PREFACE

The present investigations have been carried out towards the fulfillment of the requirements for the award of a Ph.D. degree in Physics of V.B.S. Purvanchal University, Jaunpur (U.P.), India under the supervision of Dr. R.S. Chauhan, Reader and Head, Department of Physics, R. P.G. College Jamuhai, Jaunpur (U.P.), India, and Co-supervision of Dr. Lal Jee, Ex. Reader, Department of Physics, K.N. Govt. P.G. College Gyanpur, Sant Ravidas Nagar, Bhadohi (U.P.), India.

The thesis deals with the "Some Studies in Dynamics of Gravity from the Entropy of Spacetimes". It has been divided into four chapters. The first chapter is introductory. So, we have formulated and discussed some of the techniques and results which are relevant for our subsequent investigations. Hence, we have presented Thermodynamical Perfect Fluid, Equation of Continuity and Conservation of the Matter, Relativistic Helmohltz Equation, Incompressible Thermodynamical Fluid.

In chapter II, we have presented asymptotically flat spacetime at infinity i.e. gauge invariance of the integrals of motion in the field technique. First we have given the notations used, reviewed some formulae from the field formulation of general relativity and presented necessary results which are
very useful for subsequent investigations. For a real isolated system the integrals of motion have to have the same values in both description. But the integrals of motion in the Hamiltonian and Lagrangian descriptions differ by surface terms. Hence, we have obtained explicit expressions for these surface terms and considered them under weakest fall-off conditions. In general we have given the connection between the symmetric and canonical vector densities constructed with use of the symmetric and canonical stress energy tensors, respectively, and of the Killing vectors of Minkowski spacetime. The with their help obtained the corresponding integrals of motion and given the explicit expressions for the differences of the surface terms, between these two kinds of integrals of motion. In the framework of the field formulation of general relativity the results are applied to asymptotically flat spacetimes and examined under weakest fall-off conditions. It is well known that many of the definitions of the integrals of motion in general relativity use several super potentials as presented by Misner, Thorn and Wheeler (1973) and Brown and York (1993). This leads to the differences between the integrals of motion by surface terms. We have defined the weakest fall-off conditions for the phase variables. Then using canonical vector density we obtained the Hamiltonian vector density and presented the explicit forms of the integrals of motion in the Hamiltonian descriptions for an isolated system. The use of asymptotics for the matter variables are obtained. These integrals of motion are exactly
ADM (Arnowitt-Deser-Misner) integrals. Thus we have shown that they are finite and conserved in time under the weakest fall-off conditions. The important result in fact we have given the direct relation between ADM integrals and integrals of motion in the Lagrangian description of the field technique.

In chapter III, we have presented some peculiar features of Einstein-Hilbert action such as (i) the existence of second derivatives of dynamical variables, (ii) a nontrivial connection between the surface term and the bulk term, (iii) the surface term in nonanalytic in coupling constant for the gravity when it is taken as a spin-2 perturbation around flat spacetime, (iv) the form of variation of surface term under infinitesimal coordinate transformations. We shown that the surface term may be obtained from very general considerations and one may derive the Einstein equations just from the surface term of the action. One may also obtain the bulk term(iii) the surface term and also derive the full Einstein-Hilbert action, based on purely thermodynamic considerations. It is also presented that $A_{\text{grav}} = -S + \beta E$ in any stationary spacetime in a natural manner, with horizon, showing that the true degrees of freedom of gravity resides in surface term of the action.

Our analysis provides a perspective towards gravity: (a) The horizon perceived by a congruence of observers indicates the form of the action functional to be used by these observes. This action possesses a surface term
which may be interpreted as an entropy. (b) The coordinate transformation $x^a \rightarrow x^a + \xi^a$ acquires a dynamical content. One may obtain Einstein equations purely from surface term, (c) The metric components become singular on horizon. The transformation from this coordinate system to a nonsingular coordinate system near the horizon will require the use of a coordinate transformation which itself is singular. Our analysis also provides some considerations about cosmological constant $\Lambda$ for which $T_{ab} = \rho g_{ab}$. Then $T_{ab} - \frac{1}{2} g_{ab} T = - \rho g_{ab}$ and the coupling term $N_{ab} \xi^a \xi^b$ for matter be proportional to $\xi^2$.

In the last chapter, we have shown that it is not possible to obtain the Einstein-Hilbert action starting from the standard graviton action and iterating in the coupling constant. This is because of the existence of the total divergence term in the Einstein-Hilbert action which is non-analytic in the coupling constant, when expanded in terms of the graviton field. This result is crucial because a series of previous investigations, Padmanabhan (2002) have shown that the surface term is vital in the thermodynamics of horizons and in semiclassical gravity. In fact, I started this investigation to understand how the surface term- and hence, possibly the entropy of horizons-can be interpreted in terms of graviton field in a Minkowski background. The result shows that one simply cannot understand the surface term in a standard field
theoretical language, using the graviton field. There is more to gravity than gravitons and this will be elaborated in a separate publication.

In a strictly classical theory, what matters is the equation of motion and not the form of the action principle. Hence, the fact that we cannot get the surface term in Einstein-Hilbert action is not of concern if we are only interested in the Einstein's equations. Our analysis shown that it is indeed possible to obtain the quadratic $\Gamma^2$ action (and thus the Einstein's equations) by starting from the graviton action and iterating on the coupling constant. But to do this, we need to couple $h_{ab}$ to a second rank tensor $S^{ab}$ which is different from the standard energy-momentum tensor $T^{ab}_G$ of the graviton. Indeed, as we explained, if the source of gravity at each order of iteration has to be the energy-momentum tensor of the graviton evaluated at the previous order, then the coupling in the Lagrangian cannot be the form $h_{ab} T^{ab}_G$ since the $h_{ab}$ dependence of the $T^{ab}_G$ will lead to an extra term on variation. A term in the lagrangian of the form $h_{ab} S^{ab}$ does lead to the energy-momentum tensor as the source of gravity. Identifying the nature of $S^{ab}$ and bringing it into focus has been one of the results of this chapter.

It we were only interested in pure gravity, this would have been the whole story. But, in that case, it is an unnecessary exercise. The linear spin-2 field, uncoupled to anything, is a perfectly consistent theory and we need not
try to couple it to itself. So the whole exercise has meaning only when we have both matter and spin-2 field and we try to couple them consistently. Then we need to assume that the spin-2 field couples to itself through \( S^{ab} \) while it couples to matter through \( T^{ab} \). This assumption will lead consistently to Einstein's theory and seems to be the most viable option, if we want to obtain standard gravity coupled to matter, starting from the graviton action. [Of course, in a world made of a spin-2 field coupled to matter made of only relativistic particles, spin-0 fields and spin-1 fields, one can assume that all the coupling is through \( S^{ab} \); this is because for matter made of these constituents, \( S^{ab} = T^{ab} \).]

Two facts need to be borne in mind as regards this option. First, we do not know anything about the coupling of spin-2 field to itself except through standard gravity; and the analysis in show that gravity does couple to itself through a term \( h^{ab} S^{ab} \). Second, there is no conflict with principle of equivalence even though the self-coupling term is \( h_{ab} S^{ab} \) while the coupling to external source is through \( h_{ab} T^{ab} \). What matters for principle of equivalence is the fact that the source for gravity is always the energy-momentum tensor. This is indeed assured in our approach and- as have been stressed several times- this requires a self-coupling term of the form \( h_{ab} S^{ab} \) in the lagrangian.

Every chapter has been divided in sections following decimal system: e.g. section (1.5) means fifth section of chapter first. On the same line, the
equations in different chapters are also numbered i.e. eq. (4.5) means, fifth equation of chapter four. However, references are quoted whenever necessary by writing authors’ name followed by year in round bracket. At last, references are given.