Chapter-5

FUZZY LOGIC BASED VARIABLE GAIN PID CONTROLLERS
5.1 INTRODUCTION

The analysis presented in chapters 3 and 4 highlighted the applications of various types of conventional controllers and control signals for a single machine and multi machine systems. The gains of the controllers in these studies are selected by trial and error. However, there exists a need for employing more efficient techniques for selection of these gains. Fuzzy logic based methodology is one such technique which can be employed. In this chapter an attempt is made to determine the gains of the modulation controller based on fuzzy logic.

5.2 FUZZY CONTROL

Traditional logic systems are different from fuzzy logic, which is a logical system having closer resemblance to human thinking and natural language. Fuzzy control is based on this fuzzy logic. Fuzzy control finds applications to systems for which conventional control is not suitable due to lack of quantitative data regarding input-output relations. Fuzzy logic based controller is useful for converting a linguistic set of control strategy based on expert knowledge into an automatic control strategy. When the system processes are too complex for analysis by conventional quantitative analysis, fuzzy logic based controller methodology will be very helpful.

A review of fuzzy set theory which finds extensive application in fuzzy control is briefly presented in the following sections.
5.2.1 Fuzzy Set Theory

The framework of fuzzy sets is more general than that of ordinary sets. Fuzzy set theory is helpful in dealing with problems involving source of imprecision rather than random variables [118].

Let X be a collection of objects (X is the Universal Set), then a fuzzy set A in X is defined to be a set of ordered pairs:

\[ A = \{ x, \mu_A(x) | x \in X \} \tag{5.1} \]

Where \( \mu_A(x) \) is called the membership function of x in A. The membership function \( \mu_A(x) \) denotes the degree to which x belongs to A and is normally limited to values between 0.0 and 1.0. The value of \( \mu_A(x) \) nearer to unity is the higher grade of membership of x in A, i.e. 0.0 and 1.0 denotes non-membership and full-membership respectively. If the values of the membership function are limited to be either 0.0 or 1.0, then A becomes an ordinary set.

In most of the cases it is possible to express the membership function of a fuzzy subset of the real line in terms of a standard function whose parameters may be adjusted to fit a specified membership function in a desired manner.
5.2.2 The AND operator (The intersection of two fuzzy sets)

Let A and B be two fuzzy sets with membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively, the membership function of the intersection (AND), C = A $\cap$ B, is defined by

$$\mu_C = \min (\mu_A(x), \mu_B(x)), \quad x \in X$$ (5.2)

5.2.3 The OR operator (The union of two fuzzy sets)

Let A and B be two fuzzy sets with membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively, the membership function of the union (OR), D = A $\cup$ B, is defined by

$$\mu_C = \max (\mu_A(x), \mu_B(x)), \quad x \in X$$ (5.3)

5.2.4 The NOT operator (The complement of a fuzzy set)

Let A be a fuzzy set with membership function $\mu_A(x)$. The membership function of the complement of A, is defined by:

$$\mu_A(x) = 1 - \mu_A(x), \quad x \in X$$ (5.4)

5.2.5 Fuzzy Relation

Let A and B be two fuzzy sets with membership functions $\mu_A(x)$ and $\mu_B(x)$, respectively. A fuzzy relation R from A to B can be visualized as a fuzzy graph and can be characterized by the membership function $\mu_R(x,y)$ which satisfies the composition rule as follows:
\[ \mu_B(y) = \max_x (\min(\mu_R(x, y) \mu_A(x))) \]  

(5.5)

In many cases it is convenient to express the membership function of a fuzzy subset of the real line in terms of a standard function whose parameters may be adjusted to fit a specified membership function in a suitable fashion.

### 5.3 Fuzzy Controller Design Principle

In general, FLC design consists of the following steps:

1. Selection of input and output variables.
2. Formulation of control rules.
3. Establishing fuzzification method and fuzzy membership functions.
4. Selection of the compositional rule of inference.
5. Establishing defuzzification method.

The above design methodology is illustrated in Fig 5.1.
5.4 Fuzzy Logic Based Gains of Modulation Controllers

The fuzzy logic theory discussed above can be effectively applied for determination of gains of conventional controllers.

5.4.1 Introduction

Here the variable gain PID control scheme has been adopted for the purpose of enhancing the stability of the multi machine power systems, utilizing HVDC power modulation. The gains of the three error signals discussed in chapter 4: $K_p$, $K_i$, and $K_d$ are adjusted in every sampling interval in accordance to a set of linguistic control rules and in conjunction with fuzzy logic. This feature is needed
because as the system operating condition change its performance deteriorates, if a fixed gain controller is used. The above mentioned control scheme has the advantages of a conventional PID controller and that of a rule-based controller.

Considering a conventional PID controller whose output $U_{PID}(t)$ is given by:

$$U_{PID}(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt}$$  \hspace{1cm} (5.6)

To facilitate the implementation, the control signal is discretized and at $k^{th}$ sampling it becomes:

$$U_F(k) = K_p e(k) + K_i Ie(k) + K_d De(k)$$  \hspace{1cm} (5.7)

Where, $K_p$, $K_d$, and $K_i$ are fixed gains and $e(k)$, $Ie(k)$, and $De(k)$ are the error signals derived from the AC system, as explained in chapter 4.

### 5.4.2 Control Rules and Fuzzy Labels

One approach to enhance the performance of the conventional controller of (5.7) is to vary its gains $K_p$, $K_i$ and $K_d$ as a function of $|e(k)|$, $|Ie(k)|$ and $|De(k)|$, respectively [119]. Thus if $|e(k)|$ is large/medium/small then $K_p$ is changed by a large/medium/small amount in order to reduce $e(t)$ to zero as fast as possible. This rule is also applicable to $|Ie(k)|$ and $|De(k)|$ by changing $K_i$ and $K_d$, accordingly. Therefore, in fuzzy logic terminology, it is possible to
associate three linguistic labels, viz., large, medium and small, to each of the variables $|e(k)|$, $|Ie(k)|$ and $|De(k)|$.

On the basis of above rules and the fact that the PID output of (5.7) has three terms, there are altogether 27 linguistic control rules which can modify the conventional PID gains to get a better performance. Table 5.1 shows the rule table.

Table 5.1: Rule Base for variable gain Fuzzy Logic Controller

<table>
<thead>
<tr>
<th>Rule No: (labels of universes; weighted outcome)</th>
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<tbody>
<tr>
<td>$i: (</td>
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<tr>
<td>------------------------------------------------</td>
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<tr>
<td>1: (S, S, S; Wu_1), 2: (S, S, M; Wu_2), 3: (S, S, L; Wu_3)</td>
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<tr>
<td>4: (S, M, S; Wu_4), 5: (S, M, M; Wu_5), 6: (S, M, L; Wu_6)</td>
</tr>
<tr>
<td>7: (S, L, S; Wu_7), 8: (S, L, M; Wu_8), 9: (S, L, L; Wu_9)</td>
</tr>
<tr>
<td>10: (M, S, S; Wu_{10}), 11: (M, S, M; Wu_{11}), 12: (M, S, L; Wu_{12})</td>
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<td>13: (M, M, S; Wu_{13}), 14: (M, M, M; Wu_{14}), 15: (M, M, L; Wu_{15})</td>
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<tr>
<td>16: (M, L, S; Wu_{16}), 17: (M, L, M; Wu_{17}), 18: (M, L, L; Wu_{18})</td>
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<td>19: (L, S, S; Wu_{19}), 20: (L, S, M; Wu_{20}), 21: (L, S, L; Wu_{21})</td>
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<tr>
<td>22: (L, M, S; Wu_{22}), 23: (L, M, M; Wu_{23}), 24: (L, M, L; Wu_{24})</td>
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<tr>
<td>25: (L, L, S; Wu_{25}), 26: (L, L, M; Wu_{26}), 27: (L, L, L; Wu_{27})</td>
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The rules in Table 5.1 are explained in the following manner:

If $|e(k)|$ is small and $|Ie(k)|$ is small and $|De(k)|$ is small then

$$W_{u_i}(k) = K_p \mu_i (|e(k)| e(k) + K_i \mu_i (|Ie(k)| Ie(k)) + K_d \mu_i (|De(k)|) De(k))$$

(5.8)
Similarly, rule 22 is read as:

If $|e(k)|$ is large and $|Ie(k)|$ is medium and $|De(k)|$ is small then

$$W_{u22}(k) = K_p \mu_L \left( |e(k)| \right) e(k) + K_i \mu_m \left( |Ie(k)| \right) Ie(k) + K_d \mu_s \left( |De(k)| \right) De(k)$$  \hspace{1cm} (5.9)

In the above rules, $\mu_L(\mid z \mid)$, $\mu_m(\mid z \mid)$ and $\mu_s(\mid z \mid)$ are membership functions when $\mid z \mid$ is large, medium and small, respectively, and $z$ is either $e(k)$, $Ie(k)$ or $De(k)$. Fig 5.2 shows the type of membership functions used in this formulation. Briefly, a membership function $\mu_{LL}(x)$ is used to indicate to what degree, in a scale ranging from 0 to 1, the variable $x$ satisfies the linguistic label LL.

![Membership function](image)

Fig 5.2: Membership function

### 5.4.3 Variable Gain PID controller

For a given set of $|e(k)|$, $|Ie(k)|$ and $|De(k)|$ all the 27 rules shown in Table 5.1 will be active. Furthermore, there is a degree of fulfillment $\mu_i$ also known as the truth value, of each rule for $i=1$ to 27. By applying the AND operation to the condition part of the rule, its degree of fulfillment is calculated.
Thus, for Rule 1, it is given by:

$$\mu_i = \min \left[ \mu_e \left( |e(k)| \right), \mu_i \left( |Ie(k)| \right), \mu_i \left( |De(k)| \right) \right]$$

(5.10)

For Rule 22, its degree of fulfillment is:

$$\mu_{22} = \min \left[ \mu_e \left( |e(k)| \right), \mu_i \left( |Ie(k)| \right), \mu_i \left( |De(k)| \right) \right]$$

(5.11)

Therefore, for a given set of $|e(k)|$, $|Ie(k)|$ and $|De(k)|$, there are 27 weighted outcomes $W_{ui}(k)$ and each outcome has $\mu_i$ as its degree of fulfillment for $i=1$ to 27. One commonly used method to determine the net outcome $U_F(k)$ is based on the weighted average approach. Here, this approach is adopted and thus

$$U_F(k) = \frac{\sum_{i=1}^{27} W_{ui}}{\sum_{i=1}^{27} \mu_i}, \quad \text{where}$$

$$\sum_{i=1}^{27} \mu_i = \sum_{i=1}^{27} \mu_i$$

(5.12) (5.13)

It can be shown that this fuzzy PID controller is equivalent to

$$U_F(k) = x_p K_p e(k) + x_i K_i Ie(k) + x_d K_d De(k)$$

(5.14)

Where,

$$X_p = g \left[ \mu_i \left( |e(k)| \right) - \mu_i \left( |e(k)| \right) + \mu_i \left( |e(k)| \right) \right] / \sum \mu$$

$$X_i = g \left[ \mu_i \left( |Ie(k)| \right) + \mu_i \left( |Ie(k)| \right) + \mu_i \left( |Ie(k)| \right) \right] / \sum \mu$$

$$X_d = g \left[ \mu_i \left( |De(k)| \right) + \mu_i \left( |De(k)| \right) + \mu_i \left( |De(k)| \right) \right] / \sum \mu$$

(5.15)
Block diagram of the above mentioned variable gain fuzzy PID controller is shown in Fig 5.3. Clearly, the effect of the 27 rules is to vary \( K_p \), \( K_i \) and \( K_d \) by a factor \( X_p \), \( X_i \) and \( X_d \), respectively.

![Block diagram of variable gain fuzzy PID controller](image)

Fig 5.3: Block diagram of variable gain fuzzy PID controller

Applying the variable gain fuzzy PID controller scheme discussed above to the multi machine system example of chapter 4, the plots of relative rotor angles is presented in Fig 5.4(a & b)- 5.6(a & b).
Fig 5.4(a & b): Plot of relative rotor angles with Fuzzy Variable Gain controller (with Kp varied)
Fig 5.5(a & b): Plot of relative rotor angles with Fuzzy Variable Gain controller (with Ki varied)
Fig 5.6(a & b): Plot of relative rotor angles with Fuzzy Variable Gain controller (with Kd varied)
It is seen from the above plots that fuzzy logic based variable gain controller gives better performance compared to the conventional PID controller presented in Fig 4.9(a & b) in terms of maximum overshoot and settling time. This is so as the gains are tuned at every sampling interval in accordance with the error.

5.5 SUMMARY

In this chapter, a variable gain PID control scheme has been implemented for HVDC power modulation to augment the transient stability of the multi machine AC-DC power system. In this scheme, the gains of the P-term, I-term and D-term of the PID controller are adjusted in every sampling interval in accordance to a set of linguistic control rules and in conjunction with fuzzy logic. This method utilizes 27 control rules to vary the three gains of a PID controller. The proposed control scheme has the advantages of a conventional PID controller and that of a rule-based controller. The performance of variable gain fuzzy logic controller as shown in Fig 5.4(a & b) - 5.6(a & b) is better than that of the conventional controller shown in Fig 4.9(a&b).

The plots shown in Fig 5.4(a & b) - 5.6(a & b) represent relative rotor angle variations for different initial gains of Kp, Ki and Kd since the effect of the 27 rules is to vary Kp, Ki and Kd by a factor Xp, Xi and Xd, respectively.
This control scheme possesses many advantages like lesser computational time and robustness. In the next chapter, a new fuzzy logic control scheme in which a FL controller is supplied with error signal generated from all the machines of the system and its rate, as input signals and output as the auxiliary stabilizing signal for DC power modulation is proposed.