CHAPTER 2

INVENTORY MODELS WITH TIME DEPENDENT DETERIORATION OF UNITS
2.0 Introduction:

In this chapter, two inventory models have been formulated. They are

Section 2.1 - A lot size model for deteriorating items with time dependent deterioration.
Section 2.2 - An order level lot size model for time dependent deterioration.

2.1 A LOT SIZE MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT DETERIORATION

In this section a lot-size inventory model is developed for deteriorating items with a time dependent rate of deterioration. The EOQ formula is derived under assumptions of constant demand, zero lead-time and no shortages. It is shown that the results can be reduced to known models. Analytic proof of parameter dependence is given. A numerical example is used to show the solution pattern.

2.1.1 Assumptions and Notations:

The lot-size inventory model for deteriorating items will be developed using the following additional assumptions other than those given in A.1:  
1. Shortages are not allowed.
2. The distribution of the time to deterioration of the item is as given in Equation 1.3.1
3. Q is a decision variable.

2.1.2 Mathematical Formulation:

Let \( Q(t), \, 0 \leq t \leq T \) denotes on-hand inventory of units at time \( t \). The instantaneous state of \( Q(t) \) for any instant of time , follows the differential equation

\[
\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R \quad 0 \leq t \leq T, \tag{2.1.2.1}
\]
with initial condition \( Q(0) = Q \) and boundary condition \( Q(T) = 0 \).

Equation (2.1.2.1) is the first order linear differential equation whose general solution is

\[
Q(t) = -Re^{-\alpha t} \int_0^t \alpha e^{\beta dt} + k e^{-\alpha t} \]

with boundary condition \( Q(T) = 0 \), we get particular solution as

\[
Q(t) = Re^{-\alpha t} \int_0^T e^{-\alpha t} dt \tag{2.1.2.2}
\]

\[
Q(t) = Re^{-\alpha t} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)}(T^n\beta+1-t^n\beta+1)
\]

and hence \( Q(0) = Q \) gives

\[
Q = R \sum_{n=0}^{\infty} \frac{\alpha^n T^n\beta+1}{n!(n\beta+1)} \tag{2.1.2.3}
\]

Number of units that deteriorate during \([0, T]\) is given by

\[
D(T) = Q - RT
\]

\[
= R \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n\beta+1}{n!(n\beta+1)} - T \right]
\]
Hence cost due to deterioration per time unit is

\[ CD = \frac{CD(T)}{T} \]

\[ = CR \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta^1}{n!(n\beta+1)} - T \right] \]  

(2.1.2.4)

The Ordering cost per time unit is

\[ OC = \frac{A}{T} \]  

(2.1.2.5)

Now, inventory on-hand per time unit \( I(T) = \frac{1}{T} \int_0^T Q(t) \, dt \)

\[ = \frac{R}{T} \int_0^T e^{-\alpha t} \frac{\alpha^n}{n!(n\beta+1)} (T^n \beta^1 - t^n \beta^1) \, dt \]

\[ = \frac{R}{T} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta+1)} \int_0^T e^{-\alpha t} (T^n \beta^1 - t^n \beta^1) \, dt \]

\[ = \frac{R}{2T} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^2 n}{(n!)^2 (n\beta+1)^2} T^2 (n\beta+1) \]  

Hence, inventory holding cost, \( IHC \) per time unit is

\[ = CIR \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^2 n}{(n!)^2 (n\beta+1)^2} T^2 (n\beta+1) \]  

(2.1.2.6)

Using Equations (2.1.2.4) - (2.1.2.6), we get total cost, \( K(T) \) of an inventory system per time unit as

\[ K(T) = CD + OC + IHC \]

\[ = CR \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^n \beta}{n!(n\beta+1)} - 1 \right] + \frac{A}{T} + CIR \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^2 n}{(n!)^2 (n\beta+1)^2} T^2 n\beta + 1 \]
The optimum value of cycle time $T = T_0$ can be obtained by solving $\frac{dK(T)}{dT} = 0$

\[ CR \sum_{n=0}^{\infty} \frac{n\beta \alpha^n T^n \beta + 1}{n!(n\beta + 1)} - A + \frac{CIR}{2} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}(2n\beta + 1)T^{2(n\beta + 1)}}{(n!)^2(n\beta + 1)^2} = 0 \]

using suitable numerical method. The above equation can be solved for any sets of parametric values. The series can be truncated if $\alpha^\beta$ is less than one because $0 < \alpha < 1$ while $T$ is time, the proper selection of the dimensions of time will allow the convergence of the solution.

**2.1.3 Special Cases:**

1) Ghare and Schrader’s (1963) model can be obtained by putting $\alpha = \theta$ and $\beta = 1$ in the above model.

\[ Q = R \sum_{n=0}^{\infty} \frac{\theta^n T^n + 1}{n!(n+1)} \]

\[ K(T) = CR\{ \sum_{n=0}^{\infty} \frac{\theta^n T^n}{n!(n+1)} - 1 \} + \frac{A}{T} + \frac{CIR}{2} \sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n} T^{2n+1}}{(n!)^2(n+1)^2} \]

2) By taking $\alpha = 0$, the model reduces to that of Naddor (1966)

\[ Q = RT \]

\[ K(T) = \frac{A}{T} + \frac{CIRT}{2} \]
2.1.4 Assertions:

1. Total cost of an inventory system per time unit increases with respect to the scale parameter $\alpha$

Proof:

$$\frac{dK}{d\alpha} = CR \sum_{n=0}^{\infty} \frac{n\alpha^{n-1}T^{n\beta}}{n!(n\beta+1)} + \frac{CIR}{2} \sum_{n=1}^{\infty} \frac{(-1)^n 2n\alpha^{2n-1}}{(n+1)^2 (n\beta+1)^2} T^{2n\beta+1} > 0 \quad \forall \ T.$$

2. Total cost of an inventory system per time unit increases with respect to the ordering cost per order.

Proof:

$$\frac{dK}{dA} = \frac{1}{T} > 0 \quad \forall \ T$$

3. Increase in demand increases total cost of an inventory system per time unit.

Proof:

$$\frac{dK}{dR} = C \left[ \sum_{n=0}^{\infty} \frac{\alpha^{nTn\beta}}{n!(n\beta+1)} - 1 \right] + \frac{CI}{2} \sum_{n=1}^{\infty} \frac{(-1)^n \alpha^{2n}}{(n+1)^2 (n\beta+1)^2} T^{2n\beta+1} > 0 \quad \forall \ T$$

4. Under the assumption that $0 \leq \alpha \leq 1$, neglecting $\alpha^2$ and its higher powers, increase in the shape parameter increases the total cost of an inventory system per time unit.

Proof:

$$\frac{dK}{d\beta} = \frac{R \alpha^\beta \beta^{-1}}{(\beta+1)(\beta+2)} \left[ 2 C I T^2 + C I T^2 + C (\beta+2)(\beta^2 + \beta - T) \right] > 0 \quad \forall \ T$$

2.1.5 Numerical Example and observations:

A hypothetical model is developed using the following parametric values.

$$[C, I, R, A, \alpha, \beta] = [20, 12\%, 1000, 150, 0.02, 1.5]$$

In the following tables, the effect of various parameters on optimum cycle time, optimal procurement quantity and minimum cost of an inventory system is studied.
Study of optimal ordering policies for time varying decay rate of inventory

Table 2.1.5.1 Variations in $\alpha$

$$[ C, I, R, A, \beta] = [20, 12\%, 1000, 150, 1.5]$$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
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<td>0.3185</td>
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<td>0.3053</td>
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<td>326.74</td>
<td>319.45</td>
<td>312.78</td>
<td>306.63</td>
</tr>
<tr>
<td>$K$</td>
<td>880.78</td>
<td>895.99</td>
<td>910.67</td>
<td>924.88</td>
<td>938.65</td>
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Table 2.1.5.2 Variations in $A$

$$[ C, I, R, \alpha, \beta] = [20, 12\%, 1000, 0.02, 1.5]$$

<table>
<thead>
<tr>
<th>$A$</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
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<tbody>
<tr>
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<td>0.4283</td>
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<td>334.75</td>
<td>385.10</td>
<td>429.26</td>
<td>469.02</td>
<td>505.46</td>
</tr>
<tr>
<td>$K$</td>
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<td>1019.70</td>
<td>1142.51</td>
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<td>1356.49</td>
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Table 2.1.5.3 Variations in $R$

$$[ C, I, A, \alpha, \beta] = [20, 12\%, 150, 0.02, 1.5]$$

<table>
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<tr>
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<th>1400</th>
<th>1600</th>
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<th>2000</th>
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<td>367.52</td>
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<td>1238.60</td>
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Table 2.1.5.4 Variations in $\beta$

$$[ C, I, A, R, \alpha] = [20, 12\%, 150, 1000, 0.02]$$

<table>
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<th>1.3</th>
<th>1.4</th>
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</table>
Table 2.1.5.5 Variations in $C$

$[I, A, R, \alpha, \beta] = [12\%, 150, 1000, 0.02, 1.5]$  

<table>
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<th>30</th>
<th>35</th>
<th>40</th>
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<td>274.65</td>
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<td>1075.06</td>
<td>1159.79</td>
<td>1238.60</td>
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Table 2.1.5.6 Variations in $I$

$[C, A, R, \alpha, \beta] = [20, 150, 1000, 0.02, 1.5]$  

<table>
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<th>13%</th>
<th>14%</th>
<th>15%</th>
<th>16%</th>
</tr>
</thead>
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<td>945.45</td>
<td>976.23</td>
<td>1006.11</td>
</tr>
</tbody>
</table>

**Observations:**

Table No. | Observations
---|---
2.1.5.1 | Increase in deterioration rate ($\alpha$) decreases cycle time and increases total cost of the inventory system.
2.1.5.2 | Increase in the ordering cost ($A$) increases the cycle time and the total cost of an inventory system significantly.
2.1.5.3 | Increase in demand rate ($R$) decreases the cycle time and increases the procurement quantity and the total cost of an inventory system significantly.
2.1.5.4 | Increase in the shape parameter ($\beta$) increases the cycle time and decreases the total cost of an inventory system.
2.1.5.5 | Increase in the purchase cost ($C$) decreases cycle time and procurement quantity while increases the total cost of an inventory system significantly.
2.1.5.6 Increase in the carrying charge fraction (I) per annum reduces the cycle time and the procurement quantity and increases the total cost of an inventory system.

2.2. AN ORDER LEVEL LOT-SIZE MODEL WITH TIME DEPENDENT DETERIORATION

In this section, a mathematical model is developed with same assumptions as those of section 2.1 by allowing shortages which are completely backlogged.

2.2.1 Assumptions and Notations:

The order-level lot-size inventory model for time dependent deterioration of units is developed under following additional assumptions and notations other than those given in A.1 and N.1 earlier.

1. Shortages are allowed and completely back-logged.
2. The shortage cost, \( \pi \), per unit is constant.
3. The distribution of the time for deterioration of the item is as given in 1.3.1.

2.2.2 Mathematical Formulation:

Suppose that the system carries inventory during \((0, T_j)\) and runs with shortages during \((T_j, T)\) (fig. 2.2.2.1). The instantaneous state of \( Q(t) \) which denotes on-hand inventory of units at time \( t \), follows the differential equation.

\[
\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R, \quad 0 \leq t \leq T_j \tag{2.2.2.1}
\]

\[
\frac{dQ(t)}{dt} = -R, \quad T_j \leq t \leq T
\]

with initial condition \( Q(T_j) = 0 \) and \( Q(0) = Q \)
Equation (2.2.2.1) is the first order linear differential equation whose solution using boundary condition \( Q(T_i) = 0 \) is given by

\[
Q(t) = \Re e^{-\alpha t^s} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!(n\beta + 1)} (T_{n\beta + 1} - t^{n\beta + 1})
\]  

(2.2.2.2)

and hence

\[
Q = Q(0) = R \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta + 1}}{n!(n\beta + 1)}
\]  

(2.2.2.3)

Number of units that deteriorate during \([0, T_i]\) is given by

\[
D(T) = Q - RT
\]

\[
= R \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta + 1}}{n!(n\beta + 1)} - T \right]
\]

Hence cost due to deterioration per time unit is

\[
CD = \frac{CD(T)}{T} = \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta + 1}}{n!(n\beta + 1)} - T \right]
\]  

(2.2.2.4)
The ordering cost per time unit is

\[ OC = \frac{A}{T} \]  

(2.2.2.5)

The inventory on-hand per time unit is given by

\[ I(T) = \frac{1}{T} \int_0^{T_i} Q(t)dt \]

\[ = \frac{R}{2T} \sum_{n=0}^\infty \frac{(-1)^n \alpha^{2n}}{(n\beta+1)^2} T_1^2(n\beta+1) \]

Hence, inventory holding cost, \(IHC\), per time unit is

\[ IHC = \frac{hR}{2T} \sum_{n=0}^\infty \frac{(-1)^n \alpha^{2n}}{(n\beta+1)^2} T_1^2(n\beta+1) \]  

(2.2.2.6)

The shortage cost, \(SC\), per time unit is

\[ SC = \frac{\pi R}{T} \int_{T_i}^T tdt \]

\[ = \frac{\pi R}{2T} (T-T_1)^2 \]  

(2.2.2.7)

Using equations (2.2.2.4) – (2.2.2.7), the total cost \(K(T_i, T)\) of an inventory system per time unit is given by

\[ K(T_i, T) = CD + OC + IHC + SC \]

\[ = \frac{CR}{T} \left[ \sum_{n=0}^\infty \frac{\alpha^{nT}n\beta+1}{n!(n\beta+1)} - T \right] + \frac{A}{T} + \frac{hR}{2T} \sum_{n=0}^\infty \frac{(-1)^n \alpha^{2n}}{(n\beta+1)^2} T_1^2(n\beta+1) + \frac{\pi R}{2T} (T-T_1)^2 \]  

(2.2.2.8)
Here, $T$ and $T_j$ are decision variables. The optimum value of $T_j$ and $T$ can be obtained by solving

$$\frac{\partial K(T,T_j)}{\partial T} = 0; \text{ i.e.}$$

$$\frac{CR}{T^2} \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T_{1} n \beta + 1}{n!(n\beta + 1)} - T_1 \right] - \frac{A}{T^2} - \frac{hR}{2T^2} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{n!(n\beta + 1)^2} T_1^{2n\beta + 1} \right]$$

$$- \frac{\pi R T_{1}^2}{2T^2} + \frac{\pi R}{2} = 0 \quad (2.2.2.9)$$

and

$$\frac{\partial K(T,T_j)}{\partial T_j} = 0; \text{ i.e.}$$

$$CR \left[ \sum_{n=0}^{\infty} \frac{\alpha^n T_{1} n \beta}{n!} - 1 \right] + hR \left[ \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{n!(n\beta + 1)^2} T_1^{2n\beta + 1} \right] - \pi R (T - T_1) = 0 \quad (2.2.2.10)$$

simultaneously using suitable iterative method. The obtained values of $T_j$ and $T$ will minimize the total cost of an inventory system provided

$$\left[ \frac{\partial^2 K(T,T_j)}{\partial T_j^2} \cdot \frac{\partial^2 K(T,T_j)}{\partial T^2} - \left[ \frac{\partial^2 K(T_j,T)}{\partial T \partial T_j} \right]^2 \right] > 0$$

### 2.2.3 Special Cases:

1) Shah's (1977) model can be obtained by putting $\alpha = \theta$ and $\beta = 1$ in the above model.

$$Q = R \sum_{n=0}^{\infty} \frac{\theta^n T_{1} n + 1}{n!(n + 1)}$$

$$K(T) = \frac{CR}{T} \left[ T_1 + \frac{\theta^n T_{1}^2}{2} - 2T_1 \right] + \frac{A}{T} - \frac{hR}{2T} \left[ T_1^2 - \frac{\theta^n T_{1}^4}{4} \right] + \frac{\pi R (T - T_1)^2}{2T}$$
2.2.4. Assertions:

1. Total cost of an inventory system per time unit will increase with respect to the scale parameter $\alpha$.

Proof:

$$
\frac{dK}{d\alpha} = \frac{CR}{T} \left[ \sum_{n=0}^{\infty} \frac{n\alpha^{n-1} T^{n} R}{n(n+1)} \right] + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2n\alpha^{n-1} R^2 T^{2(n+1)}}{n!(n+1)^2} \right] > 0, \quad \forall \ T, \ T_1
$$

2. Total cost of an inventory system per time unit increases with respect to the ordering cost per order.

Proof:

$$
\frac{dK}{dA} = \frac{1}{T} > 0, \quad \forall \ T
$$

3. Increase in demand increases total cost of an inventory system per time unit.

Proof:

$$
\frac{dK}{dR} = \frac{C}{T} \left[ \sum_{n=0}^{\infty} \frac{\alpha^{n} T^{n} R^{n+1}}{n(n+1)} - T_1 \right] + \frac{CIR}{2T} \left[ \sum_{n=0}^{\infty} \frac{(-1)^n 2n\alpha^{n+1} R^2 T^{2(n+1)}}{n!(n+1)^2} \right] + \frac{\pi}{2T} \left(T - T_1\right)^2 > 0, \quad \forall \ T, \ T_1
$$

2.2.5 Numerical Example and observations:

Consider an inventory system with following parameters in appropriate units.

$$
[ C, I, R, A, \pi, \alpha, \beta ] = [20, 12\%, 1000, 150, 30, 0.02, 1.5]
$$

In the following table, the effect of various parameters on optimum time for having positive inventory, optimum cycle time, optimal procurement quantity, optimum positive stock and minimum cost of an inventory system is studied.
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<th>Variable</th>
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<th>( T )</th>
<th>( Q_I )</th>
<th>( K )</th>
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Observations:

- Increase in purchase cost reduces cycle time of positive stock, increases optimum cycle time and total cost of an inventory system.
- Increase in ordering cost increases optimum cycle time, positive stock and total cost of an inventory system.
- Increase in shortage cost increases time of positive stock, and reduces total cycle time and total cost of an inventory system.
- Total cost increases significantly with increase in demand rate because procurement quantity increases drastically.
- Increase in deterioration rate ($\alpha$) reduces time of positive stock and cycle time while increases total cost of an inventory system.
- Increase in shape parameter ($\beta$) reduces all decision variables and total cost of an inventory system.

2.3 Conclusion:

In this chapter two models have been proposed and formulated. An EOQ model has been derived in section 2.1 and an order level lot-size inventory model has been derived in section 2.2 when units in the inventory system are subject to time dependent deterioration. The analytic proofs are given to support each assertion of parameter dependence in both the models. The model in section 2.2 will reduce to the model in section 2.1 when there are no shortages i.e when $T_j = 0$. 