4. MODELING AND DESIGN OF Z-SOURCE CONVERTERS

4.1. Introduction

The state space averaging technique approach has a number of advantages over circuit averaging technique, these include:

- More compact representation of equations.
- Ability to obtain more transfer functions than was possible using circuit averaging technique.
- Both DC and AC transfer functions are obtained with more ease.

Thus derivation of state space averaged models of basic converter topologies helps in solving the circuits using various methods easily.

In case we have two state space model of system for different switching states. For one switching state referred as shoot-through state with DT period (D is the shoot through duty cycle and T is the overall time period), the state equations are

\[
\begin{align*}
\dot{x} &= A_1.x + B_1.u \\
y &= C_1.x + D_1.u
\end{align*}
\]  

(4.1)

Similarly for non-shoot through state with (1-D) T period, the state equations are

\[
\begin{align*}
\dot{x} &= A_2.x + B_2.u \\
y &= C_2.x + D_2.u
\end{align*}
\]  

(4.2)

The averaged large signal model of the system is following-

\[
\begin{align*}
\dot{x} &= A.x + B.u \\
y &= C.x + D.u
\end{align*}
\]  

(4.3)

Where A, B, C, D are weighted averaged matrices of the system, which can be calculated as-

\[
\begin{align*}
\hat{A} &= A_1.D + A_2.(1-D) \\
\hat{B} &= B_1.D + B_2.(1-D) \\
\hat{C} &= C_1.D + C_2.(1-D) \\
\hat{D} &= D_1.D + D_2.(1-D)
\end{align*}
\]  

(4.4)
Now small signal mathematical model can be developed to study system dynamic response and this can be done through perturbation by small disturbances of the variables. Applying this in the state equation (4.3)
\[ \dot{x} = A \cdot x + B \cdot u \]
Hence
\[ \left( X + \dot{x} \right) = A_1 \cdot (D + \dot{d}) + A_2 \cdot (1 - D - \dot{d}) \right] \cdot (X + \dot{x}) + + [ B_1 \cdot (D + \dot{d}) + B_2 \cdot (1 - D - \dot{d}) ] \cdot (U + \dot{u}) \] (4.5)

Where \( \dot{d} \) is the perturbation for the variable D.

Assumptions-

- Product of small signal values is negligible as compared to the steady state value.
\[ \frac{\dot{x}}{X} \cdot \dot{d} << 1 \] (4.6)

- As steady state parts are steady like D.C. Hence it has no slope. i.e. \[ \frac{dx}{dt} = 0 \]
\[ \dot{X} = A \cdot X + B \cdot U = 0 \]

Hence using these assumptions-
\[ \dot{x} = \bar{A} \cdot \dot{x} + \bar{B} \cdot \dot{u} + [(A_1 - A_2) \cdot X + (B_1 - B_2) \cdot U] \cdot \dot{d} \] (4.7)

Where \[ \bar{A} = A_1 \cdot (D + A_2 \cdot (1 - D)) \] and \[ \bar{B} = B_1 \cdot (D + B_2 \cdot (1 - D)) \]

Similarly, small signal output equation will be as follows-
\[ \dot{y} = \bar{C} \cdot \dot{x} + \bar{D} \cdot \dot{u} + [(C_1 - C_2) \cdot X + (D_1 - D_2) \cdot U] \cdot \dot{d} \] (4.8)

Based on the above theory following models are developed

Now small signal mathematical model can be developed to study system dynamic response and this can be done through perturbation by small disturbances of the variables.

4.2. State Space Model of z-source Inverter (ZSI) with ideal network component

The equivalent circuit of ZSI [2] is shown in Figure 4.1. Where shoot through operation is presented by switch S1. \( I_L \) is the overall inverter input current during non-shoot through period.

Figure 4.1 Equivalent circuit of Z-Source inverter
For S1= ON i.e. during shoot-through state from the equivalent circuit Figure 4.2

BY KVL
\[ L1 \frac{dLI_1}{dt} - VC_1 = 0 \]  
\[ L2 \frac{dLI_2}{dt} - VC_2 = 0 \]  

BY KCL
\[ C_1 \frac{dVC_1}{dt} = -LI_1 \]  
\[ C_2 \frac{dVC_2}{dt} = -LI_2 \]  

State matrix
\[
\begin{bmatrix}
LI_1 \\
LI_2 \\
VC_1 \\
VC_2 
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 1/L1 & 0 & [LI_1] \\
0 & 0 & 0 & 1/L2 & [LI_2] \\
-1/C1 & 0 & 0 & 0 & [VC_1] \\
0 & -1/C2 & 0 & 0 & [VC_2] 
\end{bmatrix}
\]  

Figure 4.2 Equivalent circuit of ZSI (a) Shoot through mode (b) non shoot through mode

For S1 = OFF equivalent circuit Figure 4.2(b)

Applying KVL
\[ L1 \frac{dLI_1}{dt} + VC_2 - V_{DC} = 0 \]  
\[ L2 \frac{dLI_2}{dt} + VC_1 - V_{DC} = 0 \]  

Applying KCL
\[ C_1 \frac{dVC_1}{dt} - LI_2 + I_{DC} = 0 \]  
\[ -C_2 \frac{dVC_2}{dt} + LI_1 - I_{DC} = 0 \]
State matrix

\[
\begin{bmatrix}
I_{L_1} \\
I_{L_2} \\
V_{c_1} \\
V_{c_2}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & -1/L_1 \\
0 & 0 & -1/L_2 & 0 \\
1/C_1 & 0 & 0 & 0 \\
1/C_2 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_{L_1} \\
I_{L_2} \\
V_{c_1} \\
V_{c_2}
\end{bmatrix} +
\begin{bmatrix}
V_{DC}/L_1 \\
V_{DC}/L_2 \\
-I_{DC}/C_1 \\
-I_{DC}/C_2
\end{bmatrix}
\]

The state equations are averaged considering shoot through duty ratio D (i.e. when S1 is on).

Averaging Two Matrices as 

\[
x(t) = D(A_1x + B_1 u) + (1 - D)(A_2 x + B_2 u)
\]

The state space model becomes

\[
\begin{bmatrix}
I_{L_1} \\
I_{L_2} \\
V_{c_1} \\
V_{c_2}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & D/L & -(1 - D)/L \\
0 & 0 & -(1 - D)/L & D/L \\
-D/C & (1 - D)/C & 0 & 0 \\
(1 - D)/C & -D/C & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_{L_1} \\
I_{L_2} \\
V_{c_1} \\
V_{c_2}
\end{bmatrix} +
\begin{bmatrix}
(1 - D)V_{DC}/L \\
(1 - D)V_{DC}/L \\
-(1 - D)I_{DC}/C \\
-(1 - D)I_{DC}/C
\end{bmatrix}
\]

Above equations contain nonlinear terms. This can be overcome performing small signal analysis.

Introducing perturbation in the first equation for the variables

\[
L \frac{d(i_{L_1} + i_{L_2})}{dt} = (D + \dot{D})(V_{c_1} + \dot{V}_{c_1}) - (\dot{D} - \ddot{D})(V_{c_2} + \ddot{V}_{c_2}) + (\dot{D} - \ddot{D})(V_{DC} + \ddot{V}_{DC})
\]

Where \((1 - D) = \dot{D}\)

Taking ac term and neglecting second and higher order terms

\[
L \frac{di_{L_1}}{dt} = (D\dot{V}_{c_1} + \ddot{V}_{c_1}) - (\dot{D}\dot{V}_{c_2} - \ddot{V}_{c_2}) + (\dot{D}\dot{V}_{DC} - \ddot{V}_{DC})
\]

Considering, \(I_{L_1} = I_{L_2} = I_L, V_{c_1} = V_{c_2} = V_c\)

\[
L \frac{di_L}{dt} = 2\dot{V}_c + (2D - 1)\ddot{V}_c + \dot{D}\ddot{V}_{DC} - \ddot{D}\dot{V}_{DC}
\]

Similarly

\[
C \frac{dV_c}{dt} = -2\dot{V}_c + (1 - 2D)i_L - \dot{D}I_{DC} - \ddot{D}i_{DC}
\]

Taking Laplace transform of (4.22) and (4.23)

\[
sL_i_L(s) = 2V_c\dot{a}(s) + (2D - 1)\ddot{V}_c(s) + \dot{D}\ddot{V}_{DC}(s) + V_{DC}\dot{a}(s)
\]
\[ sC\hat{v}_c(s) = -2I_L \hat{d}(s) + (1 - 2D)\hat{i}_L(s) - \hat{d}I_{dc}(s) + I_{dc} \hat{d}(s) \]  
(4.25)

Rearranging above

\[ sL\hat{i}_L(s) + (1 - 2D)\hat{v}_c(s) = (2V_c + V_{DC})\hat{d}(s) + \hat{d}V_{dc}(s) \]  
(4.26)

\[ (2D - 1)\hat{i}_L(s) + sC\hat{v}_c(s) = (I_{dc} - 2I_L)\hat{d}(s) - \hat{d}I_{dc}(s) \]  
(4.27)

(4.26) and (4.27) in matrix form

\[
\begin{bmatrix}
  \hat{i}_L(s) \\
  \hat{v}_c(s)
\end{bmatrix}
= 
\begin{bmatrix}
  sL & (1 - 2D) \\
  sC & (2D - 1)
\end{bmatrix}
\begin{bmatrix}
  2V_c + V_{DC} \\
  (I_{dc} - 2I_L)
\end{bmatrix}
\hat{d}(s) + \hat{d}
\begin{bmatrix}
  \hat{v}_{dc}(s) \\
  -I_{dc}(s)
\end{bmatrix}
\]  
(4.28)

Or,

\[
\begin{bmatrix}
  \hat{i}_L(s) \\
  \hat{v}_c(s)
\end{bmatrix}
= 
\begin{bmatrix}
  sL & (1 - 2D) \\
  (2D - 1) & sC
\end{bmatrix}^{-1}
\begin{bmatrix}
  2V_c + V_{DC} \\
  (I_{dc} - 2I_L)
\end{bmatrix}
\hat{d}(s) + \hat{d}
\begin{bmatrix}
  sL & (1 - 2D) \\
  (2D - 1) & sC
\end{bmatrix}^{-1}
\begin{bmatrix}
  \hat{v}_{dc}(s) \\
  -I_{dc}(s)
\end{bmatrix}
\]  
(4.29)

\[
\begin{bmatrix}
  \hat{i}_L(s) \\
  \hat{v}_c(s)
\end{bmatrix}
= \frac{1}{A}
\begin{bmatrix}
  sC & (2D - 1) \\
  sL & (I_{dc} - 2I_L)
\end{bmatrix}
\hat{d}(s) + \hat{d}
\begin{bmatrix}
  sC & (2D - 1) \\
  (2D - 1) & sL
\end{bmatrix}^{-1}
\begin{bmatrix}
  \hat{v}_{dc}(s) \\
  -I_{dc}(s)
\end{bmatrix}
\]  
(4.30)

Where \( A = S^2 LC + (2D - 1)^2 \)

Transfer functions from above can be derived as

\[
\frac{\hat{i}_L(s)}{\hat{d}(s)} = \frac{(2V_c + V_{DC})sC - (2D - 1)(2I_L - I_{dc})}{S^2 LC + (2D - 1)^2} \]  
(4.31)

\[
\frac{\hat{v}_c(s)}{\hat{d}(s)} = \frac{(2V_c + V_{DC})(1 - 2D) - sL(2I_L - I_{dc})}{S^2 LC + (2D - 1)^2} \]  
(4.32)
4.3. State Space Model of z-source Inverter (ZSI) with non-ideal network component

Figure 4.3 Equivalent circuit of the Z-Source inverter with non ideal components

In non-ideal network the inductors and capacitors are considered lossy and having parasitic resistances with each of them. The capacitors $C_1$ and $C_2$ are having equivalent series resistances as $R_1$ and $R_2$. The inductors $L_1$ and $L_2$ having DC value of resistances as $r_1$ and $r_2$ respectively.

For $S_1 = \text{ON}$, equivalent circuit shown in Figure 4.4 gives

Applying KVL

\[
L_1 \frac{dI_{L1}}{dt} + (R_1 + r_1)I_{L1} - V_{C1} = 0
\]

\[L_2 \frac{dI_{L2}}{dt} + (R_2 + r_2)I_{L2} - V_{C2} = 0\]

Applying KCL

\[
C_1 \frac{dV_{C1}}{dt} = -I_{L1}
\]

\[
C_2 \frac{dV_{C2}}{dt} = -I_{L2}
\]

State equation will be

\[
\begin{bmatrix}
I_{L1} \\
I_{L2} \\
V_{C1} \\
V_{C2}
\end{bmatrix}
\begin{bmatrix}
-(R_1 + r_1)/L_1 & 0 & 1/L_1 & 0 \\
0 & -(R_2 + r_2)/L_2 & 0 & 1/L_2 \\
-1/C_1 & 0 & 0 & 0 \\
0 & -1/C_2 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_{L1} \\
I_{L2} \\
V_{C1} \\
V_{C2}
\end{bmatrix}
\]

\[
(4.37)
\]

Again for $S_1 = \text{OFF}$, the equivalent circuit of Figure 4.4 (b) gives

Applying KVL

\[
L_1 \frac{dI_{L1}}{dt} + (R_2 + r_1)I_{L1} - R_2I_{DC} + V_{C2} - V_{DC} = 0
\]

\[
L_2 \frac{dI_{L2}}{dt} + (R_1 + r_2)I_{L2} - R_1I_{DC} + V_{C1} - V_{DC} = 0
\]

\[
(4.38)
\]

\[
(4.39)
\]
Applying KCL

\[ C_1 \frac{dV_{c1}}{dt} - IL_2 + I_{DC} = 0 \]  
(4.40)

\[ -C_2 \frac{dV_{c2}}{dt} + IL_1 - I_{DC} = 0 \]  
(4.41)

Figure 4.4 Equivalent circuit of ZSI (a) shoot through mode (b) non shoot through mode

The state equations for this mode becomes

\[
\begin{bmatrix}
I_{L1} \\
I_{L2} \\
V_{c1} \\
V_{c2}
\end{bmatrix}
= 
\begin{bmatrix}
-(R2 + r1)/L1 & 0 & 0 & -1/L1 \\
0 & -(R1 + r2)/L2 & -1/L2 & 0 \\
1/C1 & 0 & 0 & 0 \\
1/C2 & 0 & 0 & 0
\end{bmatrix}
I_{L1} \\
I_{L2} \\
V_{c1} \\
V_{c2}
\]

\[ + \begin{bmatrix}
(R2I_{DC} + V_{DC})/L1 \\
(R1I_{DC} + V_{DC})/L2 \\
-I_{DC}/C1 \\
-I_{DC}/C2
\end{bmatrix} \]  
(4.42)

The state equations are averaged considering D shoot through duty ratio \( n \) (i.e. when S1 is on). Also assume \( R1=R2=R, r1=r2=r \) and \( L1=L2=L, C1=C2=C \).

Averaging Two Matrices as \( x(t) = D(A_1 x + B_1 u) + (1-D)(A_2 x + B_2 u) \)

The state space model becomes

\[
\begin{bmatrix}
I\dot{L}_1 \\
I\dot{L}_2 \\
\dot{V}_{c1} \\
\dot{V}_{c2}
\end{bmatrix}
= 
\begin{bmatrix}
-(R + r)/L & 0 & D/L & -(1-D)/L \\
0 & -(R + r)/L & -(1-D)/L & D/L \\
-D/C & (1-D)/C & 0 & 0 \\
(1-D)/C & -D/C & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_{L1} \\
I_{L2} \\
V_{c1} \\
V_{c2}
\end{bmatrix} \\
+ \begin{bmatrix}
(1-D)(RI_{DC} + V_{DC})/L \\
(1-D)(RI_{DC} + V_{DC})/L \\
-(1-D)I_{DC}/C \\
-(1-D)I_{DC}/C
\end{bmatrix} \]  
(4.43)

Above equations contain nonlinear terms. This can be linearised performing small signal analysis.

Introducing perturbation in the first equation for the variables.
\[
L \frac{d(IL_1 + i_{L1})}{dt} = -(R + r)(IL_1 + i_{L1}) + (D + \hat{d})(v_{c1} + \hat{v}_{c1})
- (\dot{\hat{d}} - \hat{\alpha})(v_{c2} + \hat{v}_{c2}) + (\dot{\hat{d}} - \hat{\alpha})(v_{DC} + \hat{v}_{DC})
+ (\dot{\hat{d}} - \hat{\alpha})R(I_{DC} + \hat{I}_{DC})
\]

(4.44)

Where \((1 - D) = \hat{D}\)

Taking ac term and neglecting smaller products.

\[
L \frac{di_{L1}}{dt} = -(R + r)i_{L1} + (Dv_{c1} + \hat{d}v_{c1}) - (\dot{\hat{d}}v_{c2} - \hat{\alpha}v_{c2}) + (\dot{\hat{d}}v_{DC} - \hat{\alpha}v_{DC})
+ R (\hat{d}I_{DC} - \hat{\alpha}I_{DC})
\]

(4.45)

Considering \(IL_1 = IL_2 = IL, v_{c1} = v_{c2} = Vc\)

\[
L \frac{di_{L}}{dt} = 2dV_c + (2D - 1)v_{c} + \dot{\hat{d}}v_{DC} - \hat{\alpha}v_{DC} - (R + r)i_{L}
+ R (\hat{d}I_{DC} - \hat{\alpha}I_{DC})
\]

(4.46)

Similarly

\[
C \frac{dv_{c}}{dt} = -2\dot{\hat{d}}l_{L} + (1 - 2D)i_{L} - \dot{\hat{d}}I_{DC} - \hat{\alpha}I_{DC}
\]

(4.47)

Taking Laplace transform of (4.46) and (4.47)

\[
sLl_{L}(s) = 2V_c \dot{\hat{d}}(s) + (2D - 1)v_{c}(s) + \dot{\hat{d}}v_{DC}(s) + v_{DC}(s) - (R + r)i_{L}(s)
+ R (\hat{d}I_{DC}(s) - \hat{\alpha}(s)I_{DC})
\]

(4.48)

\[
sCv_{c}(s) = -2\dot{\hat{d}}l_{L}(s) + (1 - 2D)i_{L}(s) - \dot{\hat{d}}I_{DC}(s) + I_{DC}(s)l_{DC}(s)
\]

(4.49)

Rearranging above

\[
(sL + R + r)i_{L}(s) + (1 - 2D)v_{c}(s)
= (2V_c + v_{DC} - RI_{DC})\dot{\hat{d}}(s) + \dot{\hat{d}}v_{DC}(s) + R\dot{\hat{d}}I_{DC}(s)
\]

(4.50)

\[
(2D - 1)i_{L}(s) + sCv_{c}(s) = (I_{DC} - 2l_{L})\dot{\hat{d}}(s) - \dot{\hat{d}}I_{DC}(s)
\]

(4.51)

(4.118) and (4.119) in matrix form

\[
\begin{bmatrix}
i_{L}(s) \\
v_{c}(s)
\end{bmatrix} =
\begin{bmatrix}
(sL + R + r) & (1 - 2D) \\
(2D - 1) & sC
\end{bmatrix}
\begin{bmatrix}
\dot{\hat{d}}(s) \\
\dot{\hat{d}}v_{DC}(s)
\end{bmatrix}
\]

\[
=\begin{bmatrix}
(2V_c + v_{DC} - RI_{DC}) \\
(I_{DC} - 2l_{L})
\end{bmatrix} \cdot \dot{\hat{d}}(s) + \begin{bmatrix}
\dot{\hat{d}} & R \dot{\hat{d}} \\
0 & -\dot{\hat{d}}
\end{bmatrix} \begin{bmatrix}
v_{DC}(s) \\
I_{DC}(s)
\end{bmatrix}
\]

(4.52)
Or,
\[
\begin{bmatrix}
i_L(s) \\
\bar{V}_c(s)
\end{bmatrix} = \left[ \begin{array}{cc}
(sL + R + r) & (1 - 2D) \\
(2D - 1) & sC
\end{array} \right]^{-1} \left[ \begin{array}{c}
(2V_c + V_{DC} - RL_{DC}) \\
(l_{DC} - 2I_c)
\end{array} \right] \cdot \bar{d}(s)
\]
\[
+ \left[ \begin{array}{cc}
(sL + R + r) & (1 - 2D) \\
(2D - 1) & sC
\end{array} \right]^{-1} \left[ \begin{array}{c}
\dot{D} \\
0
\end{array} \right] \bar{V}_{DC}(s)
\]  
(4.53)

Or,
\[
\begin{bmatrix}
i_L(s) \\
\bar{V}_c(s)
\end{bmatrix} = 1/A \left[ \begin{array}{cc}
sC & (2D - 1) \\
-(2D - 1) & (sL + R + r)
\end{array} \right]^{-1} \left[ \begin{array}{c}
(2V_c + V_{DC} - RL_{DC}) \\
(l_{DC} - 2I_c)
\end{array} \right] \cdot \bar{d}(s)
\]
\[
+ 1/A \left[ \begin{array}{cc}
sC & (2D - 1) \\
-(2D - 1) & (sL + R + r)
\end{array} \right] \left[ \begin{array}{c}
\dot{D} \\
0
\end{array} \right] \bar{V}_{DC}(s)
\]  
(4.54)

Where \( A = S^2LC + sC(R + r) + (2D - 1)^2 \)

Above matrices gives transfer functions as below
\[
\frac{i_L(s)}{\bar{d}(s)} = \frac{(2V_c + V_{DC} - RL_{DC})sC - (2D - 1)(2I_c - l_{DC})}{S^2LC + sC(R + r) + (2D - 1)^2}  
\]  
(4.55)

\[
\frac{\bar{V}_c(s)}{\bar{d}(s)} = \frac{(2V_c + V_{DC} - RL_{DC})(1 - 2D) - (sL + R + r)(2I_c - l_{DC})}{S^2LC + sC(R + r) + (2D - 1)^2}  
\]  
(4.56)

4.4. State Space Model of Quasi z-source Inverter (QZSI) with ideal network component

\[
\begin{array}{c}
\text{Figure 4.5 Equivalent circuit model of Quasi z-source inverter}
\end{array}
\]

From the equivalent circuit [54] of Figure 4.5, for \( S1 = \text{ON} \), it becomes shoot-through equivalent circuit shown in Figure 4.6(a). From this circuit applying KVL and KCL.

Applying KVL
\[
L_1 \frac{dI_1}{dt} + V_{C_2} - V_{DC} = 0 
\]  
(4.57)

\[
L_2 \frac{dI_2}{dt} - V_{C_1} = 0 
\]  
(4.58)

Applying KCL
\[
C_1 \frac{dV_{C_1}}{dt} = 0 
\]  
(4.59)

\[
C_2 \frac{dV_{C_2}}{dt} = I_1 
\]  
(4.60)
The State matrix becomes

\[
\begin{bmatrix}
I_L_1 \\
I_L_2 \\
V_{C_1} \\
V_{C_2}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & -1/L_1 \\
0 & 0 & -1/L_2 & 0 \\
0 & 0 & 0 & 0 \\
1/C_2 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_L_1 \\
I_L_2 \\
V_{C_1} \\
V_{C_2}
\end{bmatrix} +
\begin{bmatrix}
V_{DC}/L_1 \\
0 \\
0 \\
-1/L_1/L_1
\end{bmatrix}
\]

For S1 = OFF it becomes non shoot-through equivalent circuit shown in Figure 4.6 (b) from this circuit applying KVL and KCL.

Applying KVL

\[
L_1 \frac{dI_L_1}{dt} + V_{C_1} - V_{DC} = 0
\]

\[
L_2 \frac{dI_L_2}{dt} - V_{C_2} = 0
\]

Applying KCL

\[
C_1 \frac{dV_{C_1}}{dt} - I_L_1 + I_L = 0
\]

\[
C_2 \frac{dV_{C_2}}{dt} + I_L_2 - I_L = 0
\]

The State matrix becomes

\[
\begin{bmatrix}
I_L_1 \\
I_L_2 \\
V_{C_1} \\
V_{C_2}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & -1/L_1 & 0 \\
0 & 0 & 0 & 1/L_2 \\
1/C_1 & 0 & 0 & 0 \\
0 & -1/C_2 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_L_1 \\
I_L_2 \\
V_{C_1} \\
V_{C_2}
\end{bmatrix} +
\begin{bmatrix}
V_{DC}/L_1 \\
0 \\
0 \\
-1/L_1/L_1
\end{bmatrix}
\]

The state equations are averaged considering shoot through duty ratio D (i.e. when S1 is on). Averaging Two Matrices as \( x(t) = D(A_1 x + B_1 u) + (1 - D)(A_2 x + B_2 u) \)

It becomes

89
\[
\begin{bmatrix}
I_{L1} \\
I_{L2} \\
V_{C1} \\
V_{C2}
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & -(1-D)/L & -D/L \\
0 & 0 & -D/L & 1-D/L \\
1-D/C & 0 & 0 & 0 \\
D/C & -(1-D)/C & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_{L1} \\
I_{L2} \\
V_{C1} \\
V_{C2}
\end{bmatrix} + 
\begin{bmatrix}
V_{DC}/L \\
0 \\
-(1-D)IL_1/L \\
(1-D)IL_2/L
\end{bmatrix}
\tag{4.67}
\]

Again, above equations contain nonlinear terms. This can be linearised performing small signal analysis. Introducing perturbation in the first equation for the variables

\[
L \frac{d(I_{L1} + \hat{i}_{L1})}{dt} = -(D - \hat{d})(V_{c1} + \hat{V}_{c1}) - (D - \hat{d})(V_{c2} + \hat{V}_{c2}) + (V_{DC} + \hat{V}_{DC})
\tag{4.68}
\]

Where \( (1-D) = \hat{B} \)

Taking ac term and neglecting products of second order products.

\[
L \frac{d\hat{i}_{L1}}{dt} = -(D \hat{V}_{c1} + \hat{d}V_{c1}) - (D \hat{V}_{c2} - V_{c2}\hat{d}) + (V_{DC})
\tag{4.69}
\]

Similarly,

\[
L \frac{d(I_{L2} + \hat{i}_{L2})}{dt} = -(D - \hat{d})(V_{c1} + \hat{V}_{c1}) + (\hat{B} - \hat{d})(V_{c2} + \hat{V}_{c2})
\tag{4.70}
\]

Taking ac term and neglecting products of second order products.

\[
L \frac{d\hat{i}_{L2}}{dt} = -(D \hat{V}_{c1} + \hat{d}V_{c1}) - (\hat{B} \hat{V}_{c2} - V_{c2}\hat{d})
\tag{4.71}
\]

Similarly

\[
C \frac{d\hat{V}_{c1}}{dt} = -\hat{d}I_{L1} + \hat{B} \hat{i}_{L1} - \hat{d}I_{L} + \hat{d}I_{L}
\tag{4.72}
\]

\[
C \frac{d\hat{V}_{c2}}{dt} = \hat{d}I_{L1} + \hat{B} \hat{i}_{L1} - \hat{B} \hat{i}_{L2} + \hat{d}I_{L2} + \hat{B} \hat{i}_{L} + \hat{d}I_{L}
\tag{4.73}
\]

Taking Laplace transform of (4.138), (4.139), (4.140) and (4.141)

\[
sL\hat{i}_{L1}(s) = V_{c1}\hat{d}(s) - \hat{d}\hat{V}_{c1}(s) + \hat{V}_{DC}(s) + V_{c2}\hat{d}(s) - D\hat{V}_{c2}(s)
\tag{4.74}
\]

\[
sL\hat{i}_{L2}(s) = V_{c1}\hat{d}(s) - D\hat{V}_{c1}(s) + V_{c2}\hat{d}(s) - \hat{B}\hat{V}_{c2}(s)
\tag{4.75}
\]

\[
sC\hat{V}_{c1}(s) = -\hat{L}_{L1}\hat{d}(s) + \hat{B}\hat{i}_{L1}(s) - \hat{d}\hat{I}_{L}(s) + \hat{L}_{L}\hat{d}(s)
\tag{4.76}
\]

\[
sC\hat{V}_{c2}(s) = \hat{L}_{L1}\hat{d}(s) + D\hat{i}_{L1}(s) - \hat{B}\hat{i}_{L2}(s) + \hat{L}_{L2}\hat{d}(s) + \hat{B}\hat{I}_{L}(s) - \hat{I}_{L}\hat{d}(s)
\tag{4.77}
\]

In matrix form

\[
\begin{bmatrix}
\hat{i}_{L1}(s) \\
\hat{i}_{L2}(s) \\
\hat{V}_{c1}(s) \\
\hat{V}_{c2}(s)
\end{bmatrix} = 
\begin{bmatrix}
0 & \hat{B} & D & 0 \\
0 & sL & D & \hat{B} \\
-\hat{B} & 0 & sC & 0 \\
-D & \hat{B} & 0 & sC
\end{bmatrix}
\begin{bmatrix}
\hat{V}_{c1} + \hat{V}_{c2} \\
\hat{V}_{c1} + \hat{V}_{c2} \\
-\hat{L}_{L1} + \hat{L}_{L} \\
\hat{L}_{L2} - \hat{L}_{L}
\end{bmatrix} \cdot \hat{d}(s) + 
\begin{bmatrix}
\hat{V}_{DC}(s) \\
0 \\
-\hat{B}\hat{I}_{L}(s) \\
\hat{B}\hat{I}_{L}(s)
\end{bmatrix}
\tag{4.78}
\]

90
\[
\begin{bmatrix}
i_{L1}(s) \\
i_{L2}(s) \\
\bar{V}_{C1}(s) \\
\bar{V}_{C2}(s)
\end{bmatrix} =
\begin{bmatrix}
sL & 0 & D & D \\
0 & sL & D & D \\
-\tilde{D} & 0 & sC & 0 \\
-D & \tilde{D} & 0 & sC
\end{bmatrix}^{-1}
\begin{bmatrix}
V_{C1} + V_{C2} \\
V_{C1} + V_{C2} \\
-L_1 + I_1 \\
I_{L2} - I_1
\end{bmatrix},
\tilde{d}(s)
\]
\[+
\begin{bmatrix}
sL & 0 & D & D \\
0 & sL & D & D \\
-\tilde{D} & 0 & sC & 0 \\
-D & \tilde{D} & 0 & sC
\end{bmatrix}^{-1}
\begin{bmatrix}
\bar{V}_{DC}(s) \\
0 \\
-\tilde{D}I_{L1}(s) \\
\tilde{D}I_{L1}(s)
\end{bmatrix},
\tilde{d}(s)
\]
(4.79)

\[
\begin{bmatrix}
i_{L1}(s) \\
i_{L2}(s) \\
\bar{V}_{C1}(s) \\
\bar{V}_{C2}(s)
\end{bmatrix} =
\begin{bmatrix}
sC(s^2L^2C^2 - \tilde{D}^2) & s\bar{D}DC & -\tilde{D}(s^2L^2C^2 - \tilde{D}^2) & -s^2DLC \\
-2s\tilde{D}DC & sC(s^2L^2C^2 + \tilde{D}^2 + D^2) & -\tilde{D}(s^2L^2C^2 + \tilde{D}^2 + D^2) & s\bar{D}DLC \\
\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & -\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & s\bar{D}DLC & 0 \\
\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & -\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & 0 & s(\tilde{D}^2L^2C^2 + \tilde{D}^2)
\end{bmatrix}^{-1}
\begin{bmatrix}
V_{C1} + V_{C2} \\
V_{C1} + V_{C2} \\
-L_1 + I_1 \\
I_{L2} - I_1
\end{bmatrix},
\tilde{d}(s)
\]
(4.80)

Where
\[
\Delta = s^4L^2C^2 + s^2D^2L^2C^2 - \tilde{D}^4 + \tilde{D}^2D^2
\]

\[
\begin{bmatrix}
sC(s^2L^2C^2 + s\bar{D}DC - \tilde{D}^2)(V_{C1} + V_{C2}) + \tilde{D}(s^2L^2C^2 + \tilde{D}^2 + D^2)(L_1 - I_1) - s^2\bar{D}DLC(L_2 - I_2) \\
-2s\tilde{D}DC & sC(s^2L^2C^4 + \tilde{D}^2 + D^2) & -\tilde{D}(s^2L^2C^2 + \tilde{D}^2 + D^2)(L_1 - I_1) - \tilde{D}(s^2L^4C^2 + \tilde{D}^2 + D^2)(L_2 - I_2) \\
\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & -\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & s\bar{D}DLC \\
\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & -\tilde{D}(s^2L^2C^2 + \tilde{D}^2) & 0 & s(\tilde{D}^2L^2C^2 + \tilde{D}^2)
\end{bmatrix}^{-1}
\begin{bmatrix}
\bar{V}_{DC}(s) \\
0 \\
-\tilde{D}I_{L1}(s) \\
\tilde{D}I_{L1}(s)
\end{bmatrix},
\tilde{d}(s)
\]
(4.81)

So, from the above, the respective Transfer functions becomes

\[
\frac{i_{L1}(s)}{\tilde{d}(s)} = \frac{sC(s^2L^2C^2 + s\bar{D}DC - \tilde{D}^2)(V_{C1} + V_{C2}) + \tilde{D}(s^2L^2C^2 + \tilde{D}^2 + D^2)(L_1 - I_1) - s^2\bar{D}DLC(L_2 - I_2)}{s^4L^2C^2 + s^2D^2L^2C^2 - \tilde{D}^4 + \tilde{D}^2D^2}
\]
(4.82)

\[
\frac{i_{L2}(s)}{\tilde{d}(s)} = \frac{sC(s^2L^2C^2 + \tilde{D}^2 + D^2 - 2\tilde{D}D)(V_{C1} + V_{C2}) + \tilde{D}(s^2L^2C^2 + \tilde{D}^2 + D^2)(L_1 - I_1) - \tilde{D}(s^2L^4C^2 + \tilde{D}^2 + D^2)(L_2 - I_2)}{s^4L^2C^2 + s^2D^2L^2C^2 - \tilde{D}^4 + \tilde{D}^2D^2}
\]
(4.83)

\[
\frac{\bar{V}_{C1}(s)}{\tilde{d}(s)} = \frac{\tilde{D}(s^2L^2C^2 - \tilde{D}^2)(V_{C1} + V_{C2}) - sL(s^2L^2C^2 - \tilde{D}^2 + D^2)(L_1 - I_1) + s\bar{D}DLC(L_2 - I_2)}{s^4L^2C^2 + s^2D^2L^2C^2 - \tilde{D}^4 + \tilde{D}^2D^2}
\]
(4.84)

\[
\frac{\bar{V}_{C2}(s)}{\tilde{d}(s)} = \frac{(D - \tilde{D})(s^2L^2C^2 + \tilde{D}^2)(V_{C1} + V_{C2}) + sL(s^2L^2C^2 + \tilde{D}^2)(L_1 - I_1)}{s^4L^2C^2 + s^2D^2L^2C^2 - \tilde{D}^4 + \tilde{D}^2D^2}
\]
(4.85)
4.5. State Space Model of Trans Quasi z-source Inverter (Trans QZSI) and Trans z-source Inverter (Trans ZSI) with ideal network component

Figure 4.7 Equivalent circuit model of Trans Quasi Z-Source inverter (Trans QZSI)

Figure 4.8 Equivalent circuit of voltage fed Trans Qzsi (a) non shoot through state (b) Shoot through state

Figure 4.7 represents the equivalent circuit model of the Trans quasi z-source inverter (trans QZSI) [56] which uses a coupled inductor in place of two discrete inductors. The non shoot-through state and shoot-through state equivalent models are also shown in Figure 4.8. To analyze and derive the state space model from the diagram Coupled Inductors are modeled based on a literature by G. Zhu et. al [77].

Figure 4.7 is then transformed into a new model shown in Figure 4.9

Here $L_n = (1 - k^2)L$, $L$ is the self inductance value of inductances $L1$ and $L2$ and $k$ is the coefficient of coupling where $k = \frac{M}{\sqrt{L_1L_2}} \leq 1$. 

92
Also,
\[ V_{L1} = L_1 \frac{di_{L1}}{dt} - M \frac{di_{L2}}{dt}, \quad V_{L2} = L_2 \frac{di_{L2}}{dt} - M \frac{di_{L1}}{dt} \]  \hfill (4.86)

Now based on the above model equivalent circuits are drawn for non-shoot through state in Figure 4.10 (a) and for shoot through state in Figure 4.10(b) Now for non-shoot through state applying KVL for two loops

Applying KVL

\[ L_k \frac{di_{L1}}{dt} + kV_{L2} - V_{DC} + V_{C1} = 0 \]  \hfill (4.87)
\[ L_k \frac{di_{L2}}{dt} + n(L \frac{di_{L2}}{dt} - M \frac{di_{L1}}{dt}) - V_{DC} + V_{C1} = 0 \]

Or,
\[ L_k \frac{di_{L2}}{dt} - kV_{L1} - V_{C1} = 0 \]  \hfill (4.88)

Rearranging (4.155) and (4.156)
\[ (L_k - kM) \frac{di_{L1}}{dt} + kL \frac{di_{L2}}{dt} = V_{DC} - V_{C1} \]  \hfill (4.89)
\[ kL \frac{di_{L1}}{dt} - (kM + L_k) \frac{di_{L2}}{dt} = -V_{C1} \]  \hfill (4.90)

Solution of above two equations gives
\[ \frac{di_{L1}}{dt} = - \frac{(kM + 2L_k)V_{C1} - (kM + L_k)V_{DC}}{(L_k^2 - k^2M^2) + k^2L^2} \]  \hfill (4.91)
\[ \frac{di_{L2}}{dt} = \frac{(-kM)V_{C1} + kL V_{DC}}{(L_k^2 - k^2M^2) + k^2L^2} \]  \hfill (4.92)

Applying KCL,
\[ C_1 \frac{dV_{C1}}{dt} + iL_2 - iL_1 = 0 \]  \hfill (4.93)

The State equations in matrix form is written as (using state variables \( i_{L1}, i_{L2}, V_{C1} \))

\[
\begin{bmatrix}
  i_{L1} \\
  i_{L2} \\
  V_{C1}
\end{bmatrix} =
\begin{bmatrix}
  0 & 0 & -(kM + 2L_k) \\
  0 & 0 & (-nM) \\
  1/C_1 & -1/C_1 & 0
\end{bmatrix}
\begin{bmatrix}
  i_{L1} \\
  i_{L2} \\
  V_{C1}
\end{bmatrix} +
\begin{bmatrix}
  (kM + L_k)V_{DC} \\
  (L_k^2 - k^2M^2) + k^2L^2 \\
  (L_k^2 - k^2M^2) + k^2L^2
\end{bmatrix}
\]  \hfill (4.94)

Similarly for shoot through period i.e. when \( S = \text{on} \), from the circuit model in Figure 4.10 (b) the equations are written as
\[ i_{L1} = 0 \]
\[ L_k \frac{di_{L2}}{dt} + kV_{L2} - V_{DC} + V_{C1} = 0 \]  \hfill (4.95)

93
Or \[ (L_k - kM) \frac{dL_1}{dt} - V_{DC} + V_{C1} = 0, \quad C_1 \frac{dV_{C1}}{dt} - iL_1 = 0 \] (4.96)

Figure 4.10 Modified equivalent circuit of Tran’s quasi z-source inverter (a) non shoot through state (b) shoot through state

Based on the above equations, the state matrix becomes

\[
\begin{bmatrix}
iL_1 \\ il_2 \\ V_{C1}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & \frac{1}{(L_k - kM)} \\
0 & 0 & 0 \\
1/C1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
iL_1 \\ il_2 \\ V_{C1}
\end{bmatrix} + \begin{bmatrix}
V_{DC} \\
0 \\
0
\end{bmatrix}
\] (4.97)

Now, the state equations are averaged considering shoot through duty ratio \( D \) as \( x(t) = D (A_1 x + B_1 u) + (1 - D) (A_2 x + B_2 u) \)

\[
\begin{bmatrix}
iL_1 \\ il_2 \\ V_{C1}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & -\frac{D (kM + 2L_k)}{(L_k^2 - k^2M^2) + k^2l^2} + \frac{1 - D}{(L_k - kM)} \\
0 & 0 & \frac{D(-kM)}{(L_k^2 - k^2M^2) + k^2l^2} \\
1/C1 & -D/C1 & 0
\end{bmatrix}
\begin{bmatrix}
iL_1 \\ il_2 \\ V_{C1}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1 - D
\end{bmatrix}
\] (4.98)

Above equations contain nonlinear terms. This can be linearised by performing small signal analysis. Let’s assume,

\[
-\frac{(kM + 2L_k)}{(L_k^2 - k^2M^2) + k^2l^2} = W, \quad \frac{1}{(L_k - kM)} = X, \quad \frac{(-kM)}{(L_k^2 - k^2M^2) + k^2l^2} = Y, \quad \frac{(kM + L_k)}{(L_k^2 - k^2M^2) + k^2l^2} = Z_1,
\]

\[
\frac{L_k}{(L_k^2 - k^2M^2) + k^2l^2} = Z_2, \quad (1 - D) = \hat{D}
\]

So, matrix (4.98) becomes,

\[
\begin{bmatrix}
iL_1 \\ il_2 \\ V_{C1}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & WD - \hat{D}X \\
0 & 0 & DY \\
1/C1 & -D/C1 & 0
\end{bmatrix}
\begin{bmatrix}
iL_1 \\ il_2 \\ V_{C1}
\end{bmatrix} + \begin{bmatrix}
V_{DC} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
DZ_1 + \hat{D}X \\
DZ_2 \\
0
\end{bmatrix}
\] (4.99)
Introducing perturbation for the variables

\[
L \frac{d(I_{L1} + i_{t11})}{dt} = W(D + \dot{d})(V_{c1} + \dot{V}_{c1}) - X(\dot{d} - \dot{d})(V_{c1} + \dot{V}_{c1}) + Z1(D + \dot{d})(V_{dc} + \dot{V}_{dc}) + X(\dot{d}V_{dc} + \dot{V}_{dc})
\]  
(4.100)

Taking ac term and neglecting second order functions, the equation (4.117) comes as

\[
L \frac{d^2i_{t11}}{dt^2} = W(DV_{c1} + \dot{d}V_{c1}) + Z2(D + \dot{d})(V_{dc} + \dot{V}_{dc})
\]  
(4.101)

Similarly, the other equations are

\[
L \frac{d(I_{L2} + i_{t12})}{dt} = Y(D + \dot{d})(V_{c1} + \dot{V}_{c1}) + Z2(D + \dot{d})(V_{dc} - \dot{V}_{dc})
\]  
(4.102)

\[
\frac{d\dot{V}_{c1}}{dt} = \frac{1}{C1} i_{t1} - \frac{D}{C1} i_{t2} - \frac{\dot{d}}{C1} i_{t12}
\]  
(4.103)

Taking Laplace transforms of (4.118)(4.120) and (4.121)

\[
sLI_{l1} = WDV_{c1} + WV_{c1} \dot{d} + \dot{X}D\dot{V}_{c1} + \dot{X}V_{c1}\dot{d} + Z1\dot{V}_{dc} + Z1V_{dc}\dot{d}
\]  
(4.105)

\[
sLI_{l2} = YDV_{c1} + YV_{c1}\dot{d} + Z2DV_{dc} + Z2V_{dc}\dot{d}
\]  
(4.106)

Rearranging and writing in matrix form

\[
\begin{bmatrix}
i_{t11}(s) \\
i_{t12}(s) \\
\dot{V}_{c1}(s)
\end{bmatrix} = \begin{bmatrix}
0 & -DL & 0 \\
-1 & D & 0 \\
1 & 0 & C1
\end{bmatrix} \begin{bmatrix}
V_{c1}(W - X) + V_{dc}(Z1 - X) \\
\dot{V}_{c1} - Z2V_{dc} \\
0
\end{bmatrix} . \dot{d}(s) + \begin{bmatrix}
\dot{d}(s) \\
\dot{d}(s)
\end{bmatrix} . \begin{bmatrix}
\dot{V}_{dc}(s) \\
V_{dc}(s)
\end{bmatrix}
\]  
(4.108)

Or,

\[
\begin{bmatrix}
i_{t11}(s) \\
i_{t12}(s) \\
\dot{V}_{c1}(s)
\end{bmatrix} = \begin{bmatrix}
0 & -DL & 0 \\
-1 & D & 0 \\
1 & 0 & C1
\end{bmatrix} \begin{bmatrix}
V_{c1}(W - X) + V_{dc}(Z1 - X) \\
\dot{V}_{c1} - Z2V_{dc} \\
0
\end{bmatrix} . \dot{d}(s)
\]  
(4.109)
Putting the constants the \( \frac{P_c_1(s)}{d(s)} \) becomes

\[
\frac{P_c_1(s)}{d(s)} = \frac{V_c_1(W-X)+V_{DC}(Z1-X)+sL(YD+Z2)L_1}{s^2(LC_1^2+YD^2-WD+X) }
\]

Where,

\[
\frac{(kM+2L_k)}{(L_k^2-k^2M^2)+k^2L^2} = W, \quad \frac{1}{(L_k-kM)} = X, \quad \frac{(-kM)}{(L_k^2-k^2M^2)+k^2L^2} = Y, \quad \frac{(kM+L_k)}{(L_k^2-k^2M^2)+k^2L^2} = Z1,
\]

\[
\frac{L_k}{(L_k^2-k^2M^2)+k^2L^2} = Z2, \quad (1-D) = \dot{D}
\]

4.

Where

\[
\begin{align*}
\frac{P_c_1(s)}{d(s)} & = V_c_1(W-X)+V_{DC}(Z1-X)+sL(YD+Z2)L_1 \\
\frac{V_c_1(W-X)+V_{DC}(Z1-X)+Z2L_1}{sL} & = sL(YD+Z2)L_1
\end{align*}
\]

Transfer functions derived from above matrices as

\[
\begin{align*}
\frac{V_c_1(W-X)+V_{DC}(Z1-X)+Z2L_1}{sL} & = sL(YD+Z2)L_1
\end{align*}
\]

Where \( A=(S^2L_{C1}^2+YD^2-WD+XD) \)

CHAPTER 4: MODELLING AND DESIGN OF Z-SOURCE CONVERTERS
\[
P_{c1}(s) = \frac{V_{c1} \left( -\frac{D(kM + 2L)}{(l_u^2 - k^2M^2) + L^2} \right) + V_{sc} \left( \frac{D(kM + L(1 + D))}{(l_u^2 - k^2M^2) + L^2} - \frac{1}{(l_u - kM)} \right)}{(s^2LC_1 + \frac{D(2L - kMD)}{(l_u^2 - k^2M^2) + L^2} + \frac{D}{(l_u - kM)^2})}
\]

(4.11)

Development of transfer functions lead to the complete system block shown in Figure 4.11. The complete loop gain of the system becomes \( T(s) = G_1(s) \cdot G_2(s) \cdot G_3(s) \cdot H(s) \). This directs to design possible controllers for different ZSI based converter systems.

![Possible block diagram of the proposed system](image)

4.6. Design of network parameter of Z-Source inverter

4.6.1. Capacitor Design

The generalized formula we have,

\[
\Delta V_{c1} = i_{c1} \cdot \frac{\Delta t}{C_1}
\]

(4.116)

If carrier frequency is \( f_s \) then the switching frequency for capacitor=2\( f_s \) for Simple boost control method [70],

\[
\Delta t = \frac{D}{2f_s}
\]

(4.117)

In the shoot through state for the Z-Source inverter

\[
i_{c1} = i_{l2}
\]

(4.118)

From (4.116),(4.117) and (4.118) we have

\[
C_1 = \frac{i_{l2}D}{2f_s \Delta V_{c1}}
\]

(4.119)

If change in the capacitor voltage is limited by
\[ \Delta V_{C1,C2} \leq aV_{C1,C2} \quad \text{Where } a \text{ is a constant} \] (4.120)

From (4.119) and (4.120) we have

\[ C_1 \geq \frac{i_{L2}D}{2f_s aV_{C1}} \] (4.121)

In the same way the capacitance \( C_2 \) will be

\[ C_2 \geq \frac{i_{L1}D}{2f_s aV_{C2}} \] (4.122)

The parameters are chosen from the APPENDIX-A

Maximum output power of the wind power source and rectifier = 5176 watt

Average DC input voltage to \( Z \) network = \( V_{dc} = 280 \) Volt.

Mean DC input current = \( i_{in} = i_{L2} = i_{L1} = 18 \) amp.

Let 30\% ripple in the inductor current is allowed .Therefore \( \Delta I = 10.8 \) amp

Peak of the desired output peak voltage \( \sqrt{V_{pH}} = 420 \times \sqrt{(2/3)} = 343 \) volt

The peak output phase voltage \( \sqrt{V_{pH}} = M_2 V_2 = MB \frac{V_{dc}}{2} \) (4.123)

The modulation index of the inverter=0.9

Therefore boost factor required to get the desired output voltage is \( B = 2.72 \)

Now we have

\[ B = \frac{T}{T_1 - T_0} = \frac{1}{1 - 2D} \] (4.124)

Therefore required shoot through duty ratio=0.316

Voltage across the capacitors

\[ V_{c1} = V_{c2} = V_c = \frac{(1 - D)}{(1 - 2D)} \cdot V_{dc} \]

\[ = 520.43 \text{ V} \] (4.125)

Let 6\% of voltage variation from peak to peak across the capacitor voltage is allowed then

\[ \Delta V_c = 7.805 \text{ V} \]

Substituting the values in equation (4.121) we get \( C_1 \geq \frac{i_{L2}D}{2f_s aV_{C1}} = 9.1 \mu F \)

Similarly the capacitor \( C_2 \) from (4.122) should be more than or equal to \( 9.1 \mu F \).

**4.6.2. Inductor Design**

The generalized formula we have

\[ \Delta i_{L1} = \frac{V_{L1}}{L_1} \Delta t \] (4.126)

In the shoot through state for the Z-Source inverter
\[ V_{L1} = V_{L2} \]  

(4.127)

From (4.117), (4.126) and (4.127) we have

\[ L_1 = \frac{V_{L2}D}{2f_s \Delta t_{L1}} \]  

(4.128)

If the change in inductor current is limited by the value

\[ \Delta i_{L1,L2} \leq bi_{L1,L2} \] where \( b \) is a constant  

(4.129)

From (4.128) and (4.129) we have

\[ L_1 \geq \frac{V_{L2}D}{2f_s b i_{L1}} \]  

(4.130)

In the same way the inductance \( L_2 \) is calculated as

\[ L_2 \geq \frac{V_{L1}D}{2f_s b i_{L2}} \]  

(4.131)

From equation (4.130) and (4.131) after substituting the chosen value of the parameters it is found that the Z-network inductor value should be more than 0.76 mH.

### 4.6.3. Device selection

Maximum voltage across the switches and the diode

\[ V_{PN} = B \cdot V_{dc} = 2.72 \cdot 280 = 761.6 \text{ V} \]  

(4.132)

### 4.7. Design of network parameter of Quasi Z-Source inverter

#### 4.7.1. Capacitor Design

\[ \Delta V_{C1} = i_{C1} \cdot \frac{\Delta t}{C_1} \]  

(4.133)

If carrier frequency is \( f_s \) then the switching frequency for capacitor=\( 2f_s \) for Simple Boost Control method.

\[ \Delta t = \frac{D}{2f_s} \]  

(4.134)

In the shoot through state for the quasi Z-Source inverter

\[ i_{C1} = i_{L2} \]  

(4.135)

From equation (4.133), (4.134) and (4.135) we have

\[ C_1 = \frac{i_{L2}D}{2f_s \Delta V_{C1}} \]  

(4.136)

If change in the capacitor voltage is limited by
\[ \Delta V_{C1,C2} \leq aV_{C1,C2} \text{ where } a \text{ is a constant} \] (4.137)

From (4.136) and (4.137) we have

\[ C_1 \geq \frac{i_{L2}D}{2f_aV_{C1}} \] (4.138)

For QZSI we have

\[ V_{C1} = \frac{T_1}{T_1 - T_0}V_{dc} = \frac{1-D}{1-2D}V_{dc}, V_{C2} = \frac{T_0}{T_1 - T_0}V_{dc} = \frac{D}{1-2D}V_{dc} \] (4.139)

After substituting the value of shoot through duty ratio \( D = 0.316 \), we get \( V_{C1} = 520.43 \text{ V} \) and \( V_{C2} = 240.43 \text{ V} \)

Substituting the values of the inductor current \( i_{L2} = 18 \text{ amp} \). Let let 60% of the peak to peak ripple in inductor current is allowed. \( a = 0.6 \)

Therefore from equation (4.138) we get \( C_1 \geq \frac{18 \times 0.316}{2 \times 10^4 \times 0.6 \times 520.43} = 9.1 \mu F \)

In the same way the capacitance \( C_2 \) will be

\[ C_2 \geq \frac{i_{L1}D}{2f_aV_{C2}} = 19.75 \mu F \]

### 4.7.2. Inductor Design

The generalized formula we have

\[ \Delta i_{L1} = \frac{V_{L1}}{L_1} \Delta t \] (4.140)

In the shoot through state for the quasi Z-Source inverter

\[ V_{L1} = V_{C1} + V_{C2} \] (4.141)

\[ V_{L2} = V_{C1} \] (4.142)

From (4.134),(4.141)and (4.142) we have

\[ L_1 = \frac{(V_{C1} + V_{C2})D}{2f_a\Delta i_{L1}} \] (4.143)

If the change in inductor current is limited by the value

\[ \Delta i_{L1,L2} \leq b i_{L1,L2} \text{ where } b \text{ is a constant} \] (4.144)

From (4.143) and (4.144) we have

\[ L_1 \geq \frac{(V_{C1} + V_{C2})D}{2f_b b i_{L1}} = 1.11 mH \] (4.145)
In the same way the inductance $L_2$ is calculated by
\[ L_2 \geq \frac{V_{C1}D}{2f_c b t_{l2}} = 0.76\text{mH} \quad (4.146) \]

### 4.8. Design of network components of Trans ZSI

#### 4.8.1. Capacitor Design

- Capacitance $V_{L1} = V_c$  
  \[ (4.147) \]

Generalised formula
\[ \Delta V_{C1} = i_{C1} \cdot \frac{\Delta t}{C_1} \quad (4.148) \]

If carrier frequency is $f_c$ then the switching frequency for capacitor=$2f_c$ for Simple Boost Control method.
\[ \Delta t = \frac{D}{2f_c} \quad (4.149) \]

In the shoot through state for the trans quasi Z-Source inverter
\[ i_{C1} = i_{L2} - i_{L1} \quad (4.150) \]

From equations (4.148),(4.149) (4.150) we have
\[ C_1 = \frac{(i_{L2} - i_{L1})D}{2f_c \Delta V_{C1}} \quad (4.151) \]

If change in the capacitor voltage is limited by 6% peak to peak
\[ \Delta V_{C1,C2} \leq aV_{C1,C2} \text{ where } a = 0.06 \quad (4.152) \]

From (4.151) and (4.152) we have
\[ C_1 \geq \frac{(i_{L2} - i_{L1})D}{2f_c aV_{C1}} \quad (4.153) \]

Capacitor voltage
\[ V_{C1} = \frac{nDV_{dc}}{1-(1+n)d} \text{ where } n = \frac{n_2}{n_1} \geq 1 \quad (4.154) \]

Let the no of turns ratio of the coupled inductor be $n=1$,

Shoot through duty ratio is given by
\[ D = \frac{nM - M + 1}{Mn + M - 1} = 0.316 \]

Then $V_{C1} = 240.43 \text{V}$

From (4.154)
\[ C_1 \geq \frac{(i_{L2} - i_{L1})D}{2f_s a V_{c1}} = \frac{3 \times 0.316}{2 \times 10000 \times 0.06 \times 240.43} = 0.033 \, \text{mF} \] (4.155)

### 4.8.2. Inductor Design

The generalised formula we have
\[ \Delta i_{L1} = \frac{V_{L1}}{L1} \Delta t \] (4.156)

In the shoot through state for the trans quasi Z-Source inverter
\[ V_{L1} = V_{dc} + V_{C1} \] (4.157)

From (4.149),(4.156) and (4.157) we have
\[ L_1 = \frac{(V_{dc} + V_{C1})D}{2f_s \Delta i_{L1}} \] (4.158)

If the change in inductor current is limited by the value
\[ \Delta i_{L1,L2} \leq bi_{L1,L2} \] Where b is a constant
(4.159)

From (4.158) and (4.159) we have
\[ L_1 \geq \frac{(V_{dc} + V_{C1})D}{2f_s bi_{L1}} \] (4.160)

In the shoot through state for the trans quasi Z-Source inverter
\[ V_{L2} = V_{L1} \] (4.161)

So from (3.42),(3.44) and (3.45) we have
\[ L_2 \geq \frac{(V_{dc} + V_{C1})D}{2f_s b i_{L2}} \] (4.162)

From equation (4.160) and (4.162) we get
\[ L_1 = L_2 \geq 0.76 \, \text{mH} \]

### 4.9. Design of Network components of Trans-quasi ZSI

#### 4.9.1. Capacitor Design

\[ V_{L1} = \frac{1}{n} V_{c1} \] (4.163)

\[ \Delta V_{C1} = i_{C1} * \frac{\Delta t}{C1} \] (4.164)

If carrier frequency is \( f_s \) then the switching frequency for capacitor=2\( f_s \) for Simple Boost Control method.
\[ \Delta t = \frac{D}{2f_s} \quad (4.165) \]

In the shoot through state for the trans quasi Z-Source inverter

\[ i_{c1} = i_{L2} - i_{L1} \quad (4.166) \]

From equation (4.164), (4.165) and (4.166) we have

\[ C_1 = \frac{(i_{L2} - i_{L1})D}{2f_s \Delta V_{c1}} \quad (4.167) \]

Capacitor voltage

\[ V_{c1} = \frac{nDV_{dc}}{1 - (1 + n)D} \quad (4.168) \]

Considering \( n = 2 \)

\[ V_{c1} = \frac{nDV_{dc}}{1 - (1 + n)D} = 3403 \text{ V} \quad (4.169) \]

If change in the capacitor voltage is limited by

\[ \Delta V_{c1,c2} \leq aV_{c1,c2} \text{ where } a = 0.06 \quad (4.170) \]

From (4.169) and (4.170) we got

\[ C_1 \geq \frac{(i_{L2} - i_{L1})D}{2f_s a V_{c1}} = 0.023 \text{ nF} \quad (4.171) \]

**4.9.2. Inductor Design**

The generalised formula we have

\[ \Delta i_{L1} = \frac{V_{L1}}{L_1} \Delta t \quad (4.172) \]

In the shoot through state for the trans quasi Z-Source inverter

\[ V_{L1} = V_{dc} + V_{c1} \quad (4.173) \]

From (4.165), (4.172) and (4.173) we have

\[ L_1 = \frac{(V_{dc} + V_{c1})D}{2f_s \Delta i_{L1}} \quad (4.174) \]

If the change in inductor current is limited by the value

\[ \Delta i_{L1,L2} \leq b i_{L1,L2} \text{ where } b = 0.6 \quad (4.175) \]

From (4.174) and (4.175) we have

\[ L_1 \geq \frac{(V_{dc} + V_{c1})D}{2f_s b i_{L1}} = 5.3 \text{ mH} \quad (4.176) \]

In the shoot through state for the trans quasi Z-Source inverter

\[ V_{L2} = nV_{L1} \quad (4.177) \]
So from (4.175),(4.176) and (4.177) we have

\[ L_2 \geq \frac{n(V_{dc} + V_{Cl})D}{2f_b b_{L2}} = 0.01H \]  \hspace{1cm} (4.178)

4.9.3. Design of controller

The transfer function obtained by applying state space averaging technique to the Z-Source inverter.

\[
\frac{\hat{V}_c(s)}{d(s)} = \frac{(2V_c + V_{DC})(1 - 2D) - sL(2I_L - I_{DC})}{S^2LC + (2D - 1)^2}
\]  \hspace{1cm} (4.179)

The parameters taken from the APPENDIX-A of wind power conversion system using z-source Inverter.

- \( V_{dc} = 280 \) Volts
- \( V_C = 520 \) Volts
- \( I_{dc} = I_L = 18 \) Amp
- \( L_1 = L_2 = L = Z \) network inductor = 1mH
- \( C_1 = C_2 = C = Z \) network Capacitor = 10\( \mu F \)
- \( R=\)Equivalent series resistance of the capacitor=0.1 Ohm
- \( D=\)Shoot through duty ratio required to get 440 volt line output=0.316
- \( r = Dc \) series resistance of the inductor = 1 Ohm

Transfer function of the open loop system

\[
\frac{V_c(s)}{D(s)} = \frac{465.2 - 0.018s}{10^{-8}s^2 + 1.1 * 10^{-5}s + 0.135}
\]  \hspace{1cm} (4.180)

The transfer function is a second order system. The denominator order is one more than the numerator so the system is a proper system.

Design criteria

- The closed loop system should be stable.
- Rise time less than 0.2 sec
- Settling time less than 0.3 sec
- Overshoot less than 3 %
- Steady state error less than 2 %

Let’s first begin by examining the step response of the open loop system. Step response of the open loop system is found in Figure 4.12
Figure 4.12 Step response of the open loop system

Table 4.1 Step input performance of open loop system

<table>
<thead>
<tr>
<th>Peak overshoot %</th>
<th>Settling time</th>
<th>Rise time</th>
<th>Steady state error</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.2</td>
<td>0.00708</td>
<td>0.00031</td>
<td>0</td>
</tr>
</tbody>
</table>

So the open loop system is found to be a stable system.

Transfer function of the unity feedback closed loop system with only proportional controller with proportional gain $K_p = 1$ is

$$
\frac{-5.22 \times 10^{-6}s + 0.1349}{10^{(-8)}s^2 + 5.78 \times 10^{-6} + 0.2699}
$$

Examining the poles of this transfer function using the pole command as shown below, it can be seen that this closed-loop system is indeed stable since all of the poles have negative real part as shown below.

1.0e+003 * ( -0.2890 + 5.1872i), 1.0e+003 * -0.2890 - 5.1872i

Stability of this closed-loop system can also be determined using the frequency response of the open-loop system. The margin command generates the Bode plot for the given transfer function for the gain margin and phase margin of the system when the loop is closed as demonstrated below. as in Figure 4.13.
Examination of the above demonstrates that the closed-loop system is indeed stable since the phase margin and gain margin are both positive. Specifically, the phase margin equals 163 degrees and the gain margin is 0.886. It is observed that this closed-loop system is stable, but it should meet our design requirements. Therefore Step response analysis of the closed loop system has been observed in Figure 4.14.
Table 4.2 Performance of the unity feedback closed loop system with proportional gain=1

<table>
<thead>
<tr>
<th>Peak overshoot %</th>
<th>Settling time(Sec)</th>
<th>Rise time (Sec)</th>
<th>Steady state error</th>
</tr>
</thead>
<tbody>
<tr>
<td>85.1</td>
<td>0.0134</td>
<td>0.0002</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Examination of the above demonstrates that the settling time requirement of 0.3 seconds is close to our design requirement. The overshoot shown above will likely become a problem. Therefore, the overshoot must be reduced in conjunction with making the system state error lesser. We can accomplish these goals by adding a compensator to reshape the Bode plot of the open-loop system. The Bode plot of the open-loop system indicates behaviour of the closed-loop system. More specifically, the gain crossover frequency is directly related to the closed-loop system's speed of response, and the phase margin is inversely related to the closed-loop system's overshoot. Therefore, we need to add a compensator that will decrease the overshoot of the system.

Table 4.3 Closed loop system response with PI controller

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>$K_i$</th>
<th>Peak overshoot %</th>
<th>Settling time(Sec)</th>
<th>Rise time (Sec)</th>
<th>Steady state error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0</td>
<td>85.1</td>
<td>0.0134</td>
<td>0.0002</td>
<td>0.5</td>
</tr>
<tr>
<td>2.0</td>
<td>0</td>
<td>101.0</td>
<td>0.139</td>
<td>0.000155</td>
<td>0.333</td>
</tr>
<tr>
<td>0.8</td>
<td>0</td>
<td>81.2</td>
<td>0.011</td>
<td>0.000214</td>
<td>0.556</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>75.0</td>
<td>0.00925</td>
<td>0.00239</td>
<td>0.667</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>65.0</td>
<td>0.00751</td>
<td>0.0029</td>
<td>0.0909</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0</td>
<td>4.2</td>
<td>2.42</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>2</td>
<td>0</td>
<td>2.1</td>
<td>1.21</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>0</td>
<td>1.4</td>
<td>0.804</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>15</td>
<td>0</td>
<td>0.28</td>
<td>0.159</td>
<td>0</td>
</tr>
</tbody>
</table>

After several trial and error runs, the gains $K_p = 0.1, K_i = 15$, provided the desired response.

The closed loop transfer function is

Examining the poles of this transfer function using the pole command as shown below, it can be seen that this closed-loop system is indeed stable since all of the poles have negative real part.
1.0e+003 * (-0.5171 + 3.8157i), 1.0e+003 * (-0.5171 - 3.8157i) and -0.0136

The margin command generates the Bode plot for the given transfer function with annotations for the gain margin and phase margin of the system when the loop is closed as demonstrated below.

Figure 4.15 Bode plot of the closed loop system

Figure 4.16 Step response of the closed loop system

Now, we have obtained a closed-loop system with no overshoot, fast rise time, and no steady-state error. Transfer function of the compensator

\[
\frac{0.1s + 15}{s}
\]

(4.182)