + 1}{n! (n \beta + 1)} \right] - T + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2\alpha^n}{(n \beta + 1)^2} T^{2n \beta + 1} \right] \\
- \frac{3CIR}{2T^2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2\alpha^n}{(n \beta + 1)^2} T^{2n \beta + 1} \right] + \frac{2CIR}{T^3} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n \beta + 1)^2} T^{2n \beta + 1} \right] \\
- \frac{C_1 e R M^2}{T^3} + \frac{C_1 e R}{T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n \beta + 1)^2} \left( \frac{T^{2(n \beta + 1)}}{2} - M^{n \beta + 1} T^{n \beta - 1} \right) \right] \\
- \frac{2C_1 e R}{T^2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n \beta + 1)^2} \left( \frac{T^{2(n \beta + 1)}}{2} - M^{n \beta + 1} T^{n \beta - 1} \right) \right] \\
+ \frac{2C_1 e R}{T^3} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n \beta + 1)^2} \left( \frac{T^{2(n \beta + 1)}}{2} - M^{n \beta + 1} (T n \beta + 1 - \frac{M^{n \beta + 1}}{2}) \right) \right]
\]

**Scenario II**: \( T < M \) (permissible delay period is greater than the cycle time \( T \))

![Fig. 3.1.2](image)

Here the payment is made after the permissible delay period. So there is no interest charged. The interest earned per time unit is
IE = \frac{CI}{T} \int_{0}^{T} Rtdt + RT (M - T)

\frac{CR}{T} (M - \frac{T}{2})

Hence, the total variable cost, \( K_2(T) \) per time unit is

\[ K_2(T) = OC + CD + IHC - IE \]

\[ = \frac{CR}{T} \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta + 1}{n!(n\beta + 1)} - T + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n\beta + 1)^2} T^{2(n\beta + 1)} \]

\[ \frac{CI}{T} \frac{R}{T} (M - \frac{T}{2}) \]

The optimal value of \( T = T_2 \) can be obtained by solving \( \frac{dK_2(T)}{dT} = 0 \)

\[ -\frac{A}{T^2} + \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta}{n!} - 1 \right] - \frac{CIR}{2T^2} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta + 1}{n!(n\beta + 1)} - T \right] \]

\[ + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2\alpha^{2n}}{(n\beta + 1)^2} T^{2n\beta + 1} \right] - \frac{CIR}{2T^2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n\beta + 1)^2} T^{2(n\beta + 1)} \right] \]

\[ \frac{CI}{2T} + \frac{CI}{T^2} \frac{R}{T} (M - \frac{T}{2}) = 0 \]

by suitable numerical method. The cycle time \( T = T_2 \) obtained by solving the equation (3.1.12), minimizes the total cost because \( \frac{d^2K_2(T)}{dT^2} > 0 \)
\[
\frac{\partial^2 K_2(T)}{\partial T^2} = \frac{2A}{T^3} + \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n n! T^{n\beta-1}}{n!} \right] - \frac{2CR}{T^2} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n}{n!} \right] - 1
\]

\[
+ \frac{2CR}{T^3} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} \right] + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2^\alpha n!}{(n\beta+1)^2} T^{2n\beta} \right]
\]

\[
- \frac{3CIR}{2T^2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2^\alpha n!}{(n\beta+1)^2} T^{2n\beta+1} \right] + \frac{CIR}{T^3} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2^\alpha n!}{(n\beta+1)^2} T^{2(n\beta+1)} \right]
\]

\[
- \frac{CLR e}{2T^2} - \frac{CLR e}{T^3} \left( M - \frac{T}{2} \right) - \frac{CLR e}{2T^2}
\]

**Algorithm:** The optimum cycle time

\[
T_0 = \begin{cases} 
T_1^* & M \leq T \\
T_2^* & M > T \end{cases}
\]

Hence,

\[
Q_0 = \begin{cases} 
Q_1^* & M \leq T \\
Q_2^* & M > T \end{cases}
\]

and total cost of an inventory system

\[
K(T_0) = \begin{cases} 
K_1(T_1^*) & M \leq T \\
K_2(T_2^*) & M > T \end{cases}
\]

### 3.1.3 Numerical example and observations:

Consider an inventory system with following parameters in appropriate units.

\[
[A, R, C, I, I_c, I_e, \alpha, \beta, M] = [250, 2000, 20, 0.10, 0.12, 0.15, 0.02, 1.5, 15/365]
\]
Study of optimal ordering policies for time varying decay rate of inventory.

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<th>$T$</th>
<th>$Q$</th>
<th>$K$</th>
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<td>2027.37</td>
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Observations:

- Increase in the deterioration rate ($\alpha$) decreases the cycle time and the procurement quantity and increases the total cost of the inventory system.
- Increase in the demand rate ($R$) decreases the cycle time and increases the procurement quantity and the total cost of the inventory system significantly.
- Increase in the shape parameter ($\beta$) increases the cycle time and the procurement quantity and decreases the total cost of the inventory system.
- Increase in the purchase cost ($C$) decreases the cycle time and the procurement quantity while increases the total cost of the inventory system.
- Increase in the ordering cost ($A$) increases the cycle time, the procurement quantity and increases the total cost of the inventory system significantly.
- Increase in the carrying charge fraction ($I$) per unit per annum reduces the cycle time and the procurement quantity and increases the total cost of the inventory system.
- Increase in the delay period ($M$) increases the cycle time and the procurement quantity while reduces the total cost of the inventory system significantly.
- Increase in the interest charged ($I_c$) reduces the cycle time and the purchase quantity while increases the total cost of the inventory system.
- Increase in the interest earned ($I_e$) reduces the cycle time, procurement quantity and the total cost of an inventory system.
3.2 AN ORDER LEVEL LOT-SIZE MODEL WITH TIME DEPENDENT 
DETERIORATION AND PERMISSIBLE DELAY IN PAYMENTS 

The model in this section is an extension of the previous model. Shortages are 
allowed here. The rest of the assumptions remain same as that of the model in section 
3.1. The mathematical methodology to find the solution of the equations is explained in 
detail and a special case is also discussed. Sensitivity analysis is carried out at the end 
and a few directions for future research are discussed.

3.2.1 Assumptions and Notations:

The following additional assumptions and notations other than those given in A.1 
and N.1 are used to formulate the proposed model.

Shortages are allowed.

The distribution of the time for deterioration of units is as given in 1.3.1

\[ R \quad : \quad \text{The known demand rate (units per time unit).} \]

\[ Q_j \quad : \quad \text{Quantity consumed during time } T_j. \]

\[ T_j \quad : \quad \text{length of the period with positive stock of the items in the inventory.} \]

3.2.2 The Mathematical formulation:

The model has two scenarios

**Scenario I**: When permissible delay \( M \) in payments is less than the period having 
inventory stock in hand \( T_j \).

**Scenario II**: When permissible delay; \( M \), in payments is greater than the period having 
inventory stock in hand \( T_j \).

In the first case, if the customer does not pay the supplier by time \( M \), then he can 
incure an interest for the outstanding balance. In the second case, the customer would 
not only be able to use all the product he bought and get the revenue for that but also he
would be able to earn interest on that revenue until the time he has to settle the account. 
There could be some other situations involving payment of dues to the supplier either 
during the inventory in stock or shortages. All these scenarios are discussed in 
subsequent sections.

Let \( Q(t) \) be the inventory level at time \( t \). Depletion of inventory occurs due to the 
simultaneous demand and deterioration of units. The deterioration of units occurs during 
time period \((0, T_j)\) and shortages occur during time interval \((T_j, T)\). (See fig. 3.2.1) 

\[
\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \quad 0 \leq t \leq T_j \\
\frac{dQ(t)}{dt} = -R \quad T_j \leq t \leq T
\]

where at time \( t = 0, Q(0) = Q \).
The solution of eq. (3.2.2.1) is given by

$$Q(t) = R e^{-\alpha t} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)} (T_1^n \beta + 1 - t^n \beta + 1) \quad 0 \leq t \leq T_1$$  \hspace{1cm} (3.2.2.2)

Then

$$Q(0) = Q = R \sum_{n=0}^{\infty} \frac{\alpha^n T_1^n \beta + 1}{n!(n\beta+1)}$$  \hspace{1cm} (3.2.2.3)

The demand during $T_1$ is $RT_1$.

The number of units deteriorated during one cycle is given by

$$D(T) = Q - RT_1$$

$$= R \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T_1^n \beta + 1}{n!(n\beta+1)} - T_1 \right]$$  \hspace{1cm} (3.2.2.4)

Since the shortages are allowed in the present mathematical model, there are two cases for payment to be made at time $M$. These cases are:

1. Payment at or before the total depletion of inventory; i.e. $(M \leq T_1 < T)$.
2. Payment after depletion i.e. $T_1 < M$.

**Case 1. $M \leq T_1 < T$.**

![Fig. 3.2.2](image)

Here the permissible payment period ends on or before the inventory depleted completely to zero. As a result, the variable cost consists of the sum of the ordering...
cost, inventory holding cost, shortage cost, cost due to deterioration of units and the interest charged minus the interest earned.

These costs are as under.

The ordering cost, $OC = A$ \hspace{1cm} (3.2.2.5)

The cost of deterioration ($CD$) incurred to $D(T)$ units of material per cycle time $T$ is given by

$$ CD = CD(T) $$

$$ = CR \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T \right] \hspace{1cm} (3.2.2.6) $$

The inventory holding cost; $IHC$ is

$$ IHC = CI \int_{0}^{T} Q(t)dt $$

$$ = CIR \frac{(-1)^n \alpha^{2n}}{2} \sum_{n=0}^{1} \frac{(n\beta+1)^2 T^{2(n\beta+1)}}{2(n!)^2} \hspace{1cm} (3.2.2.7) $$

The interest charged per cycle for the inventory not being sold after the due date $M$ is

$$ IC = CI \int_{M}^{T} Q(t)dt $$

$$ = CIR \frac{(-1)^n \alpha^{n}}{2} \sum_{n=0}^{1} \frac{(n\beta+1)^2 T^{2(n\beta+1)}}{2(n!)^2} \left[ T^{n\beta+1} (T T^{n\beta+1} - M^{n\beta+1}) - \frac{1}{2} (T T^{2(n\beta+1)} - M^{2(n\beta+1)}) \right] \hspace{1cm} (3.2.2.8) $$

Interest earned per cycle, $IE$, during the positive inventory is given by

$$ IE = CI e \int_{0}^{M} R(t)dt = \frac{CI e R M^2}{2} \hspace{1cm} (3.2.2.9) $$

The backordered cost, $SC$, per cycle is given by

$$ SC = \pi \int_{0}^{T-T_1} R(t)dt = \frac{\pi R(T-T_1)^2}{2} \hspace{1cm} (3.2.2.10) $$
Hence, the total variable cost, \( K_1(T_j,T) \) per time unit is

\[
K_1(T_j,T) = \frac{1}{T} (OC + CD + IHC + IC + SC - IE)
\]

\[
= \frac{1}{T} \left( A + CR \left( \sum_{n=0}^{\infty} \frac{\alpha^n T_1^n \beta + 1}{n!(n\beta + 1)} - T_1 \right) + 2 \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta + 1)^2} T_{1}^{2(n\beta + 1)} \right)
\]

\[
+ \frac{CIR}{2} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n!)^2 (n\beta + 1)^2} \left[ T^n \beta + 1 \left( T_1^n n \beta + 1 - M^n \beta + 1 \right) - \frac{1}{2} \left( T_{1}^{2(n\beta + 1)} - M^2(n\beta + 1) \right) \right]
\]

\[
+ \frac{\pi R (T - T_1)^2}{2} - \frac{CIR \cdot RM^2}{2} \right)
\]

(3.2.2.11)

To evaluate the nature of the total cost function in (3.2.2.11), it is to establish whether the function is convex or not. Since \( K_1(T_j,T) \) involves higher order and summation, it is not easy to evaluate the Hessians in closed form to conclude about its positive definiteness directly. The total cost \( K_1(T_j,T) \) is evaluated over certain range of \( T_j \) and \( T \) with different sets of inventory parametric values. Therefore, the values of \( T \) and \( T_j \) which minimize \( K_1(T_j,T) \) can be obtained by simultaneously solving \( \frac{\partial K_1(T_j,T)}{\partial T_j} = 0 \) and \( \frac{\partial K_1(T_j,T)}{\partial T} = 0 \) within the stated ranges. These two partial differential equations lead to the equations (3.2.2.12) and (3.2.2.13) as shown below.

\[
\frac{\partial K_1(T_j,T)}{\partial T_j} = 0
\]

\[
\Rightarrow \frac{CIR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T_1^n \beta}{(n!)^2 (n\beta + 1)} - 1 \right] + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2 \alpha^n T_1^n \beta + 1}{(n!)^2 (n\beta + 1)} \right] - \frac{\pi R}{T} (T - T_1) \]

\[
+ \frac{CIR}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n T_1^n \beta (T^n \beta + 1 - T_1^n \beta + 1)}{n!} - \frac{CIR \cdot KT_1}{T} = 0
\]

(3.2.2.12)
and

\[ \frac{\partial K_1(T_1, T)}{\partial T} = 0 \]

\[ \therefore - CR \left( \sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n \beta + 1}}{n! (n \beta + 1)} - T_1 \right) - A - \frac{CIR}{2} \left( \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n \beta + 1)^2} T_1^{2(n \beta + 1)} \right) \]

\[ \frac{\pi R (T - T_1)^2}{2} + \pi R (T - T_1) T \]

\[ -CI C R \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n!)} \left[ \frac{T_1^{n \beta + 1}}{(n \beta + 1)} (T_1^{n \beta + 1} - M^{n \beta + 1}) - \frac{1}{2(n \beta + 1)} (T_1^{2(n \beta + 1)} - M^{2(n \beta + 1)}) \right] \]

\[ + CI C R T \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n!)} T_1^{n \beta} (T_1^{n \beta + 1} - M^{n \beta + 1}) + \frac{CIR M^2}{e T^2} = 0 \quad (3.2.2.13) \]

Since equations (3.2.2.12) and (3.2.2.13) are functions of \( T_1 \) and \( T \), and the convexity of none of the functions is assured in general, the iterative search approach must be used simultaneously to obtain pragmatic solutions for \( T_1 \) and \( T \). When the initial value of \( T_1 \) within certain feasible range is assumed in (3.2.2.12), the solution of \( T \) is immediately known. If the value of \( T \) obtained from (3.2.2.12) is then used as an initial value in (3.2.2.13), the value of \( T_1 \) is also known by a one dimensional iterative search procedure. This value of \( T_1 \) may not be equal to the value of \( T_1 \) obtained earlier from (3.2.2.12). The process of switching between the equations is repeated until two consecutive iterations give same values of \( T_1 \) and \( T \). Once \( T_1 \) and \( T \) are obtained, the optimal ordering quantity and total cost of an inventory system is calculated easily.
A Special case: \( T_I = M \).

If payment is made at the time \( T_I = M \), the ordering cost remains the same as before, and also the deterioration cost, inventory holding cost, interest earned and the shortage cost remain the same as in the earlier case. Since the payment is made when it is due at time \( T_I \), the interest charged, \( IC \), is zero. Therefore, \( \frac{\partial K_2(M, T)}{\partial M} = 0 \) after replacing \( T_I \) with \( M \) in (3.2.2.12) and \( \frac{\partial K_2(M, T)}{\partial T} = 0 \) in (3.2.2.13) as given (3.2.2.14) & (3.2.2.15) shown below.

\[
K_2(M, T) = \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n M^{n\beta + 1}}{n!(n\beta + 1)} - M \right] + \frac{A}{T} + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta + 1)^2} M^{2(n\beta + 1)} \right]
\]

\[
+ \frac{\pi R(T - M)^2}{2T} - \frac{2 \pi R T M}{2T}
\]

\[
\frac{\partial K_2(M, T)}{\partial M} = 0
\]

\[
\therefore \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n M^{n\beta + 1}}{n!} - 1 \right] + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{2(-1)^n \alpha^{2n}}{(n!)^2 (n\beta + 1)^2} M^{2(n\beta + 1)} \right] - \frac{\pi R(T - M)}{T}
\]

\[
+ \frac{\pi R M}{T} - \frac{\pi R M}{T} = 0
\]

(3.2.2.14)
\[
\frac{\partial K_2(M, T)}{\partial T} = 0
\]
\[
\therefore - \text{CR} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n M^{n\beta + 1}}{n!(n\beta + 1)} - M \right] - A - \frac{\text{CIR}}{2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta + 1)^2} M^{2(n\beta + 1)} \right] = 0
\]
\[
- \frac{\pi R (T - M)^2}{2} + \pi R (T - M) T + \frac{\text{CI}_e R M^2}{2} = 0
\]

(3.2.2.15)

**Case 2: \( T_1 < M \) (After - depletion payment)**

The deterioration cost \( CD \); the inventory holding cost, \( IHC \), and the shortage cost, \( SC \), per cycle are the same as in the earlier case. The interest charged per cycle \( IC = 0 \) when \( T_1 < M \leq T \) because the supplier can be paid in full at time \( M \), the permissible delay period.

![Graph](image)

**Fig. 3.2.4**

The interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from the cash invested during time period \( (T_1, M) \) after the inventory is exhausted at time \( T_1 \), and it is given by

\[
IE = \text{CI}_e \left[ \frac{\pi R d^2}{2} \right] + \text{CRI} \left[ e^{T_1(M - T_1)} - 1 \right]
\]

\[
= \text{CRI} e^{T_1(M - \frac{T_1}{2})}
\]

(3.2.2.16)
Incorporating these modifications in (3.2.2.11), the total variable cost per unit time, $K_3(T_1,T)$ is given by

$$K_3(T_1,T) = \frac{1}{T}(OC + CD + IHC + SC - IE)$$

$$= \frac{1}{T}\left(A + CR\sum_{n=0}^{\infty} \frac{\alpha^n T_{1}^{n\beta+1}}{n!(n\beta+1)} - T_1^2\right) + \frac{CIR}{2}\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n} T_{1}^{2(n\beta+1)}}{(n!)(n\beta+1)}$$

$$+ \frac{\pi R(T - T_1)^2}{2} - CIR_1 M - \frac{T_1}{2}$$

(3.2.2.17)

As in case 1, the total cost is minimized when $\frac{\partial K_3(T_1,T)}{\partial T_1} = 0$ (eq. 3.2.2.18) and $\frac{\partial K_3(T_1,T)}{\partial T} = 0$ (eq. 3.2.2.19) as shown below.

$$\frac{\partial K_3(T_1,T)}{\partial T_1} = 0$$

$$\therefore \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T_{1}^{n\beta}}{n!} - 1 \right] + \frac{CIR R}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n} T_{1}^{n\beta+1}}{(n!)(n\beta+1)} \right] - \frac{\pi R(T - T_1)^2}{2} - CIR_1 M - \frac{T_1}{2} = 0$$

(3.2.2.18)

$$\frac{\partial K_3(T_1,T)}{\partial T} = 0$$

$$\therefore - CR\left[ \sum_{n=0}^{\infty} \frac{\alpha^n T_{1}^{n\beta+1}}{n!(n\beta+1)} - T_1 \right] - A - \frac{CIR}{2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n} T_{1}^{2(n\beta+1)}}{(n!)(n\beta+1)} \right]$$

$$- \frac{\pi R(T - T_1)^2}{2} + \pi R(T - T_1)T = 0$$

(3.2.2.19)

Equations (3.2.2.18) and (3.2.2.19) need to be solved simultaneously for optimal values of $T_1$ and $T$ as it is done in case 1.
3.2.3 Numerical Example and observations:

Consider, the inventory parametric values \( R = 2000 \text{ units / year} \), \( A = $250 / \text{order} \), \( I = 10\% / \text{unit / year} \), \( I_e = 12\% / \text{unit / year} \), \( I_c = 15\% / \text{unit / year} \), \( \beta = 1.5 \). Following the procedure given in above section, the economic ordering policies computed for different values of \( \alpha \), \( C \) and \( \pi \) are given in Tables 1–3.

### Table 3.2.1 (Variation in \( \alpha \))

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( T )</th>
<th>( T_I )</th>
<th>( Q )</th>
<th>( Q_I )</th>
<th>( Q_2 )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.254</td>
<td>0.187</td>
<td>508</td>
<td>374</td>
<td>134</td>
<td>3593.72</td>
</tr>
<tr>
<td>0.03</td>
<td>0.232</td>
<td>0.146</td>
<td>464</td>
<td>292</td>
<td>172</td>
<td>3495.57</td>
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<tr>
<td>0.04</td>
<td>0.217</td>
<td>0.123</td>
<td>434</td>
<td>246</td>
<td>188</td>
<td>3420.88</td>
</tr>
</tbody>
</table>

### Table 3.2.2 (Variation in \( C \))

<table>
<thead>
<tr>
<th>( C )</th>
<th>( T )</th>
<th>( T_I )</th>
<th>( Q )</th>
<th>( Q_I )</th>
<th>( Q_2 )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.254</td>
<td>0.187</td>
<td>508</td>
<td>374</td>
<td>134</td>
<td>3593.72</td>
</tr>
<tr>
<td>25</td>
<td>0.231</td>
<td>0.161</td>
<td>462</td>
<td>322</td>
<td>140</td>
<td>4192.48</td>
</tr>
<tr>
<td>30</td>
<td>0.220</td>
<td>0.148</td>
<td>440</td>
<td>296</td>
<td>144</td>
<td>4865.34</td>
</tr>
<tr>
<td>35</td>
<td>0.218</td>
<td>0.132</td>
<td>436</td>
<td>264</td>
<td>172</td>
<td>5692.00</td>
</tr>
<tr>
<td>40</td>
<td>0.204</td>
<td>0.121</td>
<td>408</td>
<td>242</td>
<td>166</td>
<td>6339.54</td>
</tr>
</tbody>
</table>

### Table 3.2.3 (Variation in \( \pi \))

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>( T )</th>
<th>( T_I )</th>
<th>( Q )</th>
<th>( Q_I )</th>
<th>( Q_2 )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.254</td>
<td>0.187</td>
<td>508</td>
<td>374</td>
<td>134</td>
<td>3593.72</td>
</tr>
<tr>
<td>35</td>
<td>0.242</td>
<td>0.193</td>
<td>484</td>
<td>386</td>
<td>98</td>
<td>3421.70</td>
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<tr>
<td>40</td>
<td>0.234</td>
<td>0.211</td>
<td>468</td>
<td>422</td>
<td>46</td>
<td>3254.37</td>
</tr>
<tr>
<td>45</td>
<td>0.223</td>
<td>0.219</td>
<td>446</td>
<td>438</td>
<td>8</td>
<td>3092.23</td>
</tr>
</tbody>
</table>

Observations:

- Increase in deterioration rate reduces optimal cycle time, as a result optimal procurement quantity decreases. With increase in deterioration there is a decrease in \( T_I \) and \( Q_I \). As a result shortages increase. Because we are procuring smaller quantities there is a decrease in total inventory cost of the system.
• Increase in the purchase cost, decreases the optimal cycle time, optimal procurement quantity $Q_j, T_j, Q_j$ and increases shortages and also increases the total cost of the inventory system.
• Increase in the unit backorder cost decreases optimal cycle time, optimal procurement quantity and total cost of an inventory system whereas there is increase in $T_j$ and $Q_j$.

3.3 AN EOQ MODEL FOR DETERIORATING ITEMS WITH TWO PARAMETER WEIBULL DISTRIBUTION DETERIORATION UNDER SUPPLIER CREDITS

In this section, an inventory model is developed for deteriorating items with two parameter Weibull distribution, in which the supplier provides both cash discount and credit period to the customer. Also, in the model, lead time is zero, replenishment is infinite and shortages are not allowed.

In the model the following four cases are discussed based on the provisions of discount and permissible delay periods.

Case 1. The payment is paid at $M_1$ to get a cash discount and cycle time $T \geq M_1$
Case 2. The customer pays in full at $M_1$ to get a cash discount but cycle time $T < M_1$
Case 3. The payment is paid at time $M$ to the permissible credit and cycle time $T \geq M$
Case 4. The customer pays in full at $M$ and cycle time $T < M$

The Taylor's series approximation is used to determine the mathematical results. A numerical example is provided to verify the results obtained in the market. A few directions for future research are also given.
3.3.1 Assumptions and Notations:

The mathematical model is derived with the following additional assumptions and notations other than those given in A.1 and N.1

- The demand for the item is constant during the cycle time.
- Shortages are not allowed.
- The distribution of the time for deterioration of units is as given in 1.3.1

Notations:

- \( r \) = the cash discount rate, \( 0 < r < 1 \).
- \( M \) = The period of permissible delay in settling account, with \( M > M_i \).
- \( K(T) \) = The total relevant cost per year which consists of (a) ordering cost, (b) cost of deteriorating units, (c) inventory carrying cost (excluding interest charges), (d) cash discount earned if the payment is made at \( M_i \), (e) cost of interest charges for unsold items after the permissible credit period \( M \), minus (f) interest earned from sales revenue during the permissible delay period.

3.3.2 Mathematical Formulation:

The inventory level \( Q(t) \) gradually decreases to meet demands and partly due to deterioration. Hence the instantaneous rate of inventory level at any instant of time \( t \) can be represented by the following differential equation

\[
\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \quad 0 \leq t \leq T 
\]  
(3.3.2.1)

with the boundary conditions \( Q(0) = Q \) and \( Q(T) = 0 \). Then the solution of differential equation (3.3.2.1) is given by

\[
Q(t) = Re^{-\alpha \beta t} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)} (T^{n\beta+1} - t^{n\beta+1})
\]  
(3.3.2.2)

and the order quantity is
Study of optimal ordering policies for time varying decay rate of inventory

\[ Q = R \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta + 1}{n!(n\beta + 1)} \]  

(3.3.2.3)

Total demand during one cycle is \( RT \).

Hence, the number of units which deteriorate during a replenishment cycle is

\[ D(T) = Q - RT \]

\[ = R \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta + 1}{n!(n\beta + 1)} - T \right] \]  

(3.3.2.4)

The total relevant cost per time unit consists of the following components.

(a) \( OC = \) Cost of placing an order = \( \frac{A}{T} \)  

(3.3.2.5)

(b) \( CD = \) Cost of deteriorated unit = \[
\frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{d^n T^n \beta + 1}{n!(n\beta + 1)} - T \right]
\]  

(3.3.2.6)

(c) \( IHC = \) Cost of carrying inventory = \[
\frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(n!)^2 (n\beta + 1)^2} T^2(n\beta + 1)
\]  

(3.3.2.7)

Regarding cash discount, interests charged and earned, there are following four cases based on the customer’s two choices (i.e. : pays at \( M_1 \) or \( M \)) and the length of cycle time \( T \).

Case 1. The payment is paid at \( M_1 \) to get a cash discount and \( T \geq M_1 \) (fig. 3.3.1)

Case 2. The customer pays in full at \( M_1 \) to get a cash discount but \( T < M_1 \) (fig. 3.3.2)

Case 3. The payment is paid at time \( M \) to the permissible credit and \( T \geq M \) (fig. 3.3.3)

Case 4. The customer pays in full at \( M \) and \( T < M \) (fig. 3.3.4)

So next we derive interest earned and interest paid in each case.
Case 1: $T \geq M_1$

![Graph](image)

Hence the payment is paid to be made at time $M_1$. Using (4), the cash discount per year is given by

$$CDC = \frac{rcQ}{T} = \frac{rcR}{T} \sum_{n=0}^{\infty} \frac{\alpha^n T^{n \beta + 1}}{n!(n \beta + 1)} \tag{3.3.2.8}$$

The interest charged per year is

$$IC_1 = \frac{CIR}{T} \int_{M_1}^{T} Q(t) dt$$

$$= \frac{CIR}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n (\beta + 1)}{(n!)^2 (n \beta + 1)^2} \left[ \frac{T^2 (n \beta + 1)}{2} - M_1^{n \beta + 1} (T^{n \beta + 1} - \frac{M_1^{n \beta + 1}}{2}) \right] \tag{3.3.2.9}$$

The interest charged per year is

$$IE_1 = \frac{PI}{T} \int_{0}^{M_1} R(t) dt = \frac{PI}{T} \frac{e^{RM_1^2}}{2T} \tag{3.3.2.10}$$

From (3.3.2.5) – (3.3.2.10), the total relevant cost per year $K_1(T)$ is given by

$$K_1(T) = OC + CD + IHC + CDC + IC_1 - IE_1$$
Study of optimal ordering policies for time varying decay rate of inventory.

\[
\begin{align*}
&= A + \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T \right] + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2} T^{2(n\beta+1)} \frac{1}{2} \\
&\quad + \frac{rcR}{T} \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - \frac{PIe^{R(M_1 - \frac{T}{2})}}{2T}
\end{align*}
\]

(3.3.2.11)

Case 2: \( T < M_1 \)

In this case, the interest charges are zero, but the cash discount is same as that in case 1. The interest earned per year is

\[
IE_2 = \frac{PL_e}{T} \left[ \int_0^T R dt - RT (M_1 - T) \right] = P L_e R (M_1 - \frac{T}{2})
\]

(3.3.2.12)

As a result, the total relevant cost per year \( K_2(T) \) is

\[
K_2(T) = OC + CD + IHC + CDC - IE_2
\]

\[
= A + \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - T \right] + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2} T^{2(n\beta+1)} \\
+ \frac{rcR}{T} \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{n!(n\beta+1)} - \frac{PL_e R (M_1 - \frac{T}{2})}{2T}
\]

(3.3.2.13)
Case 3. \( T \geq M \)

Here the payment is made at time \( M \), there is no cash discount.

The interest payable per year is

\[
IC_3 = \frac{C_l R}{T} \int_0^T Q(t) \, dt \\
= \frac{C_l R}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n (\beta+1)}{(n!)^2 (n\beta+1)^2} \left[ \frac{T^2 (n\beta+1)}{2} - M \frac{n\beta+1}{2} \left( T^n \beta + 1 \frac{M^n \beta+1}{2} \right) \right]
\]

(3.3.2.14)

The interest earned per year is

\[
IE_3 = \frac{P_l R}{T} \int_0^M R \, dt = \frac{P_l R M}{2T}
\]

(3.3.2.15)

Therefore, the total relevant cost per year \( K_3(T) \) is

\[
K_3(T) = OC + CD + IHC + IC_3 - IE_3 \\
= \frac{A}{T} + \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta + 1}{n!(n\beta+1)} - T \right] + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n\beta+1)^2} \left[ \frac{T^2 (n\beta+1)}{2} - M \frac{n\beta+1}{2} \left( T^n \beta + 1 \frac{M^n \beta+1}{2} \right) \right]
\]

\[
+ \frac{C_l R}{T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n (\beta+1)}{(n!)^2 (n\beta+1)^2} \left[ \frac{T^2 (n\beta+1)}{2} - M \frac{n\beta+1}{2} \left( T^n \beta + 1 \frac{M^n \beta+1}{2} \right) \right]
\]

\[
- \frac{P_l R M^2}{2T}
\]

(3.3.2.16)
Case 4. \( T < M \)

In this case, the interest charged is zero. The interest earned per year is

\[
IE_4 = \frac{CI}{T} e^{\frac{T}{R}} \left[ \int_0^T Rdt + RT(M-T) \right] = \frac{CI}{T} e^{\frac{T}{R}} \left[ M - \frac{T}{2} \right]
\]

Hence the total relevant cost per year \( K_4(T) \) is

\[
K_4(T) = OC + CD + IHC - IE_4
\]

\[
= \frac{A}{T} + CR \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n \beta + 1}}{n!(n \beta + 1)!} - T \right] + \frac{CIR}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n!)^2 (n \beta + 1)^2} T^{2(n \beta + 1)}
\]

\[
- \frac{CI}{T} e^{\frac{T}{R}} \left[ M - \frac{T}{2} \right]
\]

Theoretical Results:

Assuming, \( \alpha \) - the rate deterioration to be very small, we get

\[
K_1(T) = \frac{A}{T} + \frac{CR \alpha T^{\beta + 1}}{\beta + 1} (1+r) + CR \left( \frac{IT}{2} + r \right) + \frac{CI}{T} e^{\frac{T}{R}} \left[ \frac{T^2}{2} - M_1(T - \frac{M_1}{2}) \right]
\]

\[
- \frac{CI e^{\frac{T}{R}} \alpha^{\beta + 1}}{T(\beta + 1)^2} \left[ \frac{T^2(\beta + 1)}{2} - M_1^{\beta + 1} (T^{\beta + 1} - \frac{M_1^{\beta + 1}}{2}) \right] - \frac{PI e^{\frac{T}{R}} M_1^2}{2T} \quad T \geq M_1
\]
Study of optimal ordering policies for time varying decay rate of inventory.

\[ K_2(T) = \frac{A}{T} + \frac{CR\alpha T^\beta}{\beta + 1} (1 + r) + CR\left(\frac{IT}{2} + r\right) - \frac{PIe R}{T} \left( M_1 - \frac{T}{2} \right) \quad T < M_1 \]  
(3.3.2.20)

\[ K_3(T) = \frac{A}{T} + \frac{CR\alpha T^\beta}{\beta + 1} + \frac{CIRT}{2} + \frac{CIR}{T} \left[ \frac{T^2}{2} - M(T - M) \right] \]

\[ - \frac{CIR}{T} + \frac{CIR^\alpha T^\beta + 1}{T(\beta + 1)^2} \left[ \frac{T^2(\beta + 1)}{2} - M\beta + 1(T\beta + 1) - \frac{M\beta + 1}{2} \right] - \frac{PIe R M^2}{2T} \]  
(3.3.2.21)

\[ K_4(T) = \frac{A}{T} + \frac{CR\alpha T^\beta}{\beta + 1} + \frac{CIRT}{2} - \frac{PIe R}{T} \left( M - \frac{T}{2} \right) \quad T < M \]  
(3.3.2.22)

The first order condition for \( K_1(T) \) in (3.3.2.19) to be minimized is

\[ \frac{dK_1(T)}{dT} = 0 \]

\[ - \frac{A}{T} + \frac{CR\alpha T^\beta}{\beta + 1} (1 + r) + \frac{CIR}{2} - \frac{CIR}{T^2} \left[ \frac{T^2}{2} - M_1(T - M_1) \right] \]

\[ + \frac{CIR}{T} (T - M_1) + \frac{CIR}{T^2} \left[ \frac{T^2(\beta + 1)}{2} - M_1\beta + 1(T\beta + 1) - \frac{M_1\beta + 1}{2} \right] \]

\[ - \frac{CIR}{T} \left[ \frac{T^2\beta + 1 - M_1^\beta + 1(T\beta + 1)}{2} \right] + \frac{PIe R M^2}{2T^2} = 0 \]  
(3.3.2.23)

For the second order condition, we obtain

\[ \frac{d^2K_1(T)}{dT^2} = \frac{2A}{T^3} + \frac{CR\alpha (\beta - 1)T^\beta - 2}{\beta + 1} (1 + r) + \frac{2CIR}{T^3} \left[ \frac{T^2}{2} - M_1(T - M_1) \right] \]

\[ - \frac{CIR}{T^2} (T - M_1) - \frac{CIR}{T^2} (T - M_1) + \frac{CIR}{T^2} \left[ \frac{T^2\beta + 1 - M_1^\beta + 1(T\beta + 1)}{2} \right] \]

\[ - \frac{2CIR}{T^3} \left[ \frac{T^2(\beta + 1)}{2} - M_1(\beta + 1)(T\beta + 1) - \frac{M_1\beta + 1}{2} \right] \]

\[ - \frac{CIR}{T^3} \left[ (2\beta + 1)T^2\beta - \beta M_1^\beta + 1(T\beta + 1)^{-1} \right] - \frac{PIe R M^2}{2T^2} \]  
(3.3.2.24)
Equation (3.3.2.23) is highly non-linear. So it can be solved by using suitable numerical method. (e.g. Newton – Raphson Method). Consequently, we obtain optimal value of $T = T_1$ for case 1. Ensure that $T_1 > M_1$.

The first order condition for case 2 is

$$\frac{dK_2(T)}{dT} = 0$$

$$-\frac{A}{T} + \frac{C R \alpha \beta T^{\beta-1}}{\beta+1} (1+r) + \frac{C I R}{2} \frac{P I e R}{T^2} (M_1 - \frac{T}{2}) + \frac{P I e R}{2T} = 0$$

(3.3.2.25)

and for the second order condition, we obtain

$$\frac{d^2 K_2(T)}{dT^2} = \frac{2A}{T^3} + \frac{C R \alpha \beta T^{\beta-1}}{\beta+1} (1+r) - \frac{2 P I e R M_1}{T^3}$$

(3.3.2.26)

Solving (3.3.2.25) by suitable numerical method, we obtain the optimal value of $T = T_2$ for case 2. Ensure that $T_2 < M_1$.

Arguing as above, the first order condition for finding the optimal value of $T = T_3$ for case 3 is

$$\frac{dK_3(T)}{dT} = 0$$

$$-\frac{A}{T^2} + \frac{C R \alpha \beta T^{\beta-1}}{\beta+1} + \frac{C I R C R}{T^2} \frac{2}{(T^2(M - \frac{T}{2}))}$$

$$+ \frac{C I R}{T} (T - M) + \frac{C I R \alpha \beta T^{\beta+1}}{T^2(\beta+1)^2} \left[ \frac{T^2(\beta+1)}{2} - M^{\beta+1}(T^{\beta+1} - \frac{M^{\beta+1}}{2}) \right]$$

$$- \frac{C I R \alpha \beta T^{\beta+1}}{T(\beta+1)} \left[ T^{2\beta+1} - M^{\beta+1}T^\beta \right] + \frac{P I e R M^2}{2T^2} = 0$$

(3.3.2.27)

and for the second order condition, we obtain
\[
\frac{d^2 K_3(T)}{dT^2} = \frac{2A}{T^3} + \frac{CR \alpha \beta (\beta - 1)T^\beta - 2}{\beta + 1} + \frac{2CI_c R}{T^3} \left[ \frac{T^2}{2} - M(T - M) \right]
\]

\[
-\frac{CI_c R}{T^2} (T - M) - \frac{CI_c R}{T^2} (T - M) + \frac{CI_c R \alpha^{\beta + 1}}{T^2 (\beta + 1)^2} \left[ T^{2\beta + 1} - M^{\beta + 1}T^\beta \right]
\]

\[
-\frac{2CI_c R \alpha^{\beta + 1}}{T^3 (\beta + 1)^2} \left[ \frac{T^{2(\beta + 1)}}{2} - M^{\beta + 1}(T^{\beta + 1} - \frac{M^{\beta + 1}}{2}) \right]
\]

\[
-\frac{CI_c R \alpha^{\beta + 1}}{T(\beta + 1)} \left[ (2\beta + 1)T^2\beta - \beta M^{\beta + 1}T^\beta - 1 \right] - \frac{PI_e RM^2}{T^3}
\]

(3.3.28)

For case 4, the first order condition is

\[
-\frac{A}{T^2} + \frac{CR \alpha \beta^{\beta - 1}}{\beta + 1} + \frac{CIR}{2} + \frac{PI_e R}{T^2} M = 0
\]

(3.3.29)

and for the second order condition, we obtain

\[
\frac{d^2 K_4(T)}{dT^2} = \frac{2A}{T^3} + \frac{CR \alpha \beta (\beta - 1)T^\beta - 2}{\beta + 1} - \frac{2PI_e RM}{T^3}
\]

Eq. (3.3.29) gives optimum value of \( T = T_4 \). Ensure that \( T_4 < M \).

### 3.3.3 Numerical Example and observations:

Consider numerical value of parameters in proper units:

\[
[A, C, I, I_e, r] = [250, 20, 10\%, 12\%, 2\%]
\]

The following tables give effect of various parameters on optimum cycle time, optimum purchase quantity and total relevant cost per year.
Table – 3.3.3.1

<table>
<thead>
<tr>
<th>$\alpha \setminus R$</th>
<th>1000</th>
<th>500</th>
<th>750</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3116</td>
<td>0.4399</td>
<td>0.3596</td>
</tr>
<tr>
<td>$Q$</td>
<td>311.89</td>
<td>220.20</td>
<td>269.95</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1870.57</td>
<td>1267.86</td>
<td>1589.13</td>
</tr>
<tr>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3076</td>
<td>0.4332</td>
<td>0.3546</td>
</tr>
<tr>
<td>$Q$</td>
<td>308.08</td>
<td>217.11</td>
<td>266.45</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1884.63</td>
<td>1279.63</td>
<td>1602.19</td>
</tr>
<tr>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3038</td>
<td>0.4269</td>
<td>0.3499</td>
</tr>
<tr>
<td>$Q$</td>
<td>304.45</td>
<td>214.19</td>
<td>263.13</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1898.43</td>
<td>1291.13</td>
<td>1614.99</td>
</tr>
</tbody>
</table>

Increase in the demand rate reduces cycle time, increases procurement quantity and total cost of the inventory system while increase in the deterioration rate decreases optimal procurement quantity and increases the total cost of the inventory system significantly.

Table – 3.3.3.2

<table>
<thead>
<tr>
<th>$\alpha \setminus \beta$</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3116</td>
<td>0.3132</td>
<td>0.3142</td>
</tr>
<tr>
<td>$Q$</td>
<td>311.89</td>
<td>313.35</td>
<td>314.34</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1870.57</td>
<td>1862.96</td>
<td>1859.48</td>
</tr>
<tr>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3076</td>
<td>0.3106</td>
<td>0.3127</td>
</tr>
<tr>
<td>$Q$</td>
<td>308.08</td>
<td>310.89</td>
<td>312.83</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1884.63</td>
<td>1869.58</td>
<td>1862.69</td>
</tr>
<tr>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3038</td>
<td>0.3082</td>
<td>0.3112</td>
</tr>
<tr>
<td>$Q$</td>
<td>304.45</td>
<td>308.54</td>
<td>311.36</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1898.43</td>
<td>1876.09</td>
<td>1865.86</td>
</tr>
</tbody>
</table>

Keeping $\alpha$ fixed, increase in shape parameter $\beta$ increases cycle time and procurement quantity while decreases the total cost of the inventory system.
Study of optimal ordering policies for time varying decay rate of inventory.

Table – 3.3.3.3

| \(\beta\ \|\ R\ |  | 500 | 750 | 1000 |
|---|---|---|---|---|---|
| 1.5 | \(T\) | 0.4399 | 0.3596 | 0.3116 |
|   | \(Q\) | 220.20 | 269.95 | 311.89 |
|   | \(K_{i}(T)\) | 1267.86 | 1589.13 | 1870.57 |
| 2 | \(T\) | 0.4417 | 0.3613 | 0.3132 |
|   | \(Q\) | 220.99 | 271.12 | 313.35 |
|   | \(K_{i}(T)\) | 1262.52 | 1582.51 | 1862.96 |
| 2.5 | \(T\) | 0.4431 | 0.3625 | 0.3142 |
|   | \(Q\) | 221.67 | 271.98 | 314.34 |
|   | \(K_{i}(T)\) | 1259.66 | 1579.27 | 1859.48 |

Increase in demand rate decreases the cycle time, increases the optimal procurement quantity and significantly increases the total cost of the inventory system. The increase in \(\beta\) decreases the total cost of an inventory system and increases the optimal procurement quantity.

Table – 3.3.3.4

| \(R\ \|\ P\ |  | 30 | 40 | 50 |
|---|---|---|---|---|---|
| 500 | \(T\) | 0.4399 | 0.4394 | 0.4390 |
|   | \(Q\) | 220.20 | 219.98 | 219.76 |
|   | \(K_{i}(T)\) | 1267.86 | 1266.71 | 1265.55 |
| 750 | \(T\) | 0.3596 | 0.3590 | 0.3585 |
|   | \(Q\) | 269.95 | 269.54 | 269.13 |
|   | \(K_{i}(T)\) | 1589.13 | 1587.01 | 1584.89 |
| 1000 | \(T\) | 0.3116 | 0.3110 | 0.3104 |
|   | \(Q\) | 311.89 | 311.26 | 310.63 |
|   | \(K_{i}(T)\) | 1870.57 | 1867.32 | 1864.06 |

Increase in the selling price decreases the number of units to be procured and as a result total cost of an inventory system decreases. For fixed selling price increase in demand increases the cost of the inventory system and the procurement quantity significantly.
Table – 3.3.3.5

<table>
<thead>
<tr>
<th>$\alpha \setminus P$</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>$T$ 0.3116, 0.3110, 0.3104</td>
<td>$Q$ 311.89, 311.26, 310.63</td>
<td>$K_j(T)$ 1870.57, 1867.32, 1864.06</td>
</tr>
<tr>
<td>0.02</td>
<td>$T$ 0.3076, 0.3070, 0.3064</td>
<td>$Q$ 308.08, 307.46, 306.84</td>
<td>$K_j(T)$ 1884.63, 1881.34, 1878.03</td>
</tr>
<tr>
<td>0.03</td>
<td>$T$ 0.3068, 0.3032, 0.3026</td>
<td>$Q$ 304.45, 303.84, 303.23</td>
<td>$K_j(T)$ 1898.43, 1895.09, 1891.74</td>
</tr>
</tbody>
</table>

For fixed $\alpha$, increase in the selling price decreases the values of the decision variables. For fixed selling price, increase in deterioration rate $\alpha$, reduces the number of units to be purchased but increases total cost of the inventory system.

Table –3.3.3.6

<table>
<thead>
<tr>
<th>$\beta \setminus P$</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>$T$ 0.3116, 0.3110, 0.3104</td>
<td>$Q$ 311.89, 311.26, 310.63</td>
<td>$K_j(T)$ 1870.57, 1867.32, 1864.06</td>
</tr>
<tr>
<td>2</td>
<td>$T$ 0.3132, 0.3126, 0.3119</td>
<td>$Q$ 313.35, 312.71, 312.08</td>
<td>$K_j(T)$ 1862.96, 1857.72, 1856.48</td>
</tr>
<tr>
<td>2.5</td>
<td>$T$ 0.3142, 0.3136, 0.3130</td>
<td>$Q$ 314.34, 313.71, 313.07</td>
<td>$K_j(T)$ 1859.48, 1856.25, 1853.02</td>
</tr>
</tbody>
</table>

For fixed shape parameter $\beta$, increase in the selling price reduces optimum quantity to be purchased and total cost of the inventory system. For fixed selling price, increase in the shape parameter $\beta$ decreases the total cost of an inventory system and increases the number of units to be purchased.
Table – 3.3.3.7

<table>
<thead>
<tr>
<th>( \alpha ) ( M )</th>
<th>15 / 365</th>
<th>30 / 365</th>
<th>45 / 365</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>( T )</td>
<td>0.3116</td>
<td>0.3107</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>311.89</td>
<td>310.94</td>
</tr>
<tr>
<td></td>
<td>( K_i(T) )</td>
<td>1870.57</td>
<td>1742.39</td>
</tr>
<tr>
<td>0.02</td>
<td>( T )</td>
<td>0.3076</td>
<td>0.3067</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>308.08</td>
<td>307.15</td>
</tr>
<tr>
<td></td>
<td>( K_i(T) )</td>
<td>1884.63</td>
<td>1756.38</td>
</tr>
<tr>
<td>0.03</td>
<td>( T )</td>
<td>0.3068</td>
<td>0.3029</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>304.45</td>
<td>303.54</td>
</tr>
<tr>
<td></td>
<td>( K_i(T) )</td>
<td>1898.43</td>
<td>1770.12</td>
</tr>
</tbody>
</table>

Increase in the delay period decreases the number of units to be purchased and the total cost of the inventory system. For fixed allowable credit period, increase in \( \alpha \), reduces the number of units to be purchased and increases the total cost of an inventory system.

Table – 3.3.3.8

<table>
<thead>
<tr>
<th>( \beta ) ( M )</th>
<th>15 / 365</th>
<th>30 / 365</th>
<th>45 / 365</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>( T )</td>
<td>0.3116</td>
<td>0.3107</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>311.89</td>
<td>310.94</td>
</tr>
<tr>
<td></td>
<td>( K_i(T) )</td>
<td>1870.57</td>
<td>1742.39</td>
</tr>
<tr>
<td>2.0</td>
<td>( T )</td>
<td>0.3132</td>
<td>0.3123</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>313.35</td>
<td>312.40</td>
</tr>
<tr>
<td></td>
<td>( K_i(T) )</td>
<td>1862.96</td>
<td>1734.80</td>
</tr>
<tr>
<td>2.5</td>
<td>( T )</td>
<td>0.3142</td>
<td>0.3133</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>314.34</td>
<td>313.39</td>
</tr>
<tr>
<td></td>
<td>( K_i(T) )</td>
<td>1859.48</td>
<td>1731.33</td>
</tr>
</tbody>
</table>

Increase in the delay period for fixed shape parameter, decreases the number of units to be procured and the total cost of the inventory system. For fixed allowable credit period, increase in the shape parameter increases the number of units to be purchased and decreases the total cost of the inventory system.
Table – 3.3.3.9

<table>
<thead>
<tr>
<th>$P \setminus M$</th>
<th>15/365</th>
<th>30/365</th>
<th>45/365</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>30</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3116</td>
<td>0.3107</td>
<td>0.3091</td>
</tr>
<tr>
<td>$Q$</td>
<td>311.89</td>
<td>310.94</td>
<td>309.36</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1870.57</td>
<td>1742.39</td>
<td>1610.91</td>
</tr>
<tr>
<td><strong>40</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3110</td>
<td>0.3081</td>
<td>0.3033</td>
</tr>
<tr>
<td>$Q$</td>
<td>311.26</td>
<td>308.40</td>
<td>303.59</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1867.32</td>
<td>1729.29</td>
<td>1581.13</td>
</tr>
<tr>
<td><strong>50</strong></td>
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<tr>
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<td>0.2975</td>
</tr>
<tr>
<td>$Q$</td>
<td>310.63</td>
<td>305.84</td>
<td>297.70</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1864.06</td>
<td>1716.08</td>
<td>1550.77</td>
</tr>
</tbody>
</table>

For fixed allowable credit period, increase in the selling price reduces the number of units to be procured and the total cost of the inventory system. For fixed selling price, increase in the allowable credit period reduces the number of units procured and the total cost of the inventory system.

Table – 3.3.3.10

<table>
<thead>
<tr>
<th>$I_c \setminus M$</th>
<th>15/365</th>
<th>30/365</th>
<th>45/365</th>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>$T$</td>
<td>0.3311</td>
<td>0.3291</td>
<td>0.3257</td>
</tr>
<tr>
<td>$Q$</td>
<td>331.40</td>
<td>329.39</td>
<td>326.00</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1797.32</td>
<td>1689.47</td>
<td>1575.35</td>
</tr>
<tr>
<td><strong>0.15</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>0.3116</td>
<td>0.3107</td>
<td>0.3091</td>
</tr>
<tr>
<td>$Q$</td>
<td>311.89</td>
<td>310.94</td>
<td>309.36</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1870.57</td>
<td>1742.39</td>
<td>1610.91</td>
</tr>
<tr>
<td><strong>0.18</strong></td>
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<td></td>
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<tr>
<td>$T$</td>
<td>0.2953</td>
<td>0.2953</td>
<td>0.2953</td>
</tr>
<tr>
<td>$Q$</td>
<td>295.50</td>
<td>295.50</td>
<td>295.50</td>
</tr>
<tr>
<td>$K_i(T)$</td>
<td>1938.57</td>
<td>1790.61</td>
<td>1642.65</td>
</tr>
</tbody>
</table>

For fixed delay period, increase in the interest charges to be paid reduces the cycle time and number of units to be procured and increases the total cost of the inventory system.
3.4 Conclusion:

In this chapter three mathematical models are formulated. Section 3.1 is a lot-size model with time dependent deterioration of units and permissible delay in payments which is extended by allowing shortages to an order level lot size model in Section 3.2. Different scenarios of permissible delay in payments are discussed. It is found that the model in Section 3.1 is sensitive to the rate of deterioration of units in the inventory system, demand rate, purchase price of a unit, ordering cost and the allowable delay period. The model in Section 3.2 is also sensitive to the backorder cost. By putting $T_1 = 0$ in the model in Section 3.2, it reduces to the model in Section 3.1. These models can be extended by introducing inflation rate, price dependent demand etc.

In Section 3.3 an attempt is made to develop an EOQ model for time dependent deteriorating items to determine the optimal ordering policy when the supplier offers cash discount and a permissible delay in payments. The Taylor series approximation is used to derive analytic results. A numerical example is provided to verify the results obtained in market. The proposed model can be extended taking demand to be a function of selling price, time varying and stock dependent. It can be generalized to allow for shortages and inflation rates.