CHAPTER 4: DIFFEOMORPHISM-BASED SYMMETRIC AND INVERSE-CONSISTENT NON-RIGID DEM REGISTRATION

4.1 INTRODUCTION

DEM registration not only aims at finding a suitable and optimal spatial transformation to enable the correspondence between reference and candidate DEM but also make it suitable for usage even when the two input DEMs have different resolutions too, as detailed in Chapter 1. Also, registration is an ill-posed problem, implying that any minor change in the input data set(s) can result in drastic change in the output result(s). One common problem that is usually a cause of discern in this domain is registration direction. The result of registration may not be same when registering candidate DEM with reference DEM as compared to registering reference DEM with candidate DEM, i.e., with respect to direction, registration is non-transitive in nature. The existing registration methods usually suffer from the problem of inconsistent registration and result in different output for forward and reverse registration direction and this asymmetry lead not only to inconsistency in registration, but also reduce the quality of registration, and hence there is an immediate need to develop a scheme that eradicates the asymmetry.

Further, not only inherent errors, like random error, as discussed in Chapter 1, exist within DEMs, but data holes i.e., regions having no data, may also be present. These pose a great challenge during registration due to the fact that the mapping functions may not have any initial data to perform correspondence to. Another problem is that of handling registration of multiple candidate DEMs.

In the literature, some of the existing works include using least-trimmed-squares estimator with Least Z-Difference (LZD) algorithm as shown by Akca and Gruen in [134], and [147].

The contents of this chapter include matter from my following research paper:

Authors Liao et al., have also used LZD as described in [155], Zhang and Cen in [174]. Generalized Least Square image matching based on the generalized Gauss-Markov model has been used for automatic co-registration of 3D point clouds was shown by Feng et al., in [144]. Their function also included minimization of sum of squares of the Euclidean distance. This distance measure is a common metric. A similar method was used in [16]. As discussed in Chapter 1 too, a very customary method includes the use of feature-based image registration – GCPs – was also used in [20] and [156]. Iterative Closest Point based matching for automatic range image registration and matching is another commonly used method and is discussed in [164]. This ICP method was shown to be useful for automated cross-sensor registration by performing ortho-rectification and geo-positioning of LIDAR DEMs. Aguilar et al., used shaded-relief matching for geo-referencing of DEMs [13]. Also, Rigid-body transformation for registration has been shown in [170], and [152]. Quality related issues were considered and elaborated in [152]. Certain parameters for evaluation that were used include average registration error, standard deviation, no. of established correspondence points, no. of iterations, and total time required. Many of these above mentioned methods have used apriori knowledge-based techniques [195].

As depicted in earlier chapters too, a critical criterion for registration is the volume of area covered under overlapping regions between the reference and the candidate images. It has been found that larger the overlapped area’s common features, the better is the performance of registration. This criterion was given priority for considering the task of geo-positioning or geo-registration and ortho-rectification for LIDAR DEMs in [162].

However, these recent techniques too are plagued with the above mentioned problems. None of the methods were able to successfully thwart the problems of asymmetry or inverse consistency, or allow multiple candidate registration, or data holes.

In this chapter we have proposed an approach to enrich DEM registration, named, Diffeomorphism-based Symmetric and Inverse-Consistent Non-rigid (DSICNR) DEM registration algorithm, to not only reduce the effects of above stated problems but also introduce symmetry and inverse-consistency in the transformation direction in the registration process. Also, the proposed approach has been shown to be robust against data holes.

Further, our proposed approach has also been tested for various types of DEMs – i.e., multi-resolution DEM sets, multi-temporal DEMs, multi-modal DEMs, and multi-view DEMs. This
approach, initially, identifies the overlapped regions of the DEM scalars so as to avoid larger computations on the regions that are not overlapping regions, thus, reducing the aggregate resource usage by the proposed DSICNR approach. The features and their ground truth deviations identified in these overlapped regions of the candidate and reference DEMs form the basis of the correspondence function. Hence the registration affects the non-overlapping regions by enforcing reorientation within these too.

Not only has the work been tested for pair-wise mapping, but also, the implicit group-wise mappings have also been established. For establishing this group-wise mapping, the various overlapping areas have been associated with each other by the in-between DEM data correspondences that are formed within the DEMs or their various regions.

Preliminaries and terms related to registration are discussed in Section 4.2. Section 4.3 contains the proposed DSICNR based DEM registration’s description, and Section 4.4 shows the related experiments and their inferences. Section 4.5 concludes the chapter.

4.2 PRELIMINARIES

In this section, relevant terms used for performing the proposed DSICNR for DEM registration are discussed including - symmetry, free-form deformation and non-rigid deformation, diffeomorphism, regularization and optimization.

The prime goal of DEM registration is to have a suitable and corresponding match between the two given input DEMs. These input DEMs are usually of the same location or should have at least some viable overlapping regions, may be from different sensors or of different resolutions, or may have been taken at different times, or may have different spatial orientation or may have been sampled from different viewpoints. As stated earlier, the overlapping regions of both the input DEMs form the basis of the registration correspondence as well as contain all the critical features or descriptors for solving the registration problem. Usually, the candidate DEM is manipulated with respect to the orientations of the reference DEM, so that overlapping areas are exactly or nearly superimposed over one another. The solution to registration is usually best approximated by the candidate DEM being iteratively transformed, relative to reference DEM,
until both DEMs’s overlapped regions have the best possible match against their direction and gradient magnitude.

In other words, DEM registration, enable the final estimation of the orientation, and shape of the candidate DEM space condition to the transformation achieved against reference DEM, such that the similarity metrics for the overlapping regions of candidate and reference DEM is very high and the difference is ideally zero. The ideal task of registration algorithm is, then, stated as a three-folds argument:

(i) It must give proper correspondence to enable candidate DEM to map w.r.t. reference DEM successfully;

(ii) The cost of registration function in terms of complexity, resource usage, and energy between the candidate and reference DEM must be minimum; whereas, with respect to issues such as generality, and accuracy, the score must be maximum;

(iii) Ideally, the difference between the overlapped regions of reference and the registered DEMs must tend towards zero. Contrarily, the similarities between the reference DEM and the registered DEM must be very high. Practically, we may say that their differences must be minimum and similarities maximum.

The manifestation of the registration framework is to reduce the energy function that may be defined in terms of certain spatial alignments or specific parameter-wise differences within the two DEMs. Alternatively, the objective of registration may be stated as - to find an optimal transformation whose cost, in terms of similarity is maximum, or in case of disparity measurements, is minimum. Registration aims for finding a deformation field that spatially aligns the candidate DEM with respect to the reference DEM. This may be written as

$$\text{Final Spatial Alignment } \Rightarrow \begin{cases} \arg \text{Max}[\text{Similarity}(\text{DEM}_{\text{ref}}, \text{DEM}_{\text{registered}})], \text{ or } \\ \arg \text{Min}[\text{Disparity}(\text{DEM}_{\text{ref}}, \text{DEM}_{\text{registered}})] \end{cases}$$

wherein, $\text{DEM}_{\text{registered}}$ is the final resultant DEM got after registration of $\text{DEM}_{\text{cand}}$ with respect to $\text{DEM}_{\text{ref}}$.

Besides introductory fundamentals, in subsequent sections we present the relevant terms used in DSICNR DEM registration approach, viz., symmetry, free-form deformation and non-rigid deformation, diffeomorphism, regularization and optimization.
4.2.1 Symmetry

In most works, the evaluation of the registration method is either done by appraising the image similarity between the reference and changed-candidate DEM pair’s overlapped region or the inconsistency measurement between them. However, these measures may not prove to be very adequate for transformations that do not modify the changed- or deformed candidate DEM data. In case of non-rigid registration, as discussed by various authors in [29], [165], [167], [181], [188], among others the registration direction i.e., the estimated mapping function from reference DEM to the candidate DEM is not identical to the inverse of the assessed transformation correspondence function from candidate DEM to the reference DEM. The resultants of such direction-dependent registrations are not consistent and may differ with respect to their similarity or disparity metrics data. Also, the correspondence mapping function, so formed, is typically deliberated to be a forward constraint. These asymmetric constrictions may amount to registration inconsistency as the resultant of which the forward registration may differ from the resultant of inverse registration. Therefore, if an inverse-consistent registration correspondence mapping is designed, it would lead to an improved registration structure as it would eradicate the emphasis on the registration direction [158], [170], and [176]. Consequently, a consistent DEM registration approach would help in overcoming the limitations put forth by forward- and inverse-direction registration, i.e., irrespective of the registration direction, the consequent registered DEM would yield identical output.

To conveniently administer the symmetry feature in our DEM registration approach, we have rephrased the initial problem as registration of residuals of the non-overlapping regions of the DEMs. Residuals are based on symmetric differences and are a concept taken from the field of set theory. It is applied based on the unions of the symmetric differences between two DEMs, say DEM<sub>A</sub> and DEM<sub>B</sub> and is represented as:

\[(DEM_{(A-B)}) \cup (DEM_{(B-A)})\], \hspace{1cm} \ldots (4.2)

where

\[DEM_{A-B} = DEM_A - DEM_B, \text{ and}\]

\[DEM_{B-A} = DEM_B - DEM_A, \hspace{1cm} \ldots (4.3)\]
Equation (4.2)’s implication is presenting a set of data values that are in DEM\textsubscript{A} but not in DEM\textsubscript{B}, along with the set of data values that are in DEM\textsubscript{B} but not in DEM\textsubscript{A}. This symmetric-difference of the input DEMs gives a coarse picture of the non-overlapping areas of the given input DEM data sets. This helps in not only reducing the computation requirements but also restricts the areas that actually contribute to forming the transformation function.

### 4.2.2 Free-Form Deformation and Non-rigid Registration

Free-Form Deformation (FFD) is a technique, borrowed from Computer Graphics [149], for assembling any shape or contour in a free-form design [159], [160], and [168]. FFD as a modeling tool was originally explored by Sederberg and Parry [166]. In this technique, a three dimensional or 3D model necessitating deformation is primarily constrained by a lattice-like configuration of control points. Subsequently, these lattice control points are employed to warp the parametric spread delimited by the lattice assembly. Finally, the deformation molds to the target model. A variation of the FFD is the rational DMS-FFD constructed on DMS-splines [173], which too has proven to be adequately robust. All these spline structures satisfy the properties of B-splines, like that of having at least C\textsubscript{2} continuity, affine invariance, convex hull property, local support, and many more. Some shortcomings of using spline based FFD as transformation model include under-shooting or over-shooting, commonly known as Gibb's phenomena, and having higher computation time [145], [182]. An important criterion is - the larger the number of topological features in the overlapped regions of the reference and candidate DEMs, the enhanced is the registration resultant and hence better is the said registration approach.

Transformations that have been used in DEM registration may be either rigid body transformation, or affine transformations or non-rigid deformations and have various parameters associated with each of them. The choice of a particular transformation is dependent on various parameters affecting their spatial or geometric orientations [149]. These parameterized transformations may typically be characterized by a combination of elementary parameters and functions. Rigid body mappings preserve the expanse of all the points contained by the body of the image or object under consideration and are permitted for only rotation and translation transformation. Hence, the rigid body transformations may be presented with the assistance of
rotation matrices and translation vectors. Affine mapping functions extend rigid body transformation to include scaling as well as shearing for global change detection.

Though the parametric transformation represent simplicity and ease of use having only a few parameters for estimation of the transformation, it lacks from providing very low accuracy of registration of local deformation. Hence, to represent these minuscule local disparities, higher-order transformation functions and parameterization suggested by Shen [26] and also used by authors Schneider and Reinartz in their work in [58] were proposed.

The cost of correspondence mapping function is shown to be associated with two important aspects – a set of global and local transformation parameters, characterized by $\mathcal{S}_{\text{global}}$ and $\mathcal{S}_{\text{local}}$ respectively. The cost of the parameters is because of the similarity measure and regularization constraints required to restrain the transformation to be smooth [149]. This has been represented as:

$$\mathcal{S} = \mathcal{S}_{\text{global}} + \mathcal{S}_{\text{local}}, \quad \ldots (4.4)$$

where $\mathcal{S}$ is used to denote transformation of the candidate DEM. The shown transformation in Equation (4.4) consisting of global transformation describing the overall motion of the DEMs under consideration and local transformation that constraints all the local discrepancies. Usually, the global transformation is performed by using rigid transformation parameterized by 6 degrees of freedom having only the global rotation and translation components performed by using 3D affine transformation using a $4 \times 4$ homogeneous transformation and may be shown as:

$$\mathcal{S}_{\text{global}} = \begin{bmatrix} a & b & c & p \\ d & e & f & q \\ g & i & j & r \\ l & m & n & s \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}, \quad \ldots (4.5)$$

where the $4 \times 4$ homogeneous matrix represents the general transformation, yielding a combination of shearing, local scaling, rotation, reflection, translation, perspective and overall scaling as shown by Hearn and Baker in [149]. In Equation (4.5), the column matrix represents each point in terms of its position coordinates ($x, y$) position and elevation, ($h$) [8]. These have been more useful when the DEMs differ only due to their affine orientation and can be easily corrected by performing composite transformations. For such cases of DEM registration, global transformation is quite suitable as well as required.
After execution of global transformation on the input DEM data, local transformation has to be administered [28], [29]. This local correspondence has been shown using the DMS-FFD model in which the vertex of a tetrahedral DEM domain may be updated recursively to obtain the deformations [173]. Further refinements have been performed using the point-based FFD [160]. The fundamental idea of FFDs is to enfold the given object under consideration within its convex hull and later reconstructing the given object within this convex hull. And as the hull is iteratively deformed, the control point mesh structures of the object, too, are iteratively updated. This required local transformation mapping using non-rigid deformation is implemented through Free-Form Deformations.

Non-rigid deformations chart the course of the values of the DEM data so that they may move independently with respect to some constraints [181], [188]. Non-rigid registration is the process of determining such transformations between the two images under consideration [136], [189]. This class of registrations is the study that has its foundation from physical models, basis function expansions, and transformation constraints [25], [188], [250]. Free-form deformations have been applied for non-rigid deformation.

Non-rigid image registration is said to have three major elements [181], [188], [203]. These constraints, also, act as the parameters while registration, as discussed in Chapter 1. These constraints are:

1. **Transformation model** – This consists of, usually related set of transformation(s), that specify the means for describing the spatial association(s) concerning the candidate DEM(s) with respect to the reference DEM. Parametric and non-parametric models have been proposed [164], [188].

2. **Similarity measures** – This is a metric to measure the volume of spatial alignment as well as statistical distance between the DEM data sets. There are two categories *i.e.*, feature-based metric and intensity-based metric apart from the usual statistical measures.

3. **Optimization and Regularization function** – These are used to modify the transformation model to allow maximization of the matching measures. In case of Diffeomorphic mapping, deformation fields such as Tikhonov regularization term or/and curvature term or/and velocity field regularization terms may be used as shown in [184], [185], [193], [253], and [255]. Other regularization factors have
been shown to be used for the elastic net regularization for fitting of linear regression models in [14], and [15].

As deliberated in earlier chapters too, the topological inconsistency between the DEM data sets may not be satisfactorily interpreted using only the affine and rigid transformation. Hence, to show these inconsistencies within our proposed DSICNR registration approach, we use non-rigid deformation for performing the local transformations. Non-rigid deformations may be designed in numerous manners including that of using geometry-based methods such as landmarks, curves, surfaces, etc., features; or physics-based models such as fluid model, elastic deformation model, etc.

4.2.3 Diffeomorphism

Refer Christensen and Johnson [180], “A Diffeomorphic transformation is a transformation that is defined to be continuous, one-to-one, onto, and differentiable”. If the correspondence function is not one-to-one or injective, the subsequent registered image may encompass a few undesirable characteristics such as folds, like wrinkles that may be formed due to folding of one part of the image onto its adjacent regions [143], [150], [151]. Diffeomorphism accomplishes a smooth differentiable, one-to-one transformation that can be inversely transformable. This permits the transformation to preserve the neighborhood feature topologies in the DEM data sets [18], [250], and [254].

As stated earlier in the chapter, diffeomorphism is a usual prerequisite for the estimation of displacement field that has to be smooth, one-to-one differentiable mapping along with being inversely differentiable. This helps in maintaining the neighborhood local topology in the input data files. The analysis is then done on the assumption that the diffeomorphic registration of DEMs would be closure for the two diffeomorphic transformations, i.e., if two underlying transformations are diffeomorphic, their subsequent conformation will also be diffeomorphic limited by a minor error threshold subjected to the discrete nature of the input DEM data sets. Diffeomorphism is implemented with the help of regularization constraints and by optimizing the parameters so used.
4.2.4 Regularization

The main prerequisite of regularization is that it should assist in differentiating the possible transformations affecting the related applications. Given the scenarios where non-parametric deformation is applied, the regularization factor constraints the estimated deformation field. To ensure that the deformations are small and local, smoothening restriction are applied on the deformation field. The parameters associated with regularization, influence the deformation’s area for the duration of registration only. Since the inconsistency within the optimal transformation cannot be predicted, regularization models are frequently acquired from the underlying characteristics of the registered data sets and belong to the classes of either diffusive regularization, or curvature-based regularization, or a linear elastic-based model or fluid-based model. The diffusive regularization build on Laplacian model was deliberated in various literatures by Modersitzki et al., [6], [8], [22], and [184] and Berthold in [15]. This measures the divergences in the deformation field. Curvature-based regularization practises second-order derivatives within the deformation to decrease the curvature of the area under consideration as explained by Bernd and Modersitzki [6], Fischer and Matsliah [106], and Modersitzki [8], and [22]. The linear elasticity-based regularization measures the strain introduced by deforming an elastic material and is estimated by means of minor deformations [19], [164]. Fluid-based model for regularization is based on the assumption that image sets have same range of intensity levels [2] and can be employed for registration of mono-modal image data sets. The regularisation is presented only for those regions in our DEM data sets where the value of the determinant of the Jacobian matrix is nearer to zero.

Oleg M. Alifanov, one of the proponents of Inverse Methods [329], said: “Solution of an inverse problem entails determining unknown causes based on observation of their effects”. In contrast to the domain of Inverse Methods is the problem sets ensuring direct correspondence, in which the solution(s) involve finding effects based on a complete depiction of their causes.

As discussed earlier, since Image Registration falls in the class of problems of Inverse Methods, inverse consistency and symmetry with respect to registration direction is a major subject in the research community as well. Also, smoothness of the curves and associated feature properties need to be conserved. Hence, a likely candidate solution, namely, diffeomorphism, related to the subject of topography, has been applied [18], [27], [187], and [196]. Diffeomorphic
transformations are smooth and invertible transformations having smooth inverses. Smoothness of the related features are preserved, properties such as disjoint sets remain disjoint, connected sets remaining connected, and the transformation between the coordinate systems is consistent. Also these transformations are able to capture large deformations without causing topological problems while maintaining performance [27], [176], [177].

4.2.5 Optimization

For lessening the non-linear functionalities, optimization is mandated in the registration process. Techniques such as gradient-based steepest descent method, Euler-Lagrange method, etc., may be used [8]. Optimizations strategies may either be optimise-discretise and discretise-optimise strategies (as deliberated in Chapter 1).

On surface data represented in DEM scalar, minor distortions can cause large deformations and produce energy. The boundary edges of this energy field is said to contain potential energy. When the two input DEM data set’s overlapped regions are identified, the super-imposition of one on the other will result in the minimization of the total potential energy of the deformation field. This, in turn, will allow the equilibrium state to stabilize in both the DEMs. As the input DEMs start to interact with one another, their associated energy is iteratively minimized. This is shown in Figure 4.1.

![Figure 4.1](image.png)

**Figure 4.1:** Reference and Candidate Images’ overlapped areas and their force lines.

With the identification of topological feature points and their minor rearrangements and movements, the aggregate energy of the system minimizes and equilibrium can be reached.

The surface of a 3D enclosure forms its boundary. Let this surface be represented by a set of spline curves. Consider that the deformation is the energy associated with the spline segments.
Let the curve be represented as \( f_3 \) in the 3D space. The potential energy associated with this curve would then be represented as

\[
E(f_3) = \min \left[ \frac{1}{2} \left\| w(x, y, h) \right\|^2 \right] dx \, dy \, dz, \quad \ldots (4.6)
\]

subject to constraints that are required to be enforced in the domain \( \Omega \), written as

\[
\nabla \times w(x, y, h) = \delta(f_3) \tau, \quad \ldots (4.7)
\]

where \( w \) is the stress that acts on the curve in the 3D space and is considered to be proportional to the deformation of the said curve. \( \tau \), represents the unit tangent vector of \( f_3 \), \( \delta(f_3) \), gives the smaller section functions, i.e., the delta function of \( f_3 \) and is zero at all places except at \( f_3 \). The curl of \( w \) is given by \( \nabla \times w \). The latter equation, Equation (4.7), implies that if there is an integration done along any segment \( S \), that is enclosed within \( f_3 \), it would be a unity, i.e.,

\[
\int_S \frac{w \, dI}{1} = 1. \quad \text{The implication of the constraint, so presented, is that the deformation is a field that is non-conservative in nature. This non-conservativeness is due to the fact that the curve would always store some potential energy. In the earlier equation, Equation (4.6), the definition of the energy factor had been done on the basis of the principle of the minimum total potential energy. Its emphasis is on the fact that any structure – curve, surface, or object would deform and be displaced to such a position that would minimize its potential energy for that object to attain its equilibrium or near-equilibrium state. These equations, depicting the energy associated with dislocation and fractures in solids, have been dealt in [330]. Equation (4.6) can be solved using Lagrange method, such that:

\[
E(f_3) = \left[ \frac{1}{2} \left\| w(x, y, h) \right\|^2 + \lambda \left( \nabla \times w(x, y, h) - \delta(f_3) \tau \right) \right] dx \, dy \, dz, \quad \ldots (4.8)
\]

where the Lagrange multiplier is a vector function and is given by \( \lambda = (\lambda_x, \lambda_y, \lambda_z) \).

The partial differential with respect to \( w \) and \( \lambda \) yield the following set of equations:

\[
\begin{align*}
\frac{\partial E}{\partial w} &= w + \nabla \times \lambda = 0 \\
\frac{\partial E}{\partial w} &= \nabla \times w - \delta(f_3) \tau = 0
\end{align*}
\quad \ldots (4.9)
\]
In Equation (4.9), the first part of the equation is found to be equivalent to \( w \cdot \nabla = 0 \), and its second part, inherently defines Equation (4.7) and can be solved as shown by Lardner in [44] as:

\[
  w = -\frac{1}{4\pi} \int_{f_3} \frac{r \times dl}{r^3},
\]

where the vector \( r = (x - x_1, y - y_1, h - h_1) \) is between a point \((x, y, h)\) and the point \((x_1, y_1, h_1)\) on the 3D surface on the spline curve \( f_3 \), with \( r = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (h - h_1)^2} \) being the distance between them and \( dl \) is the segment of the curve.

The proposed diffeomorphic, symmetric, inverse-consistent non-rigid DEM registration approach uses similar energy function for the judgment of the registration between two energy objects that move towards and interact with one another so that the aggregate associated energies can be minimized. When two DEMs are placed in such a system, their common or overlapping regions form a new interactive system wherein their movement and interaction are allowed to reach the equilibrium and stable state.

There are two basic properties that have profound impact over registration:

(a) The feature points with larger differences in the gradient magnitude contribute more to the registration drive than the lesser ones.

(b) The force is stronger when the distinctions between the DEM data sets is closer i.e., the reference surface’s feature-points are closer to its corresponding feature point in candidate surface. This suggests that when the attraction field is stronger, the registration process approaches its stability at a faster pace.

### 4.3 Proposed DSICNR Approach for DEM Registration

In this section, we propose symmetric and inverse-consistent approach for registration of input DEM data sets that overcome some accuracy issues as well [21], [53], [64], [65], [170], [314].

The various steps of registration are discussed in this section. Usually there are two or more inputs to the system – the reference DEM \( (DEM_{ref}) \), sometimes also considered the standard DEM, and a single or a set of the candidate DEM(s) \( (DEM_{cand}) \).
After finding the various overlapping regions, i.e., regions common to both the reference and candidate DEMs, a novel registration approach has been presented to reduce direction inconsistency in registration and establishing a symmetric and inverse-consistent DEM registration transformation and interdependence. This has been performed using diffeomorphic free-form non-rigid deformations. The related particulars were discussed in detail in Section 4.2. Diffeomorphic free-form deformation has allowed the main emphasis to be on the one-to-one mapping between the two image data files.

The proposed DSICNR approach also solves for cases wherein only the reference DEM has been provided and no specific candidate DEM is given to be registered. In such cases, the probable candidate DEM(s) are first found. This forms the coarse registration phase and later proceeds to the Fine Registration phase. When the reference DEM and candidate DEM(s) are given, the algorithm proceeds directly to the Fine Registration phase. The phases are discussed subsequently.

Before commencement of the registration procedure, every DEM must undergo noise filtering as its initial process. After noise filtering, the subsequent algorithm follows two key tracks:

(a) The first case is when the reference DEM is given but the candidate DEM(s) need to be selected from an existing set of DEMs. For such cases, the reference DEM is initially lead to extraction of correspondence for finding potential candidate DEM(s). This phase termed as ‘Coarse Registration phase’ that is followed by ‘Fine Registration phase’ steps and usually is based on histogram-based matching.

(b) The second case is when the candidate and reference DEMs, both, are known, then the registration proceeds towards the ‘Fine Registration phase’ directly.

For both the circumstances, either given $DEM_{ref}$ and $DEM_{cand}$ or after choosing a suitable $DEM_{cand}$, we ascertain the residuals, $DEM_{ref-cand} = DEM_{ref} - DEM_{cand}$ and $DEM_{cand-ref} = DEM_{cand} - DEM_{ref}$ . Thereafter using the set-theoretic concept of symmetric-difference, we calculate $(DEM_{(ref-cand)}) \cup (DEM_{(cand-ref)})$ . The symmetric-difference, as is known, gives a rough estimation of non-overlapping regions between $DEM_{ref}$ and $DEM_{cand}$ . The symmetric-difference values give a perception of the coarsely-dense and highly-dense regions that help in easy detection of the numerous overlaps. This technique for finding
overlapped areas is found to demonstrate robust and a good quality of registration even when the
same DEM(s) of different types - multi-temporal, multi-modal, or multi-resolution – are
considered for registration process.

In general, the aim of non-rigid deformation is to find a mapping model, $\mathcal{I}$, that is optimal in
nature and maps each voxel of the candidate elevation model, $DEM_{\text{cand}}(x, y, h)$, to the reference
elevation model, $DEM_{\text{ref}}(x, y, h)$. These transformations proclaim an energy function that
reduces the dissimilarity between the reference and the candidate DEMs. Registration
necessitates determining a correspondence mapping function. This mapping is usually
proclaimed to belong to set of ill-posed problems and requires additional constraints to regularize
it. Computation of the registration has been then done by minimizing the objective function,
stated as the energy function. This is given by:

$$E = \sum E_{\text{similarity}}(\mathcal{I}(DEM_{\text{cand}}(x, y, h)), DEM_{\text{ref}}(x, y, h)) + E_{\text{constraints}}, \quad \ldots (4.11)$$

where, $E_{\text{similarity}}$ is the similarity metric and $E_{\text{constraints}}$ is the aggregate of the constraints mandated
for regularizing the function. These constraints can be regularization parameters or optimization
parameters or an amalgam of both. Such situations fall in the domain of “inverse methods”.
Equation (4.11) helps in defining the overall energy objective function.

Hence, as per this definition, the problem of registration may be stated as that of finding the
transformations $\mathcal{I}_{\text{refcand}}$ and $\mathcal{I}_{\text{candref}}$ such that, $\mathcal{I}_{\text{refcand}}$ maps $DEM_{\text{ref}}$ to $DEM_{\text{cand}}$, and
$\mathcal{I}_{\text{candref}}$ maps $DEM_{\text{cand}}$ to $DEM_{\text{ref}}$ in the form, $\mathcal{I}_{\text{refcand}} = \mathcal{I}_{\text{candref}}^{-1}$. The
transformation for inverse consistent may be shown as:

$$\mathcal{I}_{\text{refcand}} \circ \mathcal{I}_{\text{candref}} = \mathcal{I}_{\text{candref}} \circ \mathcal{I}_{\text{refcand}} = I, \quad \ldots (4.12)$$

i.e., an identity mapping, where $\circ$ is the convolution function between the two transformations.
Basic registration diagram is shown in Figure 4.2. Equation (4.12) relates to the symmetry
constraint of the registration approach.

Topological variability mapping between two or more DEM scalar data sets is usually
insufficient using only the affine or rigid transformations. Non-rigid transformations have been
found to be capable to represent such minuscule but sometimes wise-spread deviations in the
DEM scalar data sets and have, therefore, been used for diffeomorphic transformations as
discussed in this chapter.
The proposed DSICNR approach is associated to both the local as well as the global deformations based on their corresponding transformation functions and optimizations. The registration process has been shown to be able to capture large-scale as well as minor deformations with the regions of consideration. The DEMs used for the experimentation are multi-temporal, multimodal, multi-view and multi-resolution DEMs. DEMs having diverse resolutions have been resolved by transforming both the DEMs to have the same resolution. Even though, the DEM having lower resolution may be converted to higher resolution or the DEM having high resolution may be converted to low resolution DEM, usually the conversion is to the one having lower resolution. In the former situation, too many falsely interpolated data insertion may happen in cases of DEMs having high difference in resolutions. In the later situation, since it is uncomplicated to convert a DEM of high resolution to the one having lower resolution through standard techniques like sub-sampling, high resolution DEMs are converted to have the same resolution as the DEM having low resolution. This conversion, if the initial data sets are not saved properly, can lead to data loss and must be avoided.
The evaluation of registration approach is formulated as a minimization of the cost of similarity measures such as SSD, NCC and NMI. The objective function of registration may also be stated as the maximization of similarity metric expression, in cases of SSD, NCC, and NMI in terms of their actual values and were discussed in Chapter 1.

The basic diagrammatic representation may be shown as the flowchart as used for registration in the proposed DSICNR approach is shown in figure, Figure 4.3. After the initial noise filtering, either ‘Coarse Registration’ or directly the ‘Fine Registration’ steps are performed.

The ‘Coarse Registration’ phase includes finding potential candidates that have to be registered with the given reference DEM image when the candidate DEM is not provided to the registration system. On application of this approach, a few potential candidates from the contained data sets have been chosen for the registration. The potential candidates are chosen based on the proximity of the histogram of the potential candidates to the given reference DEM. The user may then choose one of these potential candidates for the final registration as deliberated in the ‘Fine Registration’ phase.

During ‘Fine Registration’, two procedures are executed consecutively: firstly, the demarcation of overlapping areas of the candidate and reference DEMs. For the demarcation, after the symmetric difference, we employ inexact sub-graph based matching between various relief-type graphs. The related matching is discussed in detail in Chapter 3. The second procedure involves using non-rigid registration using extraction of the deformation model, and applying symmetry and diffeomorphism in the transformation mapping function. Thereafter optimization is performed to reach to a stable registration resultant.

These, depicted in Figure 4.3, follow these summarized steps as shown:

**Step 1:** Preprocessing of given DEMs via noise filtering.

Steps 2 – 4 refer to the ‘Coarse Registration’ Phase

**Step 2:** If the reference and the candidate DEMs are given as inputs to the system, then proceed to Step 4.

**Step 3:** If candidate DEM(s) are not given to the system as inputs, the proposed DSICNR approach would search for a suitable candidate DEM(s) from the existing DEM data sets.

**Step 4:** Apply symmetric-difference to indicate the density of overlapping and non-overlapping regions in the reference and candidate DEMs.
Figure 4.3: Pictorial representation of the proposed DSICNR approach for DEM registration
Steps 5 – 7 form the ‘Fine Registration’ phase.

**Step 5:** After an initial approximate misaligned area demarcation, overlapping regions are found and this is then subjected to segmentation followed by sub-graph based matching.

**Step 6:** Extract the deformation transformation mapping function which is based on non-rigid deformation model.

**Step 7:** Inverse Consistency is then imposed by the restrictions of symmetry, diffeomorphism and optimization criterion.

The Fine Registration process is based on DMS-splines as introduced by Dahmen *et al.*, [141] that use the properties of deformation of the tetrahedron lattice structure, and supports optimization of local and global cost functions. The flowchart is as shown subsequently in Figure 4.4 and its stages may be stated as follows:

**Step 1:** The candidate DEM undergoes global transformation w.r.t. the reference DEM.

**Step 2:** Construct the lattice structure, on both the candidate and the reference DEM data spaces, with each data point as a vertex within the lattice structure having some pre-assigned weights to each data point. This is followed by calculation of parametric coordinates for each vertex to be deformed. The weights have been assigned automatically. Initially all the control points have been given weights to follow the properties of having range of ± .85, and zero mean. These weights may change in their subsequent iterations.

Steps (3-6), outlined below, and are applied on candidate DEM w.r.t. the reference DEM.

**Step 3:** Once the tetrahedral model’s vertices are formed, they are, then, manipulated so that the weights of the control points are updated recursively. A trade-off between the optimized numbers of control points to be used is to be formulated so as to keep the computation time within bounds. Initially all the elevation values have been considered as control points.

**Step 4:** The new locations of the control points are then evaluated based on the tetrahedral model.

**Step 5:** The voronoi edges and vertices are removed with respect to the outlier points.
Figure 4.4: Flowchart for Fine Registration stage used in the proposed DSICNR DEM registration approach.
**Step 6:** Recursively reduce the number of points w.r.t. the internal markers already set as the minimum number of points to be used.

The iteration terminates under the following two conditions:

(i) The algorithm has reached a maximum iteration number. This is user defined and has the range of 100 – 800 depending on the size of the DEM used.

(ii) The difference in transformations, *i.e.*, error between two successive iterations, is negligibly small.

The iteration is continued till it reaches either its limit or is lesser than a pre-set threshold error level between the orientations of the candidate DEM and registered DEM is reached.

**Step 7:** Construct mesh of the candidate DEM after optimizing the non-rigid deformation functions. Then construct mesh of the reference DEM and register it with the candidate DEM's so formed in the previous step, by embedding the candidate vertices with the reference vertices.

**Step 8:** Similarity evaluation between the registered DEM and a standard DEM to evaluate the effectiveness of the proposed DSICNR approach is measured by applying KLD measure. PSNR (Peak signal to noise ratio) and CC (correlation coefficient) values have been found and have been used as initial indicators only.

We have used free-form deformation (FFD) based on spline functions so as to model the required non-rigid deformation [138], [164], and [166]. Since the splines are depicted on a regular uniform grid, the model is spatially invariant with respect to the degree of freedom that the transformation is supplicant to and does not vary spatially. However, it suffers from limitation of being inverse inconsistent. Spline-based FFD [142], [143], [151], [165], and [233] is an effectual tool for modeling 3D deformations.

Inverse-consistency [201] has been imposed by employing diffeomorphism constraint. Diffeomorphic deformation is done by approximating spline-based FFD as a composition of diffeomorphic FFDs. To make the complete composition diffeomorphic, each and every individual FFD will have to be made diffeomorphic along with the imposition that the transformation will not have local optima convergence. This has been enforced by using certain constraints on the largest deformation within each FFD.
Let the domain of the DEM volume be defined by:

\[ \Delta = \{(x, y, h) \mid 0 \leq x < X, 0 \leq y < Y, 0 \leq h < H\}, \quad \ldots (4.13) \]

where \((x, y)\) are the coordinate positions and \(h\) is the elevation value at the particular \((x, y)\) position.

The general expression for the calculation of coordinate positions along a B-spline curve in a blending-function formulation may be stated as:

\[ P(u, v) = \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{n_2} p_{k_1,k_2} B_{k_1}(u) B_{k_2}(v), \quad \ldots (4.14) \]

for \(u_{\min} \leq u \leq u_{\max}; \quad v_{\min} \leq v \leq v_{\max}; \quad 2 \leq d \leq n + 1\), where the vector values for \(p_{k_1,k_2}\) specify the positions of \((n_1 + 1) \times (n_2 + 1)\) control points and the B-spline blending functions \(B_{k_1}(u)\) & \(B_{k_2}(v)\) are polynomials of degree \(d_i + 1\) where parameter ‘\(d_i\)’ can be chosen to be any integer value in the range from 2 up to the number of control points and the subscript ‘\(i\)’ is either 1 or 2. Blending functions for B-spline curves as given in Hearn and Baker [149], may be defined by the Cox-deBoor recursion formula for ‘\(d\)’ subintervals, ‘\(n\)’ number of control points with ‘\(k\)’ subintervals and ‘\(u\)’ range of knot vectors:

\[
\begin{align*}
B_{k,1}(u) &= \begin{cases} 1 & \text{if } u_k \leq u \leq u_{k+1}, \\ 0 & \text{otherwise} \end{cases}, \\
B_{k,d}(u) &= \frac{u-u_k}{u_k+d-1-u_k} B_{k,d-1}(u) + \frac{u_{k+d-1-u_{k+1}}}{u_{k+d-1-u_{k+1}}} B_{k+1,d-1}(u), \quad \ldots (4.15)
\end{align*}
\]

The 2D B-splines have been extended for use in 3D for B-spline surfaces [154], [168], [169], [172], and [179] as is required in our work. Since B-spline surfaces exhibit the same properties as those of their corresponding B-spline curves, the tensor products of local control also have been extended to be used for 3D B-spline surfaces. It may be noted that \(\beta\)-splines may also be used for a similar purpose as it provides similar results with approximation for the same properties.

Coordinate position along a B-spline curve in a blending function may be calculated as:

\[ P(u) = \sum_{i=0}^{n} p_i B_{1,i}(u), \quad \ldots (4.16) \]

for \(u_{\min} \leq u \leq u_{\max}\) with \(u_{\min}\) and \(u_{\max}\) depending on the number of control points.

The result of FFD, which basically denotes our local transformation, has been achieved using the 2D tensor product of 1-D cubic B-spline as:
\[ \mathcal{Z}_{local} = \sum_{l=0}^{3} \sum_{m=0}^{3} B_l(u) B_m(v) \Theta_{l+j+m}, \]  
\[ \ldots (4.17) \]

where \((l,m) \in [0,3]\) denotes the index of the control point cell containing \(u, v\); \(\Theta_{l+j}\) is the position of the \(ij^{th}\) control point on the lattice image with uniform spacing \(\delta\); further, \(i = \lfloor x/\delta \rfloor - 1, \ j = \lfloor y/\delta \rfloor - 1, \ u = x/\delta - \lfloor x/\delta \rfloor, \ v = y/\delta - \lfloor y/\delta \rfloor\), and \(B_l\) represents the \(l^{th}\) basis function of the B-spline [135], [168], [172].

DMS-splines, introduced by Dahmen \textit{et al.}, [141] and used by Xu \textit{et al.}, [173] have the interesting property of tetrahedron lattice deformation. The degree 'n' for DMS-spline volume, \(S(u)\), over \(\Omega\) is defined as:

\[ S(u) = \sum_{I \in \Omega} \sum_{|\beta|=n} C^I_{\beta} N^I_{\beta}(u), \]  
\[ \ldots (4.18) \]

where \(\Omega\) is an arbitrary proper tetrahedralization of the three-dimensional space, \(R^3\); \(C^I_{\beta} \in R^3\) are the control points and \(N^I_{\beta}(u)\) are the DMS-spline basis functions [173]. If some assumed weights \(w^I_{\beta}\) are set with each of the control points, the rational DMS-spline volumes may then be defined as some combinations of a set of piecewise rational functions [150]. For our experimentation we have assumed the starting weights of all the control points to follow the properties of having range of \(\pm .85\), and zero mean.

Next, let each vertex \(V_i\) of the B-spline parametric space, be assigned a set of one or more parameterization per basis function to influence the position of the vertex. And, let \(\tilde{U}_{ij}\) be the parameterization of a vertex \(V_i\) with respect to the parametric domain of basis function. Also, let \(N^I_{\beta}(u)\) be assumed to be associated with exactly one of the \(C^I_{\beta}\) control points \(p_i\). These vectors form a vector space in which vertices are parameterized. Therefore, the extendable basis functions may be approached as being scaled and rotated 3D Gaussian distribution, which have been normalized by a constant value, say \(\alpha\), such that the basis function should be able to take the value 1 at the center of the ellipsoid [160]. This basis function would include the scaling and rotation matrices inherently.
The 3D deformation, modeled as per FFD may be written as 3D tensor product of 3D cubic B-splines, which may be represented by a patch $f_3$ defined by:

$$f_3(u,v,w) = (x,y,h) = \sum_{i=0}^{3} \sum_{j=0}^{3} \sum_{k=0}^{3} B_i(u)B_j(v)B_k(w)\emptyset_{ijk}, \quad \ldots \quad (4.19)$$

where $0 \leq u,v,w \leq 1$ and $\emptyset_{ijk}$ is the set of control lattice. This function $f_3$ is locally injective if and only if their Jacobian matrices are non-singular all over the domain [179], [194], and [202].

The Jacobian matrix of function, [6], [17], is represented as:

$$J(f_3) = \begin{bmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\
\frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} & \frac{\partial h}{\partial w}
\end{bmatrix} \quad \ldots \quad (4.20)$$

A square matrix may be considered as non-singular only if its row or column vectors are linearly independent, hence, function $f_3$ may be considered as locally injective if and only if its three 3D row vectors of $\boldsymbol{r}_1 = (\frac{\partial x}{\partial u}, \frac{\partial x}{\partial v}, \frac{\partial x}{\partial w})$, $\boldsymbol{r}_2 = (\frac{\partial y}{\partial u}, \frac{\partial y}{\partial v}, \frac{\partial y}{\partial w})$, $\boldsymbol{r}_3 = (\frac{\partial h}{\partial u}, \frac{\partial h}{\partial v}, \frac{\partial h}{\partial w})$ are linearly independent.

This Jacobian matrix determinant $det(J(f_3))$ helps in checking whether the updated deformation field, so calculated, preserves the one-to-one mapping. This has been done by checking the sign of $det(J(f_3))$; if this is positive, it indicates that the one-to-one mapping property is intact. The magnitude, if larger than a certain threshold limit, indicates the possibility of crossing within the intensity points and thereafter folding; whereas if this value is below the norm limit, it would represent an invertible transformation. To constraint the FFD to be diffeomorphic, it becomes necessary to evaluate any non-diffeomorphic transformation by penalizing it using a cost value as given by Rueckert et al., [165], [196], and [250]:

$$f_{pen} = \left\{ \begin{array}{ll}
\frac{\rho_{\text{threshold}}}{det|J(f_3)|} - 2 & \text{if} \quad |J(f_3)| \leq \rho_{\text{threshold}} \\
0 & \text{otherwise}
\end{array} \right., \quad \ldots \quad (4.21)$$

The smoothness cost function $\rho_{\text{threshold}}$ is a threshold limit of the cost of transformation when its Jacobian falls below this threshold limit. By adding this cost on Jacobian, the transformation is prevented from collapsing and helps in remaining diffeomorphic. Through experimentation it was found that there exists an inverse relationship between the penalty function and the determinant value of the Jacobian. Further, the penalty function was also identified to be directly dependent.
on the smoothness cost function. Since the vertexes have small values, the penalty function has to be much smaller and is, hence, represented as given in Equation (4.21).

Small-step approach has been employed for updating the deformation fields [180]. Another approach using log-domain parameterization can, also, be used for facilitating diffeomorphic transformations [14].

The primary inconsistency in registration is due to the characteristics of forward and the reverse transformations which are not symmetric in nature and are inverse-consistent. This arises partially due to the ill-posed nature of image registration. Other contributing factor may be the choice of the similarity criteria having asymmetric properties. These inconsistencies have been concurred by symmetrisation of the correspondence functions and similarity measure under consideration and by explicitly minimizing the consistency error using the joint estimation of the forward and reverse transformations. Hence the aggregate similarity, depicted by summation of similarity measures of the forward Similarity\textsubscript{forward} and the reverse Similarity\textsubscript{reverse} directions, and is represented as:

\[
\text{Similarity}_{\text{consistent}} = \text{Similarity}_{\text{forward}} + \text{Similarity}_{\text{reverse}}, \quad \ldots (4.22)
\]

The penalty for incorporating this consistency is the sum of the aggregate smoothness cost function \((f_{\text{pen}})\), penalty of regularization in forward \(\text{Reg}_{\text{forward}}\) and reverse \(\text{Reg}_{\text{reverse}}\) directions. This is given by

\[
\text{Pen}_{\text{consistent}} = f_{\text{pen}} + \text{Reg}_{\text{forward}} + \text{Reg}_{\text{reverse}}, \quad \ldots (4.23)
\]

The aggregate cost function is a combination of image similarity and smoothness cost penalty function:

\[
\text{Cost} = \text{Similarity}_{\text{consistent}} + \text{Weight}_{\text{parameters}} \ast \text{Pen}_{\text{consistent}}, \quad \ldots (4.24)
\]

This ‘\text{Weight}_{\text{parameters}}’ is a trade-off between the image similarity and the penalty function. In Equation (4.11), the optimized objective function, in terms of, the energy function is stated and is improvised to include the Cost parameters.

The optimization is accomplished by minimizing the cost function w.r.t the global as well as the local transformation parameters and is said to have two parts - first belonging to the similarity and other to the errors due to smoothness and consistency functions.
The inverse transformation for forming the consistent registration is depicted in Algorithm 4.1 and takes the reference and candidate DEMs as inputs to generate the forward $\mathcal{Z}_{\text{refcand}}$ and backward or reverse transformation $\mathcal{Z}_{\text{candref}}$.

**Algorithm 4.1: Inverse Transformation**

Input: Two DEMs namely, reference DEM, and candidate DEM - DEM$_{\text{ref}}$, DEM$_{\text{cand}}$.

Output: Forward transformation $\mathcal{Z}_{\text{refcand}}$, backward transformation $\mathcal{Z}_{\text{candref}}$

Step 1: Set initial forward and reverse transformations as null.

Step 2: Start iteration

for all set of spatial control points, do

- Calculate forward update displacement field such that the symmetric gradient is given as average of the sum of the gradients of the previous forward and the reverse deformations.
- Smooth update of the deformation field using Gaussian filter
- Calculate backward update as shown in the next algorithm, Algorithm 4.2
- Update intermediary forward displacement field given by the convolution of the previous forward displacement with present forward update.
- Smooth forward displacement field using Gaussian filter by applying the penalty function to the forward transformation.
- Update intermediary backward deformation field given by the convolution of the previous backward displacement with present backward update field.
- Smooth backward deformation field using Gaussian filter by applying the penalty function to the backward transformation.

Step 3: Update iteration until (deformation fields do not change) or iterations have reached their maximum limit.

Step 4: calculate the inverse of the intermediary forward and reverse transformation using next algorithm, Algorithm 4.2
Step 5: calculate forward transformation as the convolution of intermediary forward
transformation and its inverse.
Step 6: Calculate backward transformation as the convolution of intermediary forward
transformation and its inverse.
Step 7: Use their composition of the half-way transformations as the final forward and reverse
transformations and return these.

The next phase is to find the inverse consistent transformation which when convoluted with
itself results in an identity function as depicted in Figure 4.3. This is depicted in Algorithm 4.2.

**Algorithm 4.2: Inversion of deformation field**

Input: Transformation \( \Im \), Parameter: penalty function

Output: Inverse of transformation \( \Im^{-1} \)

Step 1: for all spatial control points do

- Start iteration
- Set initial spatial transformation as 0
- Repeat
  - Calculate determinant of the Jacobian matrix \( \det(J(f_3)) \)
  - If \( \det(J(f_3)) \leq 0 \) then
    - Add regularisation parameter \( e_{\text{smooth}} \) as given by Equation
      \( (4.28) \).
    - Add a three dimensional diagonal matrix with the regularization parameter
    - Update the current estimation of spatial transformation by incrementing it
      with the displacement.
  
  Increment iteration until \( E(f_3) \geq \text{penulty} \) or (iteration has reached its maximum
  value)
  
  Set the current spatial transformation as the inverse transformation

Step 2: return \( \Im^{-1} \)
Following this we take help of the strategy by Choi and Lee [179], for ensuring that the FFDs are truly diffeomorphic by using the sufficient conditions for proving that the function is injective in terms of control point displacements. If \( \Phi_{ijk}^0 = (i - 1, j - 1, k - 1) \) for \( i, j, k = 0,1,2,3 \), the function \( f_3 \) will be reduced to an identity function. Therefore, the displacement of control points \( \Phi_{ijk} \) from \( \Phi_{ijk}^0 \) is given, using recurrence formula, by:

\[
\Delta \Phi_{ijk} = \Phi_{ijk} - \Phi_{ijk}^0 = (\Delta x_{ijk}, \Delta y_{ijk}, \Delta z_{ijk}). \quad \dot{\ldots} (4.25)
\]

Then,

\[ f_3 (u, v, w) = (u, v, w) + \sum_{i=0}^{3} \sum_{j=0}^{3} \sum_{k=0}^{3} B_i(u)B_j(v)B_k(w)\Delta \Phi_{ijk}, \quad \dot{\ldots} (4.26) \]

Let \( D^u_{ijk}(u, v, w) = B'_i(u)B_j(v)B_k(w) \); \( D^v_{ijk}(u, v, w) = B_i(u)B'_j(v)B_k(w) \); \( D^w_{ijk}(u, v, w) = B_i(u)B_j(v)B'_k(w) \). Let \( K_3 \) be related to cubic B-spline basis function such that \( K_3 = \max_{0\leq u,v,w\leq 1} \{ val(u,v,w) \} \).

\[
val(u,v,w) = \sum_{i=0}^{3} \sum_{j=0}^{3} \sum_{k=0}^{3} | D^u_{ijk}(u, v, w) + D^v_{ijk}(u, v, w) + D^w_{ijk}(u, v, w) | \quad \dot{\ldots} (4.27)
\]

as per Choi and Lee [179], and \( K_3 \) is approximately 2.4794 and is computed by partitioning the domain \( \Omega_3 \{(u, v, w) | 0 \leq u, v, w \leq 1 \} \) into a highly dense grid.

The function \( val(u,v,w) \), calculated for each grid point with maximum at \( (u_0,v_0,w_0) \) for \( u_0 = 0.16406 \) or \( u_0 = 0.83593 \). The geometric interpretations are explained in Choi and Lee [179].

Since we are considering that the splines may be non-uniform as well, we have approximated these non-uniform splines to behave as uniform splines by using an added term \( e_{add} \) to the non-uniform splines. We consider the uniform splines from the path of area which gives us the maximum common area coverage at each hierarchical level.

Smooth transformation is a characteristic requirement of local transformation of DEMs. This may be seen as penalty energy or as an error function that would help in regularizing the transformation. Hence an error term \( e_{smooth} \) has been used for regularization of the transformation functions discussed in [193] and its 3-D form shown in [253] and [255] may be stated as:
\[ e_{\text{smooth}} = \frac{1}{V} \int \int \int [\nabla^2 \mathcal{J}] \, dx \, dy \, dz, \ldots \text{(4.28)} \]

with

\[
\nabla^2 \mathcal{J} = \left( \frac{\partial^2 \mathcal{J}}{\partial x^2} \right) + \left( \frac{\partial^2 \mathcal{J}}{\partial y^2} \right) + \left( \frac{\partial^2 \mathcal{J}}{\partial z^2} \right) + 2 \left( \frac{\partial^2 \mathcal{J}}{\partial xy} \right) + 2 \left( \frac{\partial^2 \mathcal{J}}{\partial xz} \right) + 2 \left( \frac{\partial^2 \mathcal{J}}{\partial yz} \right).
\]

In Equation (4.28), ‘\(V\)’ denotes volume of the image domain under consideration and ‘\(\mathcal{J}\)’ denotes the transformation function. This term is 0 for any affine transformations and has been used for error correction with respect only to non-affine transformation [14], and [171]. This measure is 3-D extension of the 2-D bending energy of a thin-plate metal and represents cost or error associated with smoothness of the non-affine transformation.

In the cases of DEM registrations, the optimality function are composed of two contending requirements - cost of the similarity function and the other associated with the error function. This function may be given as:

\[
e (\text{global, local}) = - \text{Divergence} (\text{DEM}_{ref}, \mathcal{J}(\text{DEM}_{cand})) + \alpha e_{\text{smooth}} (\mathcal{J}(\text{DEM}_{cand})), \ldots \text{(4.29)}
\]

where \(\text{DEM}_{ref}\) is the reference DEM domain and \(\mathcal{J}(\text{DEM}_{cand})\) is the transformed DEM domain, \(\alpha\) is the weighing parameter which defines the calibration between the two DEM volumes and smoothness of the transformation functions. For our work we have considered \(\alpha\) within the range of 0.01 to 0.07 as determined experimentally. This is not critical for low resolution DEMs as they have less number of control point mesh.

The direction of the steepest descent is dependent on the choice of the similarity criterion and the regularization term. Further, the iterative optimization approach so selected, resolves the optimal set for parameter \(\mu\), which is given as:

\[
\mu_{t+1} = \mu_t + \alpha_t d_t, \quad i = 0,1,2,\ldots, \ldots \text{(4.30)}
\]

with \(d_t\) as the search direction at iteration \(i\), and \(\alpha_i\) as the scalar gain factor for controlling the step size along the search direction. Assuming the existence of local minimum for the aggregate energy function, Equation (4.11), the functional derivative, \(\nabla E\), for the energy function at this local minimum must be 0, \(i.e., \nabla E = 0\). This derivative energy function, then, includes the force, \(F\), so that the candidate DEM’s regions be warped to those of reference DEM’s via transformation based on displacement in terms of changes in gradient in small-steps, given by:
Steepest gradient descent method has been used for this work, and this takes steps in the negative direction of the gradient of the cost function. This may be represented by modifying Equation (4.30) and written as:

\[ \mu_{i+1} = \mu_i - \alpha_i \nabla(\mu_i), \quad \ldots \ (4.32) \]

where \( \nabla(\mu_i) \) is the derivative of the cost function evaluated at its current \( \alpha_i \) position. The algorithm, Algorithm 4.3 discusses the steepest descent method used in the final formation of diffeomorphic transformation.

**Algorithm 4.3:** Steepest Descent Approach for final Diffeomorphic Transformation

**Input:** Two DEMs namely, reference DEM, and candidate DEM - DEM\(_{ref}\), DEM\(_{cand}\); weight of regularization

**Output:** Transformation \( \mathcal{I} \)

**Step 1:** Let initial deformation be 0.

**Step 2:** for all spatial positions, do

- Calculate Force to be applied on each spatial position as given by Equation (4.31);
- Calculate difference in regularization using Equation (4.29);
- Calculate update filed using Equation (4.32);
- Update deformation cost field using Equation (4.24);

End until (deformation field does not change) or till maximum number of iterations is reached

**Step 3:** Return transformation \( \mathcal{I} \).

In the next section, we have given the experimental results of not only a pair-wise registration, but also extended it for registration of multiple candidate DEMs. Also, a comparison of the proposed DSICNR approach judged on various noise types and levels has been performed to show its robustness to noise. Further, handling of data holes has been shown to be successfully
achieved. In addition, comparison of the proposed DSICNR approach is shown with respect to other methods too.

4.4 EXPERIMENTAL RESULTS AND DISCUSSIONS

We have tested our proposed DSICNR approach on several DEM data sets. Each set has 10 reference and 10 candidate DEMs from 7 different sensors: InSAR, CartoSAT, ASTER, LIDAR, SRTM, PRISM and QuickBird DEMs, provided by available freely from data.geocomm.com, ws.csiss.gmu.edu, edcwww.cr.usgs.gov, gdex.cr.usgs.gov/gdex/, Bhuvan’s CartoSat-1 [337] and others); RMSE 3.44m, LE 90 4.7m. DEM files having various resolutions in the ranges of 512 x 512 x 512 to 15000 x 15000 x 15000 and having units in the ranges of 30m, 1-arc-sec to 90m, 3-arc-sec are used for experimentation. These DEM data sets have varieties of terrains like plateau regions, mild hilly areas, deserts, mountain demographics, and other topological features.

The test cases encompass a wide range of percentages for overlapping regions starting from 4% overlap regions and upwards. The fact that a few false positives could occur with respect to the overlapping regions has been assumed. To lessen the consequence of these spatial disparities over the common areas, a minimum 8 grid-point value radii was considered. If these erroneous island clusters were found to be fewer than these 8 grid-point value radii, they were presumed to be cases of false positives.

Robustness testing on the DEM data sets have been done, by incorporating Salt & Pepper, Gaussian and Pulse noises having a ±3 points range and zero mean to the original data sets. Testing for small occlusions or data holes that may exist in the DEM test cases having ±30 points pixels have been done.

4.4.1 Evaluation of the proposed DSICNR approach using Similarity Metrics.

The superiority of our proposed DSICNR approach is evaluated using similarity criterion of SSD (sum of the squared distances), normalized CC (correlation coefficients), and normalized MI (Mutual Information) metric values. This evaluation has been done by depicting the assessment between a standard reference DEM and its respective reference-candidate resultant i.e., registered
DEM. Table 4.1 shows one set of average values for the DEM’s corresponding metric values. Since SSD can result in a negative value too, the SSD values shown in Table 4.1, depicts its absolute of the average only. Also the experimentation included the DEMs that were subjected to noise having zero mean and ±9 points range.

**Table 4.1**: Evaluation of the DSICNR approach based on Symmetric Metrics of SSD, NCC, and NMI with noise having 0 mean and range ± 9.

<table>
<thead>
<tr>
<th>Evaluation Parameters →</th>
<th>SSD</th>
<th>NCC</th>
<th>NMI</th>
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<tbody>
<tr>
<td>After Registration with given Overlapping area</td>
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</tr>
<tr>
<td>Less than 5% S&amp;P noise</td>
<td>1398.8</td>
<td>0.22</td>
<td>0.32</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>1465.3</td>
<td>0.25</td>
<td>0.43</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>1348.2</td>
<td>0.19</td>
<td>0.41</td>
</tr>
<tr>
<td>Between 5-8% S&amp;P noise</td>
<td>1265.6</td>
<td>0.26</td>
<td>0.73</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>1315.8</td>
<td>0.24</td>
<td>0.67</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>1130.9</td>
<td>0.28</td>
<td>0.92</td>
</tr>
<tr>
<td>Upto 20% S&amp;P noise</td>
<td>1006.3</td>
<td>0.32</td>
<td>1.3</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>1023.1</td>
<td>0.38</td>
<td>1.42</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>1100.7</td>
<td>0.49</td>
<td>1.48</td>
</tr>
<tr>
<td>Upto 35% S&amp;P noise</td>
<td>986.5</td>
<td>0.41</td>
<td>1.49</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>939.4</td>
<td>0.4</td>
<td>1.49</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>1002</td>
<td>0.55</td>
<td>1.57</td>
</tr>
<tr>
<td>Upto 45% S&amp;P noise</td>
<td>929.8</td>
<td>0.43</td>
<td>1.44</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>899</td>
<td>0.6</td>
<td>1.51</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>938.4</td>
<td>0.7</td>
<td>1.64</td>
</tr>
<tr>
<td>Upto 50% S&amp;P noise</td>
<td>859</td>
<td>0.61</td>
<td>1.56</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>891.9</td>
<td>0.66</td>
<td>1.59</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>900</td>
<td>0.76</td>
<td>1.79</td>
</tr>
<tr>
<td>Upto 60% S&amp;P noise</td>
<td>802.9</td>
<td>0.85</td>
<td>1.81</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>876.3</td>
<td>0.84</td>
<td>1.81</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>832.7</td>
<td>0.84</td>
<td>1.8</td>
</tr>
<tr>
<td>Upto 70% S&amp;P noise</td>
<td>653.6</td>
<td>0.91</td>
<td>1.86</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>687.5</td>
<td>0.89</td>
<td>1.77</td>
</tr>
<tr>
<td>Pulse noise</td>
<td>729.7</td>
<td>0.92</td>
<td>1.86</td>
</tr>
<tr>
<td>Upto 80% S&amp;P noise</td>
<td>615</td>
<td>0.97</td>
<td>1.99</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td>635</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
As the values seen in Table 4.1 emphasize, our proposed DSICNR approach for DEM registration may be stated to be quite successfully register the candidate and reference DEMs having various percentages of overlapped regions and is found to be very robust to the Salt and Pepper (S&P), Gaussian and Pulse noises.

We have also shown the resultants of the registration approach in Table 4.2 for three sets of reference, candidate and their corresponding registered DEMs.

**Table 4.2:** Three sets of DEM data files - reference, candidate and registered DEMs are shown in three columns respectively.
### 4.4.2 Comparison with some existing methods

To depict the surety of the registration approach, it has been compared to some of the existing as well some contemporary methods. These methods include techniques such as affine, rigid and non-rigid registration methods including optical flow method [3], fluid-flow method, least square method [134], linear elasticity method [187], FFD method [164], direct KLD-based method [135], Crystal Dislocation Energy based method [190], diffeomorphic deformation & pair-wise alignment.
correspondence [183], shape-based multi-sensor image registration [198], and Demons based registration [200]. The comparison of these techniques with our proposed DSICNR approach is depicted through Table 4.3.

The DEM data sets have been subjected to being tested using the numerous above mentioned methods of registrations for evaluating the registration precision. The various constraints with respect to which comparison has been performed include - percentage of overlapping region, aggregate file sizes, data-hole sizes i.e., islands of missing data within these overlapping areas, and most commonly, with varying degree of noises. The precision of our proposed DSICNR registration approach has been subjected to visual inspection and also by applying statistical evaluations [199]. Also, the various sources of DEMs used for testing has been presented in the Table 4.3. For assessment of the efficacy of landform based detection method, work by Boriah et al., [178] has been informative. The registered DEM and reference DEM have been compared in the similarity metric tables.

Table 4.3: Comparative analysis of some method for registration with the proposed DSICNR approach using three parameters - SSD, CC, and MI.

<table>
<thead>
<tr>
<th>Method</th>
<th>SSD</th>
<th>NCC</th>
<th>MI</th>
<th>DEM sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only Affine registration</td>
<td>10536.42</td>
<td>0.21</td>
<td>1.49</td>
<td>SAR</td>
</tr>
<tr>
<td>Only rigid registration</td>
<td>10023.90</td>
<td>0.31</td>
<td>1.50</td>
<td>SAR</td>
</tr>
<tr>
<td>Using optical-flow method [3]</td>
<td>9652.00</td>
<td>0.52</td>
<td>1.60</td>
<td>SAR</td>
</tr>
<tr>
<td>Using Least square method [134]</td>
<td>7567.00</td>
<td>0.59</td>
<td>1.62</td>
<td>LIDAR</td>
</tr>
<tr>
<td>Using Linear elasticity method [187]</td>
<td>7344.56</td>
<td>0.63</td>
<td>1.68</td>
<td>LIDAR</td>
</tr>
<tr>
<td>Using FFD based method [164]</td>
<td>7032.59</td>
<td>0.65</td>
<td>1.68</td>
<td>LIDAR, QUICKBIRD</td>
</tr>
<tr>
<td>KLD-based method [135]</td>
<td>7221.00</td>
<td>0.64</td>
<td>1.66</td>
<td>QUICKBIRD</td>
</tr>
<tr>
<td>Crystal Dislocation Energy based method [190]</td>
<td>2050.40</td>
<td>0.90</td>
<td>1.90</td>
<td>QUICKBIRD</td>
</tr>
<tr>
<td>Diffeomorphic pair-wise correspondence [183]</td>
<td>4256.95</td>
<td>0.71</td>
<td>1.79</td>
<td>QUICKBIRD</td>
</tr>
<tr>
<td>Shape-based registration [198]</td>
<td>5871.41</td>
<td>0.70</td>
<td>1.82</td>
<td>SPOT, QUICKBIRD</td>
</tr>
<tr>
<td>Demon based registration [200]</td>
<td>2209.00</td>
<td>0.90</td>
<td>1.90</td>
<td>LIDAR, SPOT</td>
</tr>
<tr>
<td><strong>Proposed DSICNR approach</strong></td>
<td><strong>2043.68</strong></td>
<td><strong>0.90</strong></td>
<td><strong>1.90</strong></td>
<td><strong>SAR, LIDAR, SPOT, IKONOS, QUICKBIRD</strong></td>
</tr>
</tbody>
</table>

As can be found from the above table, there is a performance gain with respect to most of the compared techniques, and ranges between 10% and 60%, barring a few. In the cases where our
The proposed DSICNR approach is seen to have least performance gain, the approach gives almost the comparable performance.

**Table 4.4:** Comparative analysis of same set of method (as shown in Table 4.3) with the proposed DSICNR approach for registration on other parameters.

<table>
<thead>
<tr>
<th>Method</th>
<th>Max. Size</th>
<th>Type of Registration</th>
<th>Min. Overlap Area</th>
<th>Noise type for Robustness checking</th>
<th>Number of iterations</th>
<th>Data Hole Size</th>
<th>Success for Data Holes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only Affine registration</td>
<td>150 x 150 x 150</td>
<td>Mono-modal</td>
<td>86%</td>
<td>Gaussian Noise, 0 mean, range ±2</td>
<td>29</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Only rigid registration</td>
<td>150 x 150 x 150</td>
<td>Mono-modal</td>
<td>83%</td>
<td>Gaussian Noise, 0 mean, range ±2</td>
<td>35</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Using optical-flow method [3]</td>
<td>150 x 150 x 150</td>
<td>Mono-modal, multi-viewpoint</td>
<td>70%</td>
<td>Gaussian Noise, 0 mean, range ±2</td>
<td>149</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Using Least square method [134]</td>
<td>300 x 300 x 300</td>
<td>Multi-modal, multi-temporal</td>
<td>60%</td>
<td>S&amp;P, Gaussian Noise, 0 mean, range ±2</td>
<td>196</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Using Linear elasticity method [187]</td>
<td>150 x 150 x 150</td>
<td>Multi-modal, multi-temporal</td>
<td>60%</td>
<td>S&amp;P, Gaussian Noise, 0 mean, range ±2</td>
<td>137</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Using FFD based method [164]</td>
<td>600 x 600 x 600</td>
<td>Multi-modal, multi-temporal</td>
<td>60%</td>
<td>S&amp;P, Gaussian Noise, 0 mean, range ±2</td>
<td>412</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>KLD-based method [135]</td>
<td>600 x 600 x 600</td>
<td>Multi-modal, multi-temporal</td>
<td>53%</td>
<td>S&amp;P, Gaussian Noise, 0 mean, range ±5</td>
<td>303</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Crystal Dislocation Energy based method [190]</td>
<td>1000 x 1000 x 1000</td>
<td>Multi-modal, multi-viewpoint</td>
<td>45%</td>
<td>S&amp;P, Gaussian Noise, 0 mean, range ±5</td>
<td>985</td>
<td>± 20 pts</td>
<td>Yes</td>
</tr>
<tr>
<td>Diffeomorphic pair-wise correspondence [183]</td>
<td>600 x 600 x 600</td>
<td>Multi-modal, multi-temporal</td>
<td>55%</td>
<td>S&amp;P, Gaussian Noise, 0 mean, range ±7</td>
<td>572</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Shape-based</td>
<td>900 x 900 x 900</td>
<td>Multi-modal</td>
<td>50%</td>
<td>S&amp;P, Gaussian</td>
<td>743</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
From the experimentation, as deduced from tables, Table 4.3, and Table 4.4, it is seen that our proposed DSICNR registration approach is accomplished effectively and in a better fashion.

### 4.4.3 Search for suitable candidate DEM when only the reference DEM is given.

When the system is initiated, after noise filtering, it checks the inputs given for registration. In the cases where candidate DEM is not provided, the system undergoes ‘Coarse Registration’ phase as was discussed in the earlier sections. As the system is capable of finding a set of suitable candidates for registration with the reference DEM, it chooses from the existing pool of DEMs. This choice is dependent not only to the proximity to the histograms of the reference and probable candidate DEMs but also to the coarse geography and topography of the DEMs along with using their latitude and longitudinal data. Following this, a coarse segmentation based matching has been performed which gives a set of much suitable candidate DEMs which is ordered as per the percentage of common area matched. Table 4.5 shows one such example of finding the probable candidate DEMs to a given reference DEM. Also shown are the percentages of overlapping regions being the probable set of candidate DEMs to the given reference DEM. Also, the final registration results too are shown in the same table. In this example, the set of probable candidate DEMs is presented in the descending order of the overlapped or common areas. False positives have been few and within the experimental assumptions.
Table 4.5: Probable candidate DEM data files.

<table>
<thead>
<tr>
<th>Reference DEM</th>
<th>Probable candidate images</th>
<th>Overlapping area</th>
<th>Candidate DEM</th>
<th>Registered DEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Candidate 1</td>
<td>&lt;7%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Candidate 2</td>
<td>10 – 15%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Candidate</td>
<td>Percentage Range</td>
<td>Image</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>------------------</td>
<td>-------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15 – 20%</td>
<td><img src="image1.png" alt="Image" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>35 – 45%</td>
<td><img src="image2.png" alt="Image" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>50 - 60%</td>
<td><img src="image3.png" alt="Image" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.4.4 Experimentation with DEMs having Data Holes or Missing Data

As discussed in Chapter 1, DEMs are derived from various sources and hence are usually destined to have random errors and in some cases data-holes. These data-holes are regions of missing or null-valued data. Data-holes occur due to the inferencing problems of radar interferometry from INSAR. These may occur because of shadows, or numerous other properties of earth’s surface.

Using our proposed DSICNR approach successful mapping of the reference and candidate DEMs can be done even when data-holes occur within them. In the cases where the data-holes were present in only one of the input DEMs, before the final mapping function, those areas of identified data-holes were simply filled with their corresponding data from the other DEM having no holes in those regions. In the cases where such data-holes where present in the concurrent locations in both the candidate and the reference DEMs, the mapping function considered these as boundary edges and surfaces and the mapping function circumvented them. Table 4.6 shows some of these cases. Candidate DEMs with various amounts of data holes are shown in the table’s column 2. For such cases, it has been assumed that the candidate and reference DEMs have at least 45% or above regions as overlapping areas.
Table 4.6: Reference and Candidate and final registered DEM having data holes.

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Reference DEM</th>
<th>Candidate DEM with certain missing values or data holes</th>
<th>Registered DEM.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1" alt="Reference DEM" /></td>
<td><img src="image2" alt="Candidate DEM" /></td>
<td><img src="image3" alt="Registered DEM" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set 2</th>
<th>Reference DEM</th>
<th>Candidate DEM with certain missing values or data holes</th>
<th>Registered DEM.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image4" alt="Reference DEM" /></td>
<td><img src="image5" alt="Candidate DEM" /></td>
<td><img src="image6" alt="Registered DEM" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set 3</th>
<th>Reference DEM</th>
<th>Candidate DEM with certain missing values or data holes</th>
<th>Registered DEM.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image7" alt="Reference DEM" /></td>
<td><img src="image8" alt="Candidate DEM" /></td>
<td><img src="image9" alt="Registered DEM" /></td>
</tr>
</tbody>
</table>

### 4.4.5 Extension of DSICNR for Registration of Multiple candidate DEMs

For implementing a symmetric and inverse-consistent diffeomorphic registration, we have relied on the use of B-spline surface for the basic model. These surfaces can be easily patched together,
rendering its extension to multiple surfaces. Our proposed DSICNR approach for DEM registration too has been shown to be extendable for more than one candidate DEM data files for registration with a single reference DEM data file and with varying degrees of overlapping regions. However, the only constraint is having an existence of if not a sizeable but a small and accountable percentage of overlapping region in all the DEMs given for registration. In Table 4.7, we have shown the resultant set for registration of two candidate DEMs with a single reference DEM. Three example DEM data sets have been shown as test cases.

<table>
<thead>
<tr>
<th>Table 4.7: Registration of a reference DEM with two candidate DEM data files.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>Set1</td>
</tr>
<tr>
<td>Set 2</td>
</tr>
</tbody>
</table>
4.5. SUMMARY

In this chapter we have presented a novel technique for non-rigid registration of pairs of candidate and reference DEM data sets. These data sets have been cross-verified with their standards, i.e., the reference DEM data. We began by formulating the correspondence mapping of residuals of candidate and reference DEM data sets based on symmetric difference of the DEMs and then identifying their substantive overlapping areas by means of fast waterfall model. These overlapping regions have been, then, subjected to non-rigid diffeomorphic registration as considered in our proposed approach.

Primarily, the algorithm uses a non-rigid deformation for the local minimization of the cost functions. After the global transformation, the DMS-spline FFD models have been used for locally deforming the candidate DEM(s). This is followed by the actual registration of the candidate DEM(s) with the reference DEM. We have opted to use a constrain-optimized number of control points so that it gives a good result set. The tradeoff, in this case is between the number of control points used and the various complexity issues. The effectiveness of the method has been tested based on the values of the minimum of KLD measure between the observed and expected joint class distribution of the registered and reference DEMs. We have also presented a comparison of some of the existing works of other authors with our proposed approach for DEM registration. The compared result sets has been shown to strengthen this finding.

Testing was done on the DEMs that were available from various sources as named in our work. Based on the experimentation done on DEMs, the proposed DSICNR approach has been found to be much effective as compared to some of the existing methods.
Alternately, we may state that our main contributions are as follows: (i) developing an efficient and robust non-rigid registration framework for aligning partially overlapping DEM models; (ii) creating an adaptive deformation model that allows for stable deformations also for parts of the surface for which good correspondences may not be available due to major variations in their values or due to presence of holes; and (iii) comprehensiveness of the model to work even on partially incomplete, i.e., data sets containing data holes and noisy data sets.

We have also used optimized regularization so as to have an accurate and smooth warping. Inaccurate mapping and overly constrained registration may happen due to over-regularization, convergence at poor local minima, or image folding. Also, incorrect registration may be resulted due to under-regularization. Thus, regularization has been shown to play a very crucial role in our proposed DSICNR registration technique.

In this chapter we have proposed a generic registration approach, called DSICNR, and, through experiments have demonstrated its usability. For the purpose of testing, we have used certain common evaluation parameters such as SSD, CC, and MI. Our work in comparison to other existing works has been shown to be quite generic in nature and compares well with them. We have been able to map small scale as well as large scale deformations through our proposed approach and have experimented on numerous terrain types too.

The various test cases have proven to be quite helpful in determining the goodness of the proposed DSICNR DEM registration approach. In cases, where the candidate DEM data file is not provided to the system, the system chooses from a given folder based on histogram-based matching. We have tested with data of varying sizes and varying noise types. As is depicted through our experimentation, the proposed DSICNR approach for the registration of DEMs has been successfully completed and gives favorable results when compared to existing techniques. The results are good even when the candidate DEM data file have data holes. The algorithm is extendable and is easily able to register successfully more than one candidate DEMs with a given reference DEM consecutively.