CHAPTER III

THE GRAVITATIONAL FIELD OF A CHARGED PARTICLE EMBEDDED IN AN EXPANDING UNIVERSE

1. Introduction

In the second chapter we have discussed the gravitational contraction of charged fluid spheres embedded in the Norstrom's exterior field described by the line-element.

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{\epsilon^2}{r^2}\right) dt^2 - \left(1 - \frac{2m}{r} + \frac{\epsilon^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

(III-1.1)

where $m$ and $\epsilon$ are the total mass and charge of the fluid distribution within a sphere. We have seen that in our model the effect of an electrostatic repulsion is only on the rate of contraction. It does not halt the contraction. From this solution one can easily derive an interesting case of the gravitational field of a charged particle in a homogeneous universe.

The gravitational field of an uncharged mass particle is described by the well-known Schwarzschild's exterior solution, while the corresponding field for a charged mass particle is described by Nordstrom's exterior field (III-1.1). Bonnor (1) has obtained the line-element describing the gravitational field of a charged mass particle. His line-element
is a transform of Nordstrom's metric (III-1.1) with \( \xi = -m \).
In all these fields the space round the particle is empty and
the geometry at large distances from the particle reduces to
flat geometry of special relativity.

McVittie\(^2\) gave the line-element describing the
gravitational field of an uncharged mass particle embedded
in an expanding universe. The space round the mass particle
is occupied by spherically symmetric distribution of matter
with non-zero density and isotropic pressure which at large
distances from the particle pass over smoothly to the cosmic
density and pressure in homogeneous cosmological models.

In this chapter we\(^3\) obtain a line-element describing
the gravitational field of a charged particle in a homogeneous
and isotropic cosmological model. The solution will be used
later for discussion of gravitational contraction of charged
fluid spheres in the back-ground of the homogeneous cosmological
model.

2. The space-time exterior metric

The line-element

\[
ds^2 = (F+G)^{-2} dt^2 - (F+G)^2 \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right)
\]

as given by (II-3.10) in chapter-II describes the interior
field of a charged fluid sphere, where
are undetermined functions of their arguments.

For the interior field described by (III—2.1), the pressure $p$, matter density $\rho$, electric field tensor $F_{14}$ and the charge density $\sigma$ as given by (II-3.11) to (II-3.14) in chapter-II are

\[ 8\pi p = -k(F+G)^{-2} - 2\dot{G}(F+G) - 3\ddot{G}^2 \quad (III-2.2) \]

\[ 8\pi \rho = -2(F+G)^{-3} \left\{ F^n(1-kr^2) + (2-3kr^2) \frac{F'}{r} \right\} \]

\[ + 3k(F+G)^{-2} + 3\dot{G}^2 \quad (III-2.3) \]

\[ F_{14} = \frac{F'}{(F+G)^2} \quad (III-2.4) \]

\[ 4\pi \sigma = 4(F+G)^{-3} \left[ F^n (1-kr^2) + (2-3kr^2) \frac{F'}{r} \right] \quad (III-2.5) \]

Let us take a particular case where the charge density $\sigma$ of the fluid vanishes. On putting $\sigma = 0$ in the equation (III-2.5) we get

\[ F^n(1-kr^2) + (2-3kr^2) \frac{F'}{r} = 0 \quad (III-2.6) \]

Integrating this equation twice we get
where \( A \) and \( m \) are arbitrary constants of integration. The constant \( A \) can as well be incorporated in the undetermined function \( G(t) \). Therefore there will be no loss of generality if we put \( A = 1 \).

Thus

\[
F = 1 + \frac{m}{r} \sqrt{1-kr^2}
\]

and the metric (III-2.1) takes the particular form

\[
ds^2 = \left(1 + \frac{m}{r} \sqrt{1-kr^2} + G\right)^2 dt^2 - \left(1 + \frac{m}{r} \sqrt{1-kr^2} + G\right)^2 r^2 d\Omega^2
\]

\[
\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)
\]

where \( G = G(t) \) is an undetermined function of time \( t \).

For this metric, equations (III-2.2) to (III-2.4) reduce to the forms

\[
8\pi \rho = -k \left(1 + \frac{m}{r} \sqrt{1-kr^2} + G\right)^2 - 2\ddot{G} \left(1 + \frac{m}{r} \sqrt{1-kr^2} + G\right) - 3\dot{G}^2
\]

(III-2.9)

\[
8\pi \rho = 3k \left(1 + \frac{m}{r} \sqrt{1-kr^2} + G\right)^2 + 3\dot{G}^2
\]

(III-2.10)

\[
F_{44} = -\frac{m}{r^2 \sqrt{1-kr^2} \left(1 + \frac{m}{2} \sqrt{1-kr^2} + G\right)^2}
\]

(III-2.11)
This shows that $F_{14}$ still remains non-zero even though the charge density $\sigma$ is zero. This non-vanishing radial electric field must be produced by the central particle which must be a charged particle. In the line-element (III-2.8), there is a singularity $r = 0$. Therefore the charged particle is at rest at the origin. At large distances from the charged particle, the electro-magnetic field and the perturbations in the density and pressure of the cosmic fluid diminish and the field approaches the undisturbed Robertson-Walker universe. Thus the line-element (III-2.8) describes the gravitational field of a charged particle embedded in a general Robertson-Walker universe.

The space round the charged particle is filled with perfect fluid. There is also an electromagnetic field present round this charged particle.

Now putting the curvature $k = 0$ in (III-2.6) we get the line-element

$$
\text{ds}^2 = \left(1 + \frac{m}{r} + G\right)^{-2} \text{dt}^2 - \left(1 + \frac{m}{r} + G\right)^2 \left(\text{dr}^2 + \frac{r^2}{\left(1 + \frac{m}{r} + G\right)^2} \text{d}\Omega^2\right)
$$

(III-2.12)

describing the gravitational field of a charged particle embedded in a homogeneous Einstein-deSitter universe.

The pressure $p$, matter density $\rho$ and the electric intensity $F_{14}$ of the surrounding smoothed out universe will now be given by
\[ 8\pi p = -2\dot{G}(1 + \frac{m}{r} + G) - 3\dot{G}^2 \quad (III-2.13) \]
\[ 8\pi \phi = 3\dot{G}^2 \quad (III-2.14) \]
\[ F_{14} = -\frac{m}{r^2(1 + \frac{m}{r} + G)^2} \quad (III-2.15) \]

Putting \( G(t) = 0 \) in (III-2.12), the present solution reduces to Bonnor's solution of a charged particle of charge \( \xi \) and mass \( m \) such that \( \xi = -m \). On putting \( m = 0 \) in (III-2.12), one gets Einstein-deSitter's universe.

Moreover at large distances from the charged particle, the electromagnetic field vanishes for both the cases and the field merges into the homogeneous and isotropic cosmological universe.
REFERENCES

