Chapter-2

TRANIENT STABILITY ANALYSIS OF AC/DC SYSTEMS
2.1 INTRODUCTION

Incorporation of HVDC transmission subsystems in AC transmission networks has been a major change in power transmission during the last few years. This change required modifications in the performance evaluation procedures notably for load flow and stability analysis. The methodologies used for AC/DC system’s load flow calculation and transient stability analysis are presented in this chapter.

2.2 AC/DC LOAD FLOW

To obtain system conditions prior to the disturbance, it is a prerequisite to do AC/DC load flow calculations in transient stability studies. A direct current link can be represented as real and reactive powers injected into the AC system at the two terminals, in the AC power flow analysis. Hence, the two terminal AC/DC buses can be represented as voltage independent constant load buses. However, this is an inadequate representation when the links contribution to AC system reactive power and voltage conditions is significant, since accurate operating mode of the link and its terminal equipment are ignored [109-112].

There are two conventional methods to solve the AC-DC load flow equations. In the first method, sequential method, the AC and DC equations are solved separately in each iteration. In this approach, the
effect of DC link is included only in the power mismatch equations. Though it is easy to implement, but convergence problems may occur in some cases. The second method is simultaneous or unified method. In each iteration of this method, both AC and DC equations are solved simultaneously. DC variables are included in the solution vector and hence it is also known as extended variable method. In this method, the dc equations are solved along with the reactive power portion of the load flow equations. The drawback with this approach is that the Jacobean cannot be completely pre-inverted and it is complex to program and hard to combine with developments in AC power flow solution techniques, such as the fast decoupled methods.

All the above mentioned difficulties are overcome with the eliminated variable method used here [113]. In this method, the real and reactive power consumption of the converters are assumed as voltage dependent loads and the DC variables are eliminated from the power flow equations by solving the DC equations numerically. This method has all the features of the above mentioned conventional techniques. The advantages of this method are:

- A reliable method for fast decoupled power flow can be easily developed based on the new method.
- In this method, it is possible to handle the AC and DC load flow equations separately which simplifies the implementation, maintenance and modification of the program.
• It is easy to handle switching between control modes due to variables hitting their limits.
• Information related to the AC and DC interactions can be obtained from this method.

The eliminated variable method is detailed in the following sections.

### 2.2.1 DC System Model

The equations describing the steady state behavior of a mono-polar DC link (Fig 2.1) can be summarized as follows:

\[
V_{dr} = \frac{3\sqrt{2}}{\pi} a_{r} V_{r} \cos \alpha_{r} - \frac{3}{\pi} X_{c} I_{d} \tag{2.1}
\]

\[
V_{di} = \frac{3\sqrt{2}}{\pi} a_{i} V_{i} \cos \gamma_{i} - \frac{3}{\pi} X_{c} I_{d} \tag{2.2}
\]

\[
V_{dr} = V_{di} + r_{d} I_{d} \tag{2.3}
\]

\[
P_{dr} = V_{di} I_{d} \tag{2.4}
\]

\[
P_{di} = V_{di} I_{d} \tag{2.5}
\]

\[
S_{dr} = k \frac{3\sqrt{2}}{\pi} a_{r} V_{r} I_{d} \tag{2.6}
\]

\[
S_{di} = k \frac{3\sqrt{2}}{\pi} a_{i} V_{i} I_{d} \tag{2.7}
\]

\[
Q_{dc} = \sqrt{S_{dr}^{2} - P_{dr}^{2}} \tag{2.8}
\]
Where, \( k \) is assumed constant and \( k \approx 0.995 \) (See list of symbols).

**AC/DC Power Flow Equations**

When the DC-link is included in the power flow equations, only the mismatch equations at the converter terminal AC buses have to be modified.

\[
\begin{align*}
\Delta P_{tr} &= P_{tr}^{spec} - P_{tr}^{ac}(\delta, v) - P_{dc}(V_{tr}, V_{tr}, x_{dc}) \\
\Delta P_{ti} &= P_{ti}^{spec} - P_{ti}^{ac}(\delta, v) + P_{di}(V_{tr}, V_{tr}, x_{dc}) \\
\Delta Q_{tr} &= Q_{tr}^{spec} - Q_{tr}^{ac}(\delta, v) - Q_{dc}(V_{tr}, V_{tr}, x_{dc}) \\
\Delta Q_{ti} &= Q_{ti}^{spec} - Q_{ti}^{ac}(\delta, v) - Q_{di}(V_{tr}, V_{tr}, x_{dc})
\end{align*}
\]

Where, \( x_{dc} \) is a vector of internal DC-variables. The DC-variables satisfy

\[
R(V_{tr}, V_{tr}, x_{dc}) = 0
\]
Where, $R$ is a set of equations given by (2.1)-(2.3) and four control specifications.

In the extended variable method, (2.15) is solved iteratively.

\[
\begin{bmatrix}
\Delta P \\
\Delta P_{i} \\
\Delta Q \\
\Delta Q_{i} \\
\Delta R
\end{bmatrix} =
\begin{bmatrix}
H & N & 0 \\
J & L & 0 \\
0 & 0 & D \\
0 & 0 & 0 \\
A & C & E
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \delta_{i} \\
\Delta V / V \\
\Delta V / V_{i} \\
\Delta x_{dc}
\end{bmatrix}
\]  

(2.15)

In the sequential method, (2.14) is solved after each iteration of (2.16).

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
H & N \\
J & L
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V / V
\end{bmatrix}
\]  

(2.16)

### 2.2.3 Control Modes

Seven variables and three independent equations, (2.1)-(2.3), are introduced when a DC-link is included. To define a unique solution, four variables are to be specified. Details of possible modes of operation are shown in Table 2.1.

Control mode A is the normal mode of operation of a two terminal DC link in which one terminal controls the current or power and the other terminal controls the voltage. Control angles and DC voltage are the specified quantities in this mode. Tap positions of the converter transformers are adjusted to meet the above specifications. The other
modes in Table 2.1 are obtained from mode A if variables hit their limits during the power flow computations, or if the time scale is such that the taps can be assumed to be fixed. The possibility of obtaining new modes when limits are encountered, depend on the control strategy adopted for the HVDC-scheme and the same must be incorporated in the computations. For modes B - D, $\alpha_r$ determines $\alpha_r$ and $\alpha_i$ determines the direct voltage, which normally is the case for current control in the rectifier. Firing angle of the rectifier determines the direct voltage for modes E – G. Constant current control is denoted with subscript I.
Table 2.1: Control modes

<table>
<thead>
<tr>
<th>Control Mode</th>
<th>Specified Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\alpha_r$, $\gamma_i$, $V_{di}$, $P_{di}$</td>
</tr>
<tr>
<td>B</td>
<td>$\alpha_r$, $\gamma_i$, $V_{di}$, $P_{di}$</td>
</tr>
<tr>
<td>C</td>
<td>$\alpha_r$, $\gamma_i$, $a_i$, $P_{di}$</td>
</tr>
<tr>
<td>D</td>
<td>$a_r$, $\gamma_i$, $a_i$, $P_{di}$</td>
</tr>
<tr>
<td>E</td>
<td>$\alpha_r$, $\gamma_i$, $a_r$, $P_{di}$</td>
</tr>
<tr>
<td>F</td>
<td>$\alpha_r$, $a_i$, $V_{di}$, $P_{di}$</td>
</tr>
<tr>
<td>G</td>
<td>$\alpha_r$, $a_i$, $a_r$, $P_{di}$</td>
</tr>
<tr>
<td>A_l</td>
<td>$\alpha_r$, $\gamma_i$, $V_{di}$, $I_d$</td>
</tr>
<tr>
<td>B_l</td>
<td>$a_r$, $\gamma_i$, $V_{di}$, $I_d$</td>
</tr>
<tr>
<td>C_l</td>
<td>$\alpha_r$, $\gamma_i$, $a_i$, $I_d$</td>
</tr>
<tr>
<td>D_l</td>
<td>$a_r$, $\gamma_i$, $a_i$, $I_d$</td>
</tr>
<tr>
<td>E_l</td>
<td>$\alpha_r$, $\gamma_i$, $a_r$, $I_d$</td>
</tr>
<tr>
<td>F_l</td>
<td>$\alpha_r$, $a_i$, $V_{di}$, $I_d$</td>
</tr>
<tr>
<td>G_l</td>
<td>$\alpha_r$, $a_i$, $a_r$, $I_d$</td>
</tr>
</tbody>
</table>
Initially, discrete tap positions are considered by assuming the taps to be continuous and at a later stage the taps are fixed at appropriate values.

### 2.2.4 The Eliminated Variable Method

In the eliminated variable method, the equations in (2.14) are, in principle, solved for $x_{dc}$.

$$x_{dc} = f(V_{tr}, V_{ti})$$  \hfill (2.17)

The active and reactive powers consumed by the converters can be expressed as functions of AC terminal voltages, $V_{tr}$ and $V_{ti}$.

$$P_{dr} = P_{dr}(V_{tr}, V_{ti}, x_{dc})$$

$$= P_{dr}(V_{tr}, V_{ti}, f(V_{tr}, V_{ti}))$$

$$= P_{dr}(V_{tr}, V_{ti})$$  \hfill (2.18)

If all real and reactive powers are written as functions of $V_{tr}$ and $V_{ti}$, (2.15) can be replaced by (2.19).

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H \\ J \end{bmatrix} \begin{bmatrix} M' \\ L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V/V \end{bmatrix}$$  \hfill (2.19)

\begin{align*}
N'(tr,tr) &= V_{tr} \frac{\partial P_{ac}}{\partial V_{tr}} + V_{tr} \frac{\partial P_{ac}(V_{tr}, V_{ti})}{\partial V_{tr}} \\
N'(tr,ti) &= V_{tr} \frac{\partial P_{ac}}{\partial V_{ti}} + V_{tr} \frac{\partial P_{ac}(V_{tr}, V_{ti})}{\partial V_{ti}} \\
N'(ti,tr) &= V_{tr} \frac{\partial P_{ac}}{\partial V_{tr}} - V_{tr} \frac{\partial P_{ac}(V_{tr}, V_{ti})}{\partial V_{tr}}
\end{align*}  \hfill (2.20, 2.21, 2.22)
\[ N'(\bar{t}_i, \bar{t}_i) = V_a \frac{\partial P_{ac}}{\partial V_a} - V_a \frac{\partial P_{di}(V_r, V_a)}{\partial V_a} \] (2.23)

\( L' \) is modified analogously. In the eliminated variable method, when a DC-link is included in the power flow, four mismatch equations and up to eight elements of the Jacobian have to be changed. Also the DC variables are not included in the solution vector. The partial derivatives required by (2.19); eg., \( \partial P_{dr}(V_{tr}, V_{ti})/\partial V_{tr} \) is the derivative of \( P_{dr} \) w.r.t. \( V_{tr} \), assuming \( V_{ti} \) as constant. The DC variables, however, are not kept constant as opposed to \( \partial P_{dr}(V_{tr}, V_{ti}, x_{dc})/\partial V_{tr} \), which is used in (2.15). Although (2.19) looks like (2.16), it is mathematically more similar to (2.15). The Jacobean in (2.19) is well-conditioned than the one in (2.15).

2.2.5 Analytical Elimination

To illustrate the procedure, the analytical elimination is carried out in detail for some representative modes. It is sufficient to find \( P_d \) and \( S_d \) at each converter, since \( Q_d \) can be computed with (2.8) or (2.9).

Control Mode A: \[ \begin{bmatrix} a_r & \gamma_i & V_{di} & P_{di} \end{bmatrix} \]

Since both the voltage and power at the inverter are specified, the direct current can be computed with (2.5), and \( P_{dr} \) can then be found by combining (2.3), (2.4) and (2.5).

\[ P_{dr} = P_{di} + R_d I_d^2 \] (2.24)
If we combine (2.1), (2.6) and (2.24), we obtain

\[ S_{dr} = k \frac{P_{di} + \left( R_d + \frac{3}{\pi} X_e \right) I_d^2}{\cos \alpha_r} \]

\[ = k_a \left( P_{di} + P_l + Q_l \right) \quad (2.25) \]

Analogously, for \( S_{di} \):

\[ S_{di} = \frac{k}{\cos \gamma_i} \left( P_{di} + Q_l \right) = k_i \left( P_{di} + Q_l \right) \quad (2.26) \]

Thus, all real and reactive powers consumed by the converters can be pre-computed, and including the dc-link in the power flow is trivial for this control mode. The same is true for any specification of the form \([\alpha_r \gamma_i x_1 x_2]\), where \( x_1 \) and \( x_2 \) are any two variables of \([P_{dr} \ P_{di} \ V_{dr} \ V_{di} \ I_d]\).

**Control Mode B:** \([\alpha_r \gamma_i V_{di} P_{di}]\)

This mode occurs if the tap changer at the rectifier hits a limit in control mode A under current control in the rectifier. Since \( P_{di} \) and \( V_{di} \) are specified, \( I_d, V_{dr}, P_{dr} \) and \( S_{di} \) are computed as for mode A. Since \( \alpha_r \) is specified, \( S_{dr} \) is computed with (2.6) instead of (2.25).

\[ V_{\alpha} \frac{\partial S_{dr}}{\partial V_{\alpha}} = V_{\alpha} \left( k \frac{3\sqrt{2}}{\pi} \alpha_r I_d \right) = S_{dr} \quad (2.27) \]

\[ V_{\alpha} \frac{\partial Q_{dr}}{\partial V_{\alpha}} = \frac{S_{dr}^2}{Q_{dr}} \quad (2.28) \]
The formulas for mode B I are identical; the only difference is that \( P_{di} \) rather than \( I_d \) is computed with (2.5). In general, when two of the variables of \([P_{dr} \ P_{di} \ V_{dr} \ V_{di} \ I_d]\) are specified, the other three can be computed from (2.3)-(2.5).

**Control Mode C:**

\[ \begin{bmatrix} a_r & \gamma_i & a_i & P_{di} \end{bmatrix} \]

These specifications are valid e.g. if the tap changer at the inverter hits a limit in mode A under current control in the rectifier.

Combining (2.2) and (2.5) gives

\[
P_{di} = \frac{3\sqrt{2}}{\pi} a_i V_n \cos \gamma_i I_d - \frac{3}{\pi} X_c I_d^2
\]

(2.29)

If we solve for \( I_d \), we obtain

\[
I_d = c_1 V_n - \sqrt{(c_1 V_n)^2 - c_2 P_{di}}
\]

(2.30)

\[
\frac{\partial I_d}{\partial V_n} = c_1 - \frac{c_1^2 V_n}{\sqrt{(c_1 V_n)^2 - c_2 P_{di}}}
\]

(2.31)

Where,

\[
c_1 = \frac{a_i \cos \gamma_i}{\sqrt{2 X_c}}
\]

(2.32)

\[
c_2 = \frac{\pi}{3 X_c}
\]

(2.33)

Define \( \partial I_i \) as

\[
\partial I_i = \frac{V_n \partial I_d}{I_d \partial V_n}
\]

(2.34)

Since \( P_{di} \) is specified, both its partials are zero. \( P_{dr} \) is given by (2.24), and its partial derivatives by:
\[ V_{tr} \frac{\partial P_{dr}}{\partial V_{tr}} = 0 \]  
\hfill (2.35)

\[ V_{ai} \frac{\partial P_{dr}}{\partial V_{ai}} = 2 R_d I_d^2 \frac{V_{ai}}{I_d} \frac{\partial I_d}{\partial V_{ai}} = 2 P_i \partial I_i \]  
\hfill (2.36)

Since \( a_i \) is specified, \( S_{di} \) is computed with (2.7), and the partial derivatives of \( Q_{di} \) are given by:

\[ V_{tr} \frac{\partial Q_{di}}{\partial V_{tr}} = 0 \]  
\hfill (2.37)

\[ V_{ai} \frac{\partial Q_{di}}{\partial V_{ai}} = \frac{S_{di}^2}{Q_{di}} (1 + \partial I_i) \]  
\hfill (2.38)

\( Q_{dr} \) and its partial derivatives are computed from (2.25).

\[ V_{ai} \frac{\partial S_{dr}}{\partial V_{ai}} = 2 \partial I_i k_a (P_i + Q_i) \]  
\hfill (2.39)

\[ V_{tr} \frac{\partial Q_{dr}}{\partial V_{tr}} = 0 \]  
\hfill (2.40)

\[ V_{ai} \frac{\partial Q_{dr}}{\partial V_{ai}} = \frac{2 \partial I_i}{Q_{dr}} \left[ k_a S_{dr} (Q_i + P_i) - P_i P_{dr} \right] \]  
\hfill (2.41)

**Other Modes**

The partial derivatives for the other control modes can be derived analogously; if the tap changer controlling the control angle is specified (modes B, D, F, G), only the reactive power at that converter will depend on corresponding AC voltage. All the active and reactive powers will depend on the AC voltage at that terminal, if the tap changer controlling the direct voltage is specified (modes C, D, E, G).
Equations (2.30) or (2.43) are used to find the direct current for constant power control.

If the tap changer position is specified at a converter, $S_d$ is computed with (2.6) or (2.7), otherwise (2.25) or (2.26) are used. All the eight partial derivatives for all possible modes listed in Table 2.1 are summarized in Table 2.2 and Table 2.3.
Table 2.2: Partial derivatives for modes with the direct voltage determined by \( a_i \)

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \frac{\partial P_{di}}{\partial V_{tr}} )</th>
<th>( v_{ir} \frac{\partial Q_{dr}}{\partial V_{tr}} )</th>
<th>( v_{ir} \frac{\partial P_{dr}}{\partial V_{ti}} )</th>
<th>( v_{ir} \frac{\partial Q_{dr}}{\partial V_{ti}} )</th>
<th>( \frac{\partial Q_{di}}{\partial V_{tr}} )</th>
<th>( v_{ir} \frac{\partial P_{di}}{\partial V_{ti}} )</th>
<th>( v_{ir} \frac{\partial Q_{di}}{\partial V_{ti}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A_i</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>( \frac{S_{dr}^2}{Q_{dr}} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B_i</td>
<td>0</td>
<td>( \frac{S_{dr}^2}{Q_{dr}} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>( 2R_i \alpha_i )</td>
<td>( \frac{2\alpha_i}{Q_{dr}} \left[ k_\alpha S_{dr} \left( P_{i} + Q_{i} \right) - P_{dr} P_{di} \right] )</td>
<td>0</td>
<td>0</td>
<td>( \frac{S_{di}^2}{Q_{di}} \left( 1 + \alpha_i \right) )</td>
</tr>
<tr>
<td>C_i</td>
<td>0</td>
<td>0</td>
<td>( P_{di} + Q_{i} )</td>
<td>( \frac{P_{di} + Q_{i}}{Q_{dr}} \left[ k_\alpha S_{dr} - P_{dr} \right] )</td>
<td>0</td>
<td>0</td>
<td>( P_{di} + Q_{i} )</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>( \frac{S_{dr}^2}{Q_{dr}} )</td>
<td>( 2R_i \alpha_i )</td>
<td>( \frac{\alpha_i}{Q_{dr}} \left[ S_{dr}^2 - 2P_{di} P_{dr} \right] )</td>
<td>0</td>
<td>0</td>
<td>( \frac{S_{di}^2}{Q_{di}} \left( 1 + \alpha_i \right) )</td>
</tr>
<tr>
<td>D_i</td>
<td>0</td>
<td>( \frac{S_{dr}^2}{Q_{dr}} )</td>
<td>( P_{di} + Q_{i} )</td>
<td>( \frac{P_{di} + Q_{i}}{Q_{dr}} P_{dr} )</td>
<td>0</td>
<td>0</td>
<td>( P_{di} + Q_{i} )</td>
</tr>
</tbody>
</table>
Table 2.3: Partial derivatives for modes with the direct voltage determined by $a_r$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\frac{\partial P_{dr}}{\partial V_{tr}}$</th>
<th>$\frac{\partial Q_{dr}}{\partial V_{tr}}$</th>
<th>$\frac{\partial P_{di}}{\partial V_{ti}}$</th>
<th>$\frac{\partial Q_{di}}{\partial V_{ti}}$</th>
<th>$\frac{\partial P_{di}}{\partial V_{ti}}$</th>
<th>$\frac{\partial Q_{di}}{\partial V_{ti}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>$2P_I \Delta I_r$</td>
<td>$Q_{dr}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E_I</td>
<td>$P_{dr} + Q_I$</td>
<td>$Q_{dr}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$P_{dr} + Q_I$</td>
<td>$0$</td>
</tr>
<tr>
<td>F</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>F_I</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>G</td>
<td>$2P_I \Delta I_r$</td>
<td>$Q_{dr}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>G_I</td>
<td>$P_{dr} + Q_I$</td>
<td>$Q_{dr}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$P_{dr} + Q_I$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

$\Delta I_r$ in Table 2.3 is defined as

$$\Delta I_r = \frac{V_{tr} \Delta I_d}{I_d \frac{\partial V_{tr}}{\partial I_d}} \quad (2.42)$$
Where,
\[ I_d = c_3 V_r - \sqrt{(c_3 V_r)^2 - c_4 P_{di}} \]  \hspace{1cm} (2.43)

\[ c_3 = \frac{3a_c \cos \alpha}{\sqrt{2(\pi R_d + 3X_c)}} \]  \hspace{1cm} (2.44)

\[ c_4 = \frac{\pi}{\pi R_d + 3X_c} \]  \hspace{1cm} (2.45)

2.3 TRANSIENT STABILITY STUDIES

Transient stability studies provide information related to the capability of a power system to remain in synchronism during major disturbances resulting from either the loss of generation or transmission facilities, sudden or sustained load changes, or momentary faults [114]. Specifically, these studies provide the changes in the voltages, currents, powers, speeds, and torques of the machines of the power system, as well as the changes in system voltages and power flows, during and immediately following a disturbance. In the planning of new facilities the degree of stability of a power system is an important factor. In order to provide the reliability required by the dependence on continuous electric service, it is necessary that power systems be designed to be stable under any conceivable disturbance.

The power system transient performance can be obtained from the network equations. The performance equation using the bus frame of reference in either the impedance or admittance form has been used in transient stability calculations.
The operating characteristics of synchronous machines are described by a set of differential equations. The number of differential equations required for a machine depends on the details needed to represent accurately the machine performance.

Transient stability analysis outcome is the resultant solution of the network algebraic equations and the machine differential equations. The solution of the network equations retains the identity of the system and thereby provides access to system voltages and currents during the transient period.

As compared with rotor long-time constants, the AC and DC-transmission systems respond rapidly to network and load changes. The time constants associated with the network variables are extremely small and can be neglected without significant loss of accuracy. Also stator time constants of the synchronous machine may be taken as zero.

The DC link is assumed here to maintain normal operation throughout the disturbance. This approach is not valid for larger disturbances such as converter faults, DC-line faults and AC faults close to the converter stations, these disturbances can cause commutation failures and alter the normal conduction sequence [105].
2.3.1 Generator Representation

The synchronous machine is represented by a voltage source that is constant in magnitude but changes in angular position, behind its transient reactance. The effect of saliency is neglected in this representation and it assumes constant flux linkages and a small change in speed. If the machine rotor speed is assumed constant at synchronous speed, then $M$ is constant. The machine accelerating power is equal to the difference between the mechanical power and the electrical power [6] if the rotational losses are neglected. The classical model can be described by the following set of differential and algebraic equations:

Differential:

\[
\frac{d\delta}{dt} = \omega - 2\pi f
\]

\[
\frac{d^2\delta}{dt^2} = \frac{d\omega}{dt} = \frac{\pi f}{H} \left( P_m - P_e \right)
\]

(2.46)

Algebraic:

\[
E' = E_t + r_a I_t + j x_d' I_t
\]

(2.47)

Where, $E'$=voltage back of transient reactance

$E_t$=machine terminal voltage

$I_t$=machine terminal current

$r_a$=armature resistance

$x_d'$=transient reactance


2.3.2 Load Representation

Power system loads, other than motors, can be treated in several ways during the transient period. The commonly used representations are either static impedance or admittance to ground, constant real and reactive power, or a combination of these representations. The parameters associated with static impedance and constant current representations are obtained from the load flow solution for the power system prior to a disturbance. The initial value of the current for a constant current representation is obtained from

\[
I_{po} = \frac{P_{lp} - jQ_{lp}}{E_p^*} \tag{2.48}
\]

The static admittance \( Y_{po} \) used to represent the load at bus P, can be obtained from

\[
Y_{po} = \frac{I_{po}}{E_p} \tag{2.49}
\]

Where, \( E_p \) is the calculated bus voltage, \( P_{lp} \) and \( Q_{lp} \) are the scheduled real and reactive bus loads. Corresponding to each load bus, using the
expression for $Y_{po}$ discussed above, the diagonal elements of the admittance matrix are modified.

### 2.3.3 HVDC System Representation

For the representation of DC systems in stability studies standard models have not been developed [1], since each DC system tends to have unique characteristics to meet the specific needs of its application.

**a) Converter model**

**i) Simplified model**

Here valve switching is neglected and the converter is represented by the average DC voltage equation. This model is similar to that used in power flow analysis. The transformer tap is assumed to be constant as the tap changer dynamics are very slow [65]. This model is inaccurate during severe disturbances such as unsymmetrical faults and cannot handle commutation failures.

**ii) Detailed model**

Here, the valve switching is incorporated and the model is free from the drawbacks associated with the simplified model. However the transient simulation of converter now requires integration step size as small as 50 – 100 µs. This implies heavy computation burden, so it is
used only for short duration (say 0.2 sec) immediately after the disturbance.

b) Converter Controller Models

i) Response type model

The dynamics of the CEA and CC are neglected and only the steady-state controller characteristics are represented. The main feature of this type of controller model is that the configuration and the parameters of the controller are assumed to be designed at a later stage based on the requirement.

ii) Detailed Representation

It requires the analysis of actual control circuitry and the establishment of a dynamic equivalent with a frequency response which matches the actual controller response. This is used along with the detailed converter model.

c) DC Network Model

i) Resistive network

Here DC network is represented as resistive network ignoring energy storage elements. This approach is valid when DC lines are short and for back to back HVDC links where the smoothing reactors are of moderate size.
ii) **Transfer Function Representation**

The DC network can be represented as a transfer function as shown in Fig 2.3. Here the time constant $T_{dc}$ represents the delay in establishing the DC current after a step change in the current order is given.

$$
\frac{I_{REF}}{1 + sT_{dc}} \rightarrow I_D
$$

*Fig 2.3: Transfer Function Model*

iii) **Dynamic Representation**

As the frequency bandwidth of the response model considered in the transient stability studies is modest, it is adequate to represent the dc network by a simple equivalent circuit of the type shown in Fig 2.4. Even here, the shunt branches may be neglected.

*Fig 2.4: Equivalent Circuit*

### 2.3.4 Runge-Kutta Method

In the application of the Runge-Kutta fourth-order approximation, the changes in the internal voltage angles and machine speeds, for the simplified machine representation, are determined from (2.50):
\[
\Delta \delta_{i(+\Delta t)} = \frac{1}{6} \left( k_{ij} + 2k_{ji} + 2k_{ji} + k_{ii} \right)
\]
\[
\Delta \omega_{i(+\Delta t)} = \frac{1}{6} \left( l_{ii} + 2l_{ii} + 2l_{ii} + l_{ii} \right)
\]  
(2.50)

Where, \( i=1,2,,\text{no. of generators} \).

The \( k \)'s and \( l \)'s are the changes in \( \delta_i \) and \( \omega_i \) respectively, obtained using derivatives evaluated at predetermined points. For this procedure the network equations are to be solved four times.

### 2.3.5 Algorithm for AC-DC Transient Stability Study

In a practical power system, the HVDC link interconnects two or more independent AC systems and the stability assessment is carried out for each of them separately, taking into account the power constraints at the converter terminal. In the case of a synchronous AC system with a DC link as part of it, the converter constraints will apply to each of the nodes containing a converter terminal. The steps of transient stability program are given below [115]:

1) The initial bus voltages are obtained from the AC/DC load flow, prior to the disturbance.

2) After the AC/DC load flow solution is obtained, the machine currents and voltages behind transient reactance are calculated.

3) The initial speed is equated to \( 2\pi f \) and the initial mechanical power is equated to the real power output of each machine prior to the disturbance.
4) The network data is modified for the new representation. Extra nodes are added to represent the generator internal voltages. Admittance matrix is modified to incorporate the load representation.

5) Set time, \( t=0 \).

6) For any switching operation or change in fault condition, network data is to be modified accordingly and the AC/DC load flow is to be performed.

7) Machine differential equations are to be solved using Runge-Kutta method to find the changes in the internal voltage angles and machine speeds.

8) Internal voltage angles and machine speeds are to be updated and stored for plotting.

9) AC/DC load flow is to be performed again to get the new output powers of the machines.

10) Advance time, \( t=t+\Delta t \).

11) Check for time limit. If \( t \leq t_{\text{max}} \) repeat the process from step 6, else plot the graphs of internal voltage angle variations and stop the process.
From the plots obtained using the procedure detailed above, it can be judged whether the system is stable or unstable. For the analysis of multi machine system stability, the plot of relative angles is to be evaluated. The flow chart for transient stability study depicting the above algorithm is given in Fig 2.5.
Fig 2.5: Flow Chart for Transient Stability Study
The transient stability analysis detailed above is initially applied for a Single Machine Infinite bus system with parallel AC & DC links and then for a typical Multi Machine System with a single DC link and these are detailed in the subsequent chapters.