For a surface of revolution

\[ z = f(r) , \]  

the line-element

\[ d\sigma^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2 , \]  

can be expressed as

\[ d\sigma^2 = \rho^2 \left[ d\mu^2 + d\phi^2 \right] , \]  

where

\[ \mu = \int \sqrt{1 + \left( \frac{df}{d\rho} \right)^2} \, d\rho . \]

According to the equation (4) \( \mu \) is a function of \( \mu \).

Thus a line element on a surface of revolution can be expressed as

\[ d\sigma^2 = F(\mu) \left[ d\mu^2 + d\phi^2 \right] . \]  

If we consider an oblate spheroid then \( Z \) depends upon \( \rho \) in the following manner.

\[ \frac{Z^2}{a^2 (1 - \epsilon^2)} + \frac{\rho^2}{a^2} = 1 , \]

\( \epsilon \) being the eccentricity of the curve which generates the surface of revolution.

Using (4) and (6), we find that the equation for the quadratic differential form for an oblate
spheroid turns to be

$$d \sigma^2 = a^2 \left[ \frac{1 - \delta^2}{1 - \varepsilon^2 \delta^2} \right] \left[ du^2 + d\phi^2 \right], \quad (7)$$

where

$$\frac{1 - \delta}{1 + \delta} \left( \frac{1 + \varepsilon \delta}{1 - \varepsilon \delta} \right) = e^{2\mu}. \quad (8)$$

Thus we have determined the choice of the function \( \alpha \) required to represent the oblate spheroidal surface. The equation which gives \( \alpha \) is

$$e^\alpha = \frac{1 - \delta^2}{1 - \varepsilon^2 \delta^2}. \quad (9)$$

In the sixth chapter we have treated \( \varepsilon \) to depend upon \( x^4 \). Such a value of \( \alpha \) also contributes the terms

$$- \frac{1}{2} \varepsilon e^\alpha \frac{\partial^2 x}{\partial (x^2)^2} + \varepsilon \frac{1}{2} \frac{\partial^2 x}{\partial x^4} \quad \text{to} \quad g_{44} (x^2 = \mu).$$

Straightforward calculations show that the contribution is

$$\left[ \frac{1 - \varepsilon^2 \delta^2}{1 - \varepsilon^2} \right]^2 + x^1 \varepsilon \frac{d\varepsilon}{dx^4} \log \frac{1 - \varepsilon \delta}{1 + \varepsilon \delta}. \quad (10)$$

If we take \( \varepsilon = 0 \), above calculations give

$$x^2 = \mu = \log \tan \frac{\theta}{2} \quad \text{and} \quad \delta = \cos \theta.$$  

Thus the line-element \((6.34)\) reduces to the form (taking \( x^1 = \lambda, \ x^3 = \phi \))

$$ds^2 = - \lambda^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \left[ \frac{-2M}{\lambda^2} \right] dt^2 + 2 dr dt. \quad (11)$$