Virtually every physical, engineering or biological process can be modelled using differential equations together with an additional set of constraints. The complexity or non-existence of analytic solution of real life boundary value problems forces one, to take the recourse of the numerical techniques. Checks like error analysis, stability analysis and computational cost analysis are of utmost importance to establish the credibility of any numerical technique.

This study has focused on the cubic Hermite collocation method (CHCM) for the solution of ordinary differential equations and partial differential equations. In this technique orthogonal collocation on finite element is used with cubic Hermite as basis function. The $C^1$ continuity of Hermite polynomials ensures that the solution and its first derivative are automatically continuous at the boundary of the elements. This results in a significant saving of the computational effort.

Different linear and nonlinear second order ordinary differential equations are solved, subject to Dirichlet, Neumann and Robin boundary conditions by taking zeros of the shifted Legendre and the shifted Chebyshev polynomials as the collocation points. It is observed that the present scheme achieves fourth order of convergence for the zeros of shifted Legendre polynomials. The results are found to be superior than the finite difference method, finite element method, finite volume method, B-spline method, polynomial and nonpolynomial spline methods, which demonstrate the effectiveness of the scheme.

Diffusion dispersion models, depicting the removal of solutes from the packed bed, are analyzed with respect to linear and nonlinear adsorption isotherms. The models are
solved numerically and a comparison is made using the experimental data. Mathematically it is shown that the solute removal process can best be described by the nonlinear isotherm. Also the asymptotic convergence of CHCM in continuous time for the parabolic PDEs is established of order 4 for shifted Legendre roots and order 2 for Chebyshev roots. Numerical examples are used to verify the order of convergence as predicted by the theoretical analysis.

Burgers’ nonlinear model is used for the conceptual understanding of turbulence, mass transport, heat conduction, environment, elasticity, wave formation etc. It is a tool to describe relation between convection and diffusion. Numerical solution of Burgers’ equation is obtained by CHCM, without any transformation and with Hopf-Cole transformation. Linear stability analysis shows that the scheme based on Crank-Nicolson approximation in time is unconditionally stable. The method is applied on some test problems, with different choice of collocation points, to validate the accuracy of the method even for high Reynolds number.

Kuramoto-Sivashinsky equation is the simplest example of a nonlinear partial differential equation that exhibits extremely rich dynamical behavior. The solution and analysis of the KS equation is performed by CHCM. A bound for maximum norm of the semi discrete solution is derived by using Lyapunov functional. Using $L_2$ and $L_\infty$ error norms properties, accuracy of the new method is established through comparison with the existing techniques to prove its applicability and superiority. The theoretical order of convergence is also verified using test problems.