HALL EFFECT ON THERMAL STABILITY OF COUPLE STRESS FLUID PERMEATED WITH SUSPENDED PARTICLES

4.1.1 INTRODUCTION

Chandrasekhar [7] has given the theory of Bénard convection in a viscous, Newtonian fluid layer heated from below. Chandra [161] observed that in an air layer, convection occur at much lower gradients than predicted if the layer depth was less than 7mm, and called this motion, “Columnar instability”. However, for layers deeper than 10mm, a Bénard type cellular convection was observed. Thus, there is a contradiction between the theory and the experiment. The use of Boussinesq approximation has been made through out which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Sharma [162] has considered the effect of rotation and magnetic field on the thermal instability in compressible fluids. The fluid has been considered to be Newtonian in all the above studies while Scanlon and Segel [119] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because of the heat capacity of the pure fluid.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The theory of couple-stress fluids is proposed by Stokes [46]. One of the applications of the couple stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid. According to the theory of Stokes [46], couple stresses are found to appear in noticeable magnitude in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluid. Walicki and Walicka [51] modelled synovial fluid as a couple stress fluids in human joints. Goel et. al. [163] has studied the hydromagnetics stability of an unbounded couple stress binary fluid mixture having vertical temperature and concentration gradients with rotation. An electrically conducting couple stress fluid heated
from below in a porous medium in the presence of uniform horizontal magnetic field has also been submitted by Sharma and Sharma [164]. The use of magnetic field is being made for the clinically purposes in detection and cure of certain diseases with the help of magnetic field devices/ instruments. The problem of a couple stress fluid heated from below in a porous medium is considered by Sharma and Sharma [165] and Sharma and Thakur [166].

Recent space craft observations have confirmed that the suspended particles play a significant role in the dynamics of the atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather. Further, environmental pollution is the main cause of suspended particles to enter the human body. The metal dust which filters into the blood stream of those working near furnace causes extensive damage to the chromosomes and genetic mutation so observed are likely to breed censer as malformations in the coming progeny. Therefore, it is very essential to study the blood flow with suspended particles. Considering blood as couple stress fluid and suspended particles as micro-organisms, Rathod and Thippeswamy [167] have studied the gravity flow of pulsatile blood through closed regular inclined channel with microorganisms. Sunil et. al. [168] have studied the effect of suspended particles on couple stress fluid heated and soluted from below in a porous medium and found that suspended particles have destabilizing effect on the system.

If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. The tendency of electric current to flow across an electric field in the presence of magnetic field is called Hall effect. The Hall currents are likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. Sharma and Gupta [169] investigated the effect of Hall currents on thermosolutal instability of rotating plasma and established the destabilizing influence of Hall effects. Thermal instability of compressible finite larmor radius Hall plasma has been studied by Sharma and Sunil [170] in a porous medium. Sharma and Sharma [171] have studied the effect of suspended particles on couple stress fluid heated from below in the presence of rotation and magnetic field. Sherman and Sutton [172] have considered the effect of Hall currents on the efficiency of a magneto-hydro-fluid dynamics (MHD) generator while Gupta [101] has seen the effect of Hall currents on the thermal instability of electrically conducting fluid in the presence of uniform vertical magnetic field.
Singh and Dixit [173] have studied the effect of Hall currents on the thermal instability of a compressible couple stress fluid with suspended particles while Kumar and Kumar [174] have seen the combined effect of suspended particles, magnetic field and rotation on a couple stress fluid heated from below. Aggarwal and Makhija [175] have studied the thermal stability of couple stress fluid in the presence of magnetic field and rotation. The same authors [176] have also seen the combined effect of magnetic field and rotation on couple stress fluid heated from below in the presence of suspended particles. Aggarwal and Verma [177] have studied the effect of suspended particles, magnetic field and rotation on the thermal stability of a ferromagnetic fluid.

Since the couple stress fluid play a significant role in industrial applications, it would be of much interest to examine the stability conditions of couple stress fluid. Since the effect of Hall currents on couple stress fluid permeated with suspended particles seems to be uninvestigated so far, hence in this paper, we shall discuss the effect of Hall currents on thermal stability of couple stress fluid in the presence of suspended particles.

4.1.2 MATHEMATICAL FORMULATION

Consider a static state in which an incompressible, Stokes couple stress fluid layer of thickness \( d \) heated from below so that a uniform temperature and density at the bottom surface \( z = 0 \) are \( T_o, \rho_o \) respectively and at the upper surface \( z = d \) are \( T_d, \rho_d \) and a uniform adverse temperature gradient \( \beta = \left( \frac{dT}{dz} \right) \) is maintained and the layer is acted upon by the gravity field \( g(0,0,-g) \), a uniform magnetic field \( H(0,0,H) \) as shown in figure 4.1.1.

Let \( p, \rho, T, \alpha, v, \mu', k, \) and \( \vec{q}(u,v,w) \) denote respectively pressure, density, temperature, thermal coefficient of expansion, kinematic viscosity, couple-stress viscosity, thermal diffusivity and velocity of the fluid. \( \vec{q}_s(l,r,s) \) and \( N(x,t) \) denote the velocity and number density of suspended particles, respectively. \( K = 6\pi \eta' \), where \( \eta' \) is the particle radius, is a constant and \( \vec{x} = (x,y,z) \). Then the equations of motion, continuity and heat conduction of couple-stress fluid are

\[
\rho \frac{D\vec{v}}{Dt} = -\nabla p + \rho \vec{g} - \nabla \tau - \eta' \vec{v}'
\]

\[
\nabla \cdot \vec{v} = 0
\]

\[
\rho c_v \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla \cdot (\kappa \nabla T)
\]
\[
\frac{\partial \tilde{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p + g \alpha \theta - \left( \nabla \cdot \left( \frac{\mu'}{\rho_0} \tilde{q} \right) + \frac{KN_0}{\rho_0} (\tilde{q}_d - \tilde{q}) \right) + \frac{\mu_s}{4\pi \rho_0} \left[ (\nabla \times \tilde{h}) \times \tilde{H} \right],
\]

(4.1.1)

\[
\nabla \cdot \tilde{q} = 0,
\]

(4.1.2)

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force and pressure force on the particles are neglected. Inter particle reactions are ignored, for it is assumed that, distances between particles are quite large compared with their diameter. If \( mN \) is the mass of the particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

\[
mN_0 \frac{\partial \tilde{q}_d}{\partial t} = KN_0 (\tilde{q} - \tilde{q}_d),
\]

(4.1.3)

\[
\frac{\partial N}{\partial t} + \nabla \left( N \tilde{q}_d \right) = 0,
\]

(4.1.4)
Let $C_v$ and $C_{pt}$ denote the heat capacity of fluid at constant volume, heat capacity of the particles, respectively. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives

$$\left[ \frac{\partial}{\partial t} + (\bar{q} \cdot \nabla) \right] T + \frac{mNC_{pt}}{\rho_0 C_v} \left( \frac{\partial}{\partial t} + q_d \cdot \nabla \right) T = k_r \nabla^2 T. \tag{4.1.5}$$

The kinematic viscosity $\nu$, couple stress viscosity $\mu'$, thermal diffusivity $k_r$ and coefficient of thermal expansion $\alpha$ are all assumed to be constants. The Maxwell's equations in the presence of Hall currents yield

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times (\bar{q} \times \vec{H}) + \eta \nabla^2 \vec{h} - \frac{C}{4\pi N' e} \nabla \times \left[ (\nabla \times \vec{h}) \times \vec{H} \right], \tag{4.1.6}$$

$$\nabla \cdot \vec{h} = 0, \tag{4.1.7}$$

where $\eta, N'$ and $e$ stand for the electrical resistivity, electron number density and the charge of an electron.

The equation of state for the fluid is

$$\rho = \rho_0 [1 - \alpha(T - T_0)]. \tag{4.1.8}$$

The basic motionless solution is

$$q = (0,0,0), \quad q_d = (0,0,0), \quad T = T_0 - \beta z, \quad \rho = \rho_0 (1 + \alpha \beta z), \quad N = N_0 \text{ constant}. \tag{4.1.9}$$

Assume small perturbations around the basic solution and let $\delta \rho, \delta \rho, \theta$ and $h(h_x, h_y, h_z)$ denote respectively the perturbations in fluid pressure $p$, density $\rho$, temperature $T$ and magnetic field $\vec{H}$. The change in density $\delta \rho$ caused mainly by the perturbation $\theta$ in temperature is given by

$$\delta \rho = -\alpha \rho_0 \theta. \tag{4.1.10}$$

Then the linearized perturbation equations of couple stress fluid become

$$\frac{\partial \bar{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta \rho + g \alpha \theta - \left( \nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \bar{q}$$

$$+ \frac{KN_0}{\rho_0} (\bar{q}_d - \bar{q}) + \frac{\mu_e}{4\pi \rho_0} \left[ (\nabla \times \vec{h}) \times \vec{H} \right], \tag{4.1.11}$$

$$\nabla \cdot \bar{q} = 0, \tag{4.1.12}$$

$$mN_0 \frac{\partial \bar{q}_d}{\partial t} = KN_0 (\bar{q} - \bar{q}_d). \tag{4.1.13}$$
\[(1 + h_i) \frac{\partial \theta}{\partial t} = \beta (w + h_i s) + \kappa \nabla^2 \theta, \quad \text{(4.1.14)}\]

\[\nabla \vec{h} = 0, \quad \text{(4.1.15)}\]

\[\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{q} \times \vec{H}) + \eta \nabla^2 \vec{h} - \frac{C}{4\pi N_e} \nabla \times \left[ (\nabla \times \vec{h}) \times \vec{H} \right], \quad \text{(4.1.16)}\]

where \(\eta\) stands for electrical resistivity, \(\kappa = \frac{q}{\rho_0 C_v}\) and \(h_i = \frac{mN_0 C_p}{\rho_0 C_v}\).

Eliminating \(\vec{q}_d\) in equation (4.1.11) with the help of (4.1.13), writing the scalar components of resulting equations and eliminating \(u, v, h_x, h_y\) and \(\delta p\) between them, by using equation (4.1.12) and equation (4.1.15), we obtain:

\[\left( \frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \left[ \frac{\partial}{\partial t} \nabla^2 w - g \alpha \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \frac{\mu H}{4\pi \rho_0} \frac{\partial}{\partial z} \nabla^2 h_z \right] + \frac{mN_0}{\rho_0} \frac{\partial}{\partial t} \nabla^2 w = \left( \frac{m}{k} \frac{\partial}{\partial t} + 1 \right) \left[ \frac{\rho_0 \nabla^2}{w} - \frac{\mu'}{\rho_0} \nabla^2 \right] \nabla^2 w, \quad \text{(4.1.17)}\]

\[\left( H + \frac{\partial}{\partial t} - \kappa \nabla^2 \right) \theta = \beta \left( \frac{m}{k} \frac{\partial}{\partial t} + H_i \right) w, \quad \text{(4.1.19)}\]

\[\left( \frac{\partial}{\partial t} - \eta \nabla^2 \right) h_z = H \frac{\partial w}{\partial z} - \frac{cH}{4\pi N'e} \frac{\partial \zeta}{\partial z}, \quad \text{(4.1.20)}\]

\[\left( \frac{\partial}{\partial t} - \eta \nabla^2 \right) \zeta = H \frac{\partial \zeta}{\partial z} + \frac{cH}{4\pi N'e} \frac{\partial h_z}{\partial z}. \quad \text{(4.1.21)}\]

### 4.1.3 Normal Mode Analysis

Analyzing the disturbances into normal modes and assume that the perturbation quantities are of the form

\[ [w, h_z, \theta, \zeta, \xi] = [W(z), K(z), \Theta(z), Z(z), X(z),] \exp(ik_x x + ik_y y + \alpha), \quad \text{(4.1.22)}\]

where \(k_x, k_y\) are wave numbers along \(x\) and \(y\) directions respectively, \(k(= \sqrt{k_x^2 + k_y^2})\) is the resultant wave number of the disturbances and \(\alpha\) is the growth rate.

Using expression (4.1.22), equations (4.1.17-4.1.21) in non-dimensional form become
\[
\left[ \sigma' + F(D^2 - a^2) - 1 \right] (D^2 - a^2) W = -\frac{g \alpha a^2 d^2}{\nu} \Theta + \frac{\mu_e H d}{4 \pi \rho_0 \nu} (D^2 - a^2) DK,
\] (4.1.23)

\[
\left[ \sigma' - d^2 \left[ 1 - F(D^2 - a^2) \right] \right] Z = \frac{\mu_e H d}{4 \pi \rho_0 \nu} DX,
\] (4.1.24)

\[
(D^2 - a^2 - H_1 p_1 \sigma) \Theta = \frac{\beta d^2}{\kappa} \left( H_1 + \frac{\tau_1 v_0}{\kappa} \right) W,
\] (4.1.25)

\[
(D^2 - a^2 - p_2 \sigma) K = -\frac{H d}{\eta} DW + \frac{c H d}{4 \pi N' e \eta} DX,
\] (4.1.26)

\[
(D^2 - a^2 - p_2 \sigma) X = -\frac{H d}{\eta} DZ + \frac{c H}{4 \pi N' e \eta d} (D^2 - a^2) DK,
\] (4.1.27)

where, we have non-dimensionalized various parameters as follows

\[
a = kd, \quad \sigma = \frac{nd^2}{\nu}, \quad \tau_1 = \frac{\tau_0}{d^2}, \quad p_1 = \frac{v_0}{\kappa}, \quad p_2 = \frac{v_0}{\eta},
\]

\[
F = \frac{\mu'}{\nu \rho_0 d^2}, \quad \sigma' = \frac{n' d^2}{\nu}, \quad H_1 = 1 + h_1, \quad n' = n \left[ 1 + \frac{m N_0 K}{mn + K} \right].
\]

After eliminating \( \Theta, Z, X \) and \( K \) from equations (4.1.23)-(4.1.27), we obtain

\[
\left[ \sigma' - d^2 \left[ 1 - F(D^2 - a^2) \right] \right] (D^2 - a^2) W + \frac{Ra^2}{(D^2 - a^2 - H_1 p_1 \sigma)} \left( H_1 + \frac{\tau_1 v_0}{\kappa} \right) W
\]

\[
+ Q \left[ \begin{array}{c}
\frac{(D^2 - a^2 - p_2 \sigma) \left( \sigma' - d^2 \left[ 1 - F(D^2 - a^2) \right] \right) + Q D^2}{(D^2 - a^2 - p_2 \sigma) \left( \sigma' - d^2 \left[ 1 - F(D^2 - a^2) \right] \right)} \\
+ Q \left( D^2 - a^2 - p_2 \sigma \right) D^2 \\
- M \left( \sigma' - d^2 \left[ 1 - F(D^2 - a^2) \right] \right) (D^2 - a^2) D^2
\end{array} \right] DW = 0,
\] (4.1.28)

where

\[
Q = \frac{\mu_e H d^2}{4 \pi \rho_0 \nu \eta} \quad \text{is the Chandrasekhar number},
\]

\[
R = \frac{g \alpha \beta d^4}{u \kappa} \quad \text{is the thermal Rayleigh number},
\]

\[
M = \left( \frac{H}{4 \pi N' e \eta} \right) \quad \text{is the non-dimensional number accounting for Hall currents}.
\]

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Consider the case in which both the boundaries are free, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The boundary conditions, appropriate for the problem, are

\[ W = 0 = Z = \Theta \text{ and } D^2 W = 0, D^4 W = 0 \quad \text{at} \quad z = 0 \text{ and } z = 1. \quad (4.1.29) \]

The proper solution of equation (4.1.29) characterizing the lowest mode is

\[ z W = \sin \pi z \quad (4.1.30) \]

where \( z W \) is constant. Substituting the proper solution (4.1.30) in equation (4.1.28), we obtain the dispersion relation

\[
\frac{R_i x}{(1 + x + i H_1 p_1 \sigma_1)} \left( H_1 + \frac{\tau_1 \nu}{\kappa} \right)
= Q_1 \left[ \frac{(1 + x + i p_2 \sigma_1) [i \sigma' + (1 + F_1 (1 + x))] + Q_1}{(1 + x + i p_2 \sigma_1)^2 [i \sigma' + (1 + F_1 (1 + x))] + Q_1 (1 + x + i p_2 \sigma_1)} - M [i \sigma' + (1 + F_1 (1 + x))] (1 + x) \right].
\]

where, \( R_i = \frac{R}{\pi^2}, \quad i \sigma_1 = \frac{\sigma}{\pi^2}, \quad Q_1 = \frac{Q}{\pi^2} \) and \( F_1 = \pi^2 F \).

### 4.1.4 STATIONARY CONVECTION

At stationary convection, when the stability sets, the marginal state will be characterized by \( \sigma = 0 \). Thus, putting \( \sigma = 0 \) in equation (4.1.31), we get

\[
R_i = \frac{Q_1}{x H_1} \left[ \frac{(1 + x) [1 + F_1 (1 + x)] + Q_1}{(1 + x) [1 + F_1 (1 + x)] + Q_1 - M_1 [1 + F_1 (1 + x)]} \right]
+ \frac{(1 + x)^2 [1 + F_1 (1 + x)]}{x H_1}.
\]

The above relation expresses the modified Rayleigh number \( R_i \) as a function of the parameters \( Q_1, \ H_1, \ F_1, \ M_1 \) and dimensionless wave number \( x \). To study the effect of magnetic field, suspended particles, couple-stress and Hall currents, we examine the nature of \( \frac{dR_i}{dQ_1}, \ \frac{dR_i}{dH_1}, \ \frac{dR_i}{dF_1} \) and \( \frac{dR_i}{dM_1} \) analytically. Equation (4.1.32) gives...
\[
\frac{dR_1}{dQ_1} = \frac{1}{xH_1} \left[ \frac{(1+x)[1+F_i(1+x)] + Q_i}{(1+x)[1+F_i(1+x)] + Q_i - M_i[1+F_i(1+x)]} \right] + \frac{1}{xH_1} \frac{M_i Q_i[1+F_i(1+x)]}{[(1+x)[1+F_i(1+x)] + Q_i - M_i[1+F_i(1+x)]]^2}, \tag{4.1.33}
\]

which shows that magnetic field has a stabilizing or destabilizing effect according to \((1+x)[1+F_i(1+x)] + Q_i > M_i[1+F_i(1+x)]\). This result is also verified from figures 4.1.2 and 4.1.3. This result is also same as obtained by Aggarwal and Makhija [176] and Kumar and Kumar [174].

\[
\frac{dR_1}{dH_1} = -\frac{Q_i}{xH_1^2} \left[ \frac{(1+x)[1+F_i(1+x)] + Q_i}{(1+x)[1+F_i(1+x)] + Q_i - M_i[1+F_i(1+x)]} \right] - \frac{(1+x)^2[1+F_i(1+x)]}{xH_1^2}, \tag{4.1.34}
\]

which clearly shows that suspended particles have destabilizing effect on thermal stability in a couple stress fluid. This result is also evident from figures 4.1.4 and 4.1.5 and is same as observed by Aggarwal and Makhija [176], Kumar and Kumar [174] and Singh and Dixit [173].

\[
\frac{dR_1}{dF_i} = \frac{Q_i(1+x)}{xH_1} \left[ \frac{(1+x)(1+F_i(1+x)) + Q_i - M_i[1+F_i(1+x)]}{(1+x)[1+F_i(1+x)] + Q_i - M_i[1+F_i(1+x)]^2} + 1 \right]. \tag{4.1.35}
\]

From equation (4.1.35), it follows that couple stress has stabilizing effect on couple stress fluid which is clear from figure 4.1.6 and 4.1.7.

\[
\frac{dR_1}{dM_1} = \frac{Q_i}{xH_1} \left[ \frac{(1+x)[1+F_i(1+x)] + Q_i}{[(1+x)[1+F_i(1+x)] + Q_i - M_i[1+F_i(1+x)]^2} \right]. \tag{4.1.36}
\]

From equation (4.1.36), we see that Hall currents have stabilizing effect on the system which is also evident from figure 4.1.8 and 4.1.9. This result is same as obtained by Singh and Dixit [173].
Figure 4.1.2 - Variation of $R_1$ and $Q_1$

Figure 4.1.3 - Variation of $R_1$ and $x$

Figure 4.1.4 - Variation of $R_1$ and $H_1$

Figure 4.1.5 - Variation of $R_1$ and $x$
Figure 4.1.6 - Variation of $R_1$ and $F_1$

Figure 4.1.7 - Variation of $R_1$ and $\lambda$

Figure 4.1.8 - Variation of $R_1$ and $M_1$

Figure 4.1.9 - Variation of $R_1$ and $\chi$
Table 4.1.1: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $H_1$.

<table>
<thead>
<tr>
<th>$H_1$</th>
<th>$M_1 = 5$, $Q_1 = 100$, $F_1 = 40$</th>
<th>$M_1 = 50$, $Q_1 = 20$, $F_1 = 2$</th>
<th>$M_1 = 100$, $Q_1 = 100$, $F_1 = 4$</th>
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<tr>
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<td>$x_c$</td>
<td>$R_c$</td>
<td>$x_c$</td>
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<tr>
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<td>0.90</td>
</tr>
<tr>
<td>70</td>
<td>0.85</td>
<td>51295.56</td>
<td>0.80</td>
</tr>
<tr>
<td>90</td>
<td>0.80</td>
<td>47035.85</td>
<td>0.65</td>
</tr>
<tr>
<td>110</td>
<td>0.70</td>
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<td>0.55</td>
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<tr>
<td>130</td>
<td>0.65</td>
<td>37399.18</td>
<td>0.45</td>
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Table 4.1.2: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $F_1$.

<table>
<thead>
<tr>
<th>$F_1$</th>
<th>$H_1 = 0.06$, $Q_1 = 300$, $M_1 = 30$</th>
<th>$H_1 = 0.06$, $Q_1 = 350$, $M_1 = 30$</th>
<th>$H_1 = 0.06$, $Q_1 = 400$, $M_1 = 30$</th>
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<td>14.31</td>
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Table 4.1.3: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $M_1$.

<table>
<thead>
<tr>
<th>$M_1$</th>
<th>$H_1 = 300$, $Q_1 = 50, F_i = 20$</th>
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</tr>
</tbody>
</table>

4.1.5 STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Now to determine under what conditions the principle of exchange of stabilities (PES) is satisfied (i.e. $\sigma$ is real and the marginal states are characterized by $\sigma = 0$) and the oscillations come into play, we multiply equation (4.1.23) with $W^+$ and integrate over the range of $z$ and making use of equations (4.1.24 - 4.1.27) together with the boundary conditions (4.1.29) and then we get

$$
(1 - \sigma')I_1 - FI_2 + \frac{ga\kappa d^2}{\beta U} \left( I_3 + H_1 p_1 \sigma^* I_4 \right) - \frac{\mu \eta}{4\pi\rho_o U} \left( I_5 + p_2 \sigma^* I_6 \right) + \frac{\mu \eta d^2}{4\pi\rho_o U} \left( I_7 + p_2 \sigma^* I_8 \right) + d^2 \left[ (\sigma' - 1)I_9 - FI_{10} \right] = 0
$$

(4.1.37)

where

$$
I_1 = \int_0^1 \left( |D^2 |^2 + \alpha^2 |W|^2 \right) dz,
I_2 = \int_0^1 \left( |D^2 W|^2 + 2\alpha^2 |D W|^2 + \alpha^4 |W|^2 \right) dz,
$$
\[ I_3 = \int_0^1 \left( |D\Theta|^2 + a^2|\Theta|^2 \right) dz, \quad I_4 = \int_0^1 |\Theta|^2 dz, \]
\[ I_5 = \int_0^1 \left( |D^2 K|^2 + 2a^2|DK|^2 + a^4|K|^2 \right) dz, \quad I_6 = \int_0^1 (|DK|^2 + a^2|K|^2) dz, \]
\[ I_7 = \int_0^1 (|DX|^2 + a^2|x|^2) dz, \quad I_8 = \int_0^1 |x|^2 dz, \]
\[ I_9 = \int_0^1 |Z|^2 dz, \quad I_{10} = \int_0^1 (|DZ|^2 + a^2|Z|^2) dz \]

and \( \sigma^* \) is complex conjugate of \( \sigma \). The integrals \( I_1 - I_{12} \) are all positive definite. Putting \( \sigma = i\sigma_i \) (\( \sigma^* = -i\sigma_i \)) in equation (4.1.37) and equating imaginary parts, we obtain
\[ \sigma_i \left[ I_1 + \frac{g\alpha k^2 a^2}{\beta \tau_i \Omega^2} I_3 + \frac{g\alpha \kappa a^2}{\beta \Omega} p_1 I_4 - \frac{\mu \eta}{4\pi \rho_\Omega} p_2 I_6 + \frac{\mu \eta d^2}{4\pi \rho_\Omega} p_2 I_8 - d^2 I_9 \right] = 0. \]

(4.1.38)

It is clear from equation (4.1.38) that \( \sigma_i \) may be zero or non-zero, which implies that modes may be non-oscillatory or oscillatory. In the absence of magnetic field (hence Hall currents) and suspended particles, equation (4.1.38) reduces to
\[ \sigma_i \left[ I_1 + \frac{g\alpha k^2 a^2}{\beta \tau_i \Omega^2} I_3 + \frac{g\alpha \kappa a^2}{\beta \Omega} p_1 I_4 + \frac{\mu \eta d^2}{4\pi \rho_\Omega} p_2 I_8 \right] = 0. \]

(4.1.39)

The terms in the bracket are positive definite. Thus \( \sigma_i = 0 \) means that the oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied in the absence of magnetic field (hence Hall currents) and suspended particles.

4.1.6 CONCLUSIONS

The effect of Hall currents and suspended particles has been considered on the thermal stability of a couple-stress fluid. The effect of various parameters such as magnetic field, suspended particles, couple-stresses and Hall currents has been investigated analytically as well as graphically. The principal conclusions from the analysis are:
1. It is found that magnetic field has a stabilizing or destabilizing effect according to \( (1 + \chi)[1 + F_i(1 + \chi)] + Q_i > 0 \) or \( M_i[1 + F_i(1 + \chi)] \). This result is also verified from figure 4.1.2 and 4.1.3.
2. The suspended particles have destabilizing effect on the system.
3. The couple stress and Hall currents have stabilizing effect on the system.
4. The critical Rayleigh numbers and the associated wave numbers are found for stationary convection for various parameters involved and it has been found that it increases with the increase in magnetic field, couple stresses and Hall currents parameter and decreases with the increase in suspended particles parameter thereby confirming the stabilizing role of magnetic field, couple stresses and Hall currents parameter and destabilizing role of suspended particles.
5. The principle of exchange of stabilities is satisfied in the absence of magnetic field (hence Hall currents) and suspended particles.
4.2.1 INTRODUCTION

Chandrasekhar [7] has given the detailed account of the theoretical and experimental results on the onset of thermal stability (Bénard Convection) in an incompressible, viscous Newtonian fluid layer under varying assumptions of hydrodynamics and hydromagnetics. Veronis [178] has investigated the thermosolutal convection in a layer of fluid heated from below and subjected to a stable solute gradient, the solute being salt.

The physics is quite similar in the stellar case in that Helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries and therefore, it is desirable to consider a fluid acted on by solute gradient and free boundaries.

When a current carrying conductor is placed into a magnetic field, a voltage will be generated perpendicular to both the current and the field. The principle is known as the Hall effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasmas. Sherman and Sutton [172] have considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta [101] studied the thermal instability of fluid in the presence of Hall currents. The thermosolutal convection problems arise in oceanography, limnology and engineering. In all the above studies, the fluid is considered to be Newtonian.

There is a growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Sharma and Rani [179] have considered the effect of suspended particles on thermosolutal convection in the presence of porous medium and found that suspended particles and medium permeability have destabilizing effect on the system.

There are many elastico-viscous fluids that cannot be characterized by these constitutive relations. Two such classes of fluids are Rivlin-Ericksen and Walters’ (model B’) fluid. Walters [180] has proposed the constitutive equations for such elastico-viscous fluids. The mixture of polymethyl methacrylate and pyride at $25^0C$ containing 30.5 gm of...
polymer per litre behaves very nearly as the Walters’ (model B’) visco-elastic fluid and which is proposed by Walters [181]. Rivlin and Ericksen [45] have proposed a theoretical model of such another elastico-viscous fluid and other polymers are used in agriculture, communication appliances and in biomedical applications. Johri [182] has discussed the visco-elastic Rivlin-Ericksen incompressible fluid under time-dependent pressure gradient. Srivastava and Singh [183] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channel of different cross-sections in the presence of time-dependent pressure gradient. Sharma and Kishor [184] have studied the Hall effect on thermosolutal instability of Rivlin-Ericksen fluid with varying gravity field in porous medium. Sunil et al. [185] have seen thermosolutal instability of compressible Rivlin-Ericksen fluid with Hall currents. Aggarwal [186] have considered the effect of rotation on thermosolutal convection in a Rivlin-Ericksen fluid permeated with suspended particles in porous medium. Gupta and Sharma [187] have considered the effect of Hall currents and rotation on stability of a layer of compressible Rivlin-Ericksen fluid heated and soluted from below. Rana and Kumar [188] have seen the thermal instability of Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended particles under variable gravity field in porous medium. Gupta and Sharma [44] have studied the thermosolutal effect of compressibility, rotation, magnetic field and Hall currents on the Rivlin-Ericksen elastico-viscous fluid

When the fluid permeates a porous material, the gross effect is represented by the Darcy’s law. As a result of this macroscopic law, the usual viscous term in the equations of Rivlin-Ericksen elastico-viscous fluid motion is replaced by \( -\left( \frac{1}{k_1} \right) \left( \mu + \mu' \frac{\partial}{\partial t} \right) \), where \( \mu \) and \( \mu' \) are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, \( k_1 \) is the medium permeability and \( q \) is the Darcian (filter) velocity of the fluid. The problem of thermosolutal stability in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consists of a dusty ‘snowball’ of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The effect on Hall currents on thermosolutal stability in porous medium is likely to be important in many astrophysical situations and atmospheric physics.

In the present section, we have extended the results reported by Gupta and Sharma [44] to include the effect of suspended particles in porous medium.
4.2.2 FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Here we consider an infinite, horizontal, compressible electrically conducting Rivlin-Ericksen elasto-viscous fluid layer of thickness \(d\), heated and soluted from below, so that the temperatures, densities and solute concentrations at the bottom surface \(z=0\) are \(T_0, \rho_0\) and \(C_0\) and at the upper surface \(z=d\) are \(T_d, \rho_d\) and \(C_d\) respectively, in a porous medium and that a uniform temperature gradient \(\beta = \left| \frac{dT}{dz} \right|\) and a uniform solute gradient \(\beta' = \left| \frac{dC}{dz} \right|\) are maintained. The gravity field \(\vec{g}(0,0,-g)\) and a uniform vertical magnetic field \(\vec{H}(0,0,H)\) pervade the system as shown in figure 4.2.1.

Let \(p, \rho, T, C, \alpha, \alpha', g, \eta, \mu, N, e\) and \(\vec{q}(u, v, w)\) denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron and fluid velocity. Then the equations of motion and continuity governing the flow are
Here we have assumed uniform size of fluid particles, spherical shape and small relative velocities between the fluid and particles. Then the net effect of the suspended particles on the fluid through porous medium is equivalent to an extra body force term per unit volume \( \frac{KN}{\varepsilon} (\bar{q}_d - \bar{q}) \). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion of particles. The distances between particles are assumed to be so large compared with their diameter that interparticle reactions need not be accounted for. The effects of pressure, gravity and Darcian force on the suspended particles (assumed large distances apart) are negligibly small and therefore ignored. If \( mN \) is the mass of particles, under the above assumptions, are

\[
mN \left\{ \frac{\partial \bar{q}_d}{\partial t} + \frac{1}{\varepsilon} (\bar{q}_d \cdot \nabla) \bar{q}_d \right\} = KN (\bar{q} - \bar{q}_d),
\]

(4.2.3)

\[
\varepsilon \frac{\partial N}{\partial t} + \nabla (N \bar{q}_d) = 0.
\]

(4.2.4)

Since the volume fraction of the particles is assumed small, the effective properties of the suspension are taken to be those of the pure (clean) fluid. If we assume that the particles and the fluid are in thermal and solute equilibrium, then the equation of heat and solute conduction gives

\[
[p\varepsilon + \rho_c c_i (1 - \varepsilon)] \frac{\partial T}{\partial t} + \rho_c (\bar{q} \cdot \nabla) T + mN c_{\text{pr}} \left( \varepsilon \frac{\partial}{\partial t} + \bar{q}_d \cdot \nabla \right) T = q \nabla^2 T ,
\]

(4.2.5)

\[
[p\varepsilon + \rho_c c_i (1 - \varepsilon)] \frac{\partial C}{\partial t} + \rho_c (\bar{q} \cdot \nabla) C + mN c_{\text{pr}} \left( \varepsilon \frac{\partial}{\partial t} + \bar{q}_d \cdot \nabla \right) C = q' \nabla^2 C .
\]

(4.2.6)

Since density variations are mainly due to variations in temperature and solute concentration, the equation of state for the fluid is given by

\[
\rho = \rho_0 \left[ 1 - \alpha (T - T_0) + \alpha' (C - C_0) \right],
\]

(4.2.7)
where the suffix zero refers to values at the reference level \( z = 0 \) and in writing equation (4.2.1), use has been made of the Boussinesq approximation. The magnetic permeability \( \mu_e \), the kinematic viscosity \( \mu \), the kinematic viscoelasticity \( \mu' \), the thermal diffusivity \( \kappa \) and the solute diffusivity \( \kappa' \) are all assumed to be constants. \( K' = 6\pi\eta' \), \( \eta' \) being particle radius, is the Stoke’s drag coefficient, \( \bar{x} = (x, y, z) \), \( E = \varepsilon + (1 - \varepsilon) \left( \frac{\rho_j C_j}{\rho_0 C_f} \right) \) is constant and \( E' \) is a constant analogous to \( E \) but corresponding to solute rather than heat; \( \rho_s \), \( \rho_0 \), \( C_s \) and \( C_f \) denote the density and heat capacity of solid (porous) matrix and fluid respectively, \( \bar{q}_d(x,t) \) denote filter velocity.

The Maxwell’s equations yield

\[
\frac{d\vec{H}}{dt} = (\vec{H}, \nabla) \hat{j} + \eta \nabla^2 \vec{H} - \frac{c}{4\pi Ne} \nabla \times \left[ (\nabla \times \vec{H}) \times \vec{H} \right],
\]

\( \nabla \cdot \vec{H} = 0, \) \hspace{1cm} (4.2.8)

where \( \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \) stands for the convective derivative.

The state variables pressure \( p \), density \( \rho \) and temperature \( T \) are expressed in the form (Spiegel and Veronis [116])

\[
f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t),
\]

where \( f_m \) is the constant space distribution of \( f \), \( f_0 \) is the variation in the absence of motion and \( f'(x, y, z, t) \) is the fluctuation resulting from motion.

The initial state is, therefore, a state in which the pressure, density, temperature, solute concentration, velocity and magnetic field at any point in the fluid are given by

\[
\bar{q} = (0, 0, 0), \quad \bar{q}_d = (0, 0, 0), \quad T = T_0 - \beta z, \quad C = C_0 - \beta' z, \quad \rho = \rho_0 (1 + \alpha \beta z - \alpha' \beta' z) \text{ and } N = N_0 = \text{constant}.
\]

(4.2.11)

Consider a small perturbation on the steady state solution and let \( \delta p \), \( \delta \rho \), \( \theta \), \( \gamma \), \( h(h_x, h_y, h_z) \) and \( \vec{q}(u, v, w) \) denote respectively the perturbations in fluid pressure \( p \), density \( \rho \), temperature \( T \), solute concentration \( C \) and magnetic field \( \vec{H}(0,0,H) \) and velocity \( \vec{q}(0,0,0) \). The change in density \( \delta \rho \) caused mainly by the perturbation \( \theta \) and \( \gamma \) in temperature and concentration is given by

\[
\delta \rho = -\rho_0 (\alpha \theta - \alpha' \gamma).
\]

(4.2.12)
Then the relevant linearized hydromagnetic perturbation equations become

\[
\frac{1}{\varepsilon} \frac{\partial \tilde{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p + g (\alpha \theta - \alpha' \gamma) \tilde{y} - \frac{1}{k_i} \left( v + v' \frac{\partial}{\partial t} \right) \tilde{q} + \frac{\mu_e}{4\pi \rho_0} \left( \nabla \times \tilde{h} \right) \times \tilde{H} + \frac{KN_0}{\rho_0 \varepsilon} \left( \tilde{q}_d - \tilde{q} \right),
\]  

(4.2.13)

\[ \nabla \tilde{q} = 0, \]  

(4.2.14)

\[ mN_0 \frac{\partial \tilde{q}_d}{\partial t} = KN_0 (\tilde{q} - \tilde{q}_d), \]  

(4.2.15)

\[ \varepsilon \frac{\partial N}{\partial t} + N_0 (\nabla \tilde{q}_d) = 0, \]  

(4.2.16)

\[ (E + \varepsilon \varepsilon) \frac{\partial \theta}{\partial t} = \left( \beta - \frac{\varepsilon}{C_p} \right) \left( w + hs \right) + \kappa \nabla^2 \theta, \]  

(4.2.17)

\[ (E' + \varepsilon \varepsilon) \frac{\partial \gamma}{\partial t} = \left( \beta - \frac{\varepsilon}{C_p} \right) \left( w + hs \right) + \kappa' \nabla^2 \gamma, \]  

(4.2.18)

\[ \nabla \tilde{h} = 0, \]  

(4.2.19)

\[ \varepsilon \frac{\partial \tilde{h}}{\partial t} = \nabla \times (\tilde{q} \times \tilde{H}) + \eta \varepsilon \nabla^2 \tilde{h} - \frac{e c}{4\pi N' e} \nabla \times \left[ (\nabla \times h) \times H \right]. \]  

(4.2.20)

Eliminating \( \tilde{q}_d \) between (4.2.13) and (4.2.15)

\[
\left[ \frac{1}{\varepsilon} \left( 1 + \frac{\varepsilon v'}{k_i} + \frac{mN_0}{\rho_0 (1 + \tau\frac{\partial}{\partial t})} \right) \frac{\partial}{\partial t} + \frac{v}{k_i} \right] \tilde{q} = -\frac{1}{\rho_0} \nabla \delta p + g (\alpha \theta - \alpha' \gamma) \tilde{y} + \frac{\mu_e}{4\pi \rho_0} \left( \nabla \times \tilde{h} \right) \times \tilde{H} \]  

(4.2.21)

Writing scalar components of equation (4.2.21), eliminating \( u, v \) and \( \delta p \) between them by using equations (4.2.14) and (4.2.17) – (4.2.20), we obtain

\[
\left[ \frac{1}{\varepsilon} \left( 1 + \frac{e v'}{k_i} + \frac{mN_0}{\rho_0 (1 + \tau\frac{\partial}{\partial t})} \right) \frac{\partial}{\partial t} + \frac{v}{k_i} \right] \nabla^2 w = g \left( \alpha \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \theta - \alpha' \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \gamma \right) + \frac{\mu_e H}{4\pi \rho_0} \frac{\partial^2 \tilde{h}_z}{\partial z^2}, \]  

(4.2.22)

\[
\left[ \frac{1}{\varepsilon} \left( 1 + \frac{e v'}{k_i} + \frac{mN_0}{\rho_0 (1 + \tau\frac{\partial}{\partial t})} \right) \frac{\partial}{\partial t} + \frac{v}{k_i} \right] \nabla^2 \xi = \frac{\mu_e H}{4\pi \rho_0} \frac{\partial \tilde{h}_z}{\partial z}, \]  

(4.2.23)
\[ (E + h\nu) \frac{\partial \Phi}{\partial t} = \beta \left( \frac{G-1}{G} \right) \left( \frac{1 + h}{1 + \tau_n} \right) \nu + \kappa \nabla^2 \Phi, \quad (4.2.24) \]
\[ (E' + h\nu) \frac{\partial \gamma}{\partial t} = \beta \left( \frac{G-1}{G} \right) \left( \frac{1 + h}{1 + \tau_n} \right) \nu + \kappa \nabla^2 \gamma, \quad (4.2.25) \]
\[ \varepsilon \frac{\partial h_z}{\partial t} = H \frac{\partial \omega}{\partial z} + \eta \nabla^2 h_z - \frac{\varepsilon cH}{4\pi N'e} \frac{\partial \xi}{\partial z}, \quad (4.2.26) \]
\[ \varepsilon \left[ \frac{\partial}{\partial t} - \eta \nabla^2 \right] \xi = H \frac{\partial}{\partial z} \zeta + \frac{\varepsilon cH}{4\pi N'e} \frac{\partial}{\partial z} \nabla^2 h_z, \quad (4.2.27) \]

where \( \zeta = \frac{\partial \nu}{\partial x} - \frac{\partial \mu}{\partial y} \) is the z-component of vorticity and \( \xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} \) is the z-component of current density.

### 4.2.3 NORMAL MODE ANALYSIS AND DISPERSION RELATIONS

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form
\[ [w, h_z, \theta, \gamma, \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), X(z)] \exp \left( ik_x x + ik_y y + m \right), \quad (4.2.28) \]
where \( k_x, k_y \) are wave numbers along x and y directions respectively, \( k = \sqrt{k_x^2 + k_y^2} \) is the resultant wave number of the disturbances and \( n \) is the growth rate which is, in general, a complex constant.

Using expression (4.2.28), equations (4.2.22) – (4.2.27) can be written as
\[ \left[ \frac{\sigma}{\varepsilon} \left\{ 1 + \frac{M}{1 + \tau_i \sigma} \right\} + \frac{1 + F\sigma}{P_1} \right] \left( D^2 - a^2 \right) W + \frac{ga^2 d^2}{v} (\alpha \Theta - \alpha' \Gamma) - \frac{\mu_c H d}{4\pi \rho_o v} (D^2 - a^2) DK = 0, \quad (4.2.29) \]
\[ \left[ \frac{\sigma}{\varepsilon} \left\{ 1 + \frac{M}{1 + \tau_i \sigma} \right\} + \frac{1 + F\sigma}{P_1} \right] Z = \frac{\mu_c H d}{4\pi \rho_o v} DX, \quad (4.2.30) \]
\[ \left( D^2 - a^2 - E_1 p_1 \right) \Theta = -\frac{\beta d^2}{\kappa} \left( \frac{G-1}{G} \right) \left( \frac{H_1 + \tau_i \sigma}{1 + \tau_i \sigma} \right) W, \quad (4.2.31) \]
\[ \left( D^2 - a^2 - E_2 p_1 \right) \Gamma = -\frac{\beta d^2}{\kappa'} \left( \frac{G-1}{G} \right) \left( \frac{H_1 + \tau_i \sigma}{1 + \tau_i \sigma} \right) W, \quad (4.2.32) \]
\[
(D^2 - a^2 - p_2\sigma)K = -\frac{Hd}{\varepsilon_1} DW + \frac{cHd}{4\pi N'\varepsilon_1} DX,
\]

\[
(D^2 - a^2 - p_2\sigma)X = -\frac{Hd}{\varepsilon_1} DZ - \frac{cH}{4\pi N'\varepsilon_1 d} (D^2 - a^2)DK,
\]

where we have non-dimensionalized various parameters as follows.

\[ a = kd, \quad \sigma = \frac{nd^2}{v}, \quad p_1 = \frac{v}{\kappa}, \quad p_2 = \frac{v}{\eta}, \quad F = \frac{v'}{d^2} \]

\[ H = 1 + h, \quad \tau = \frac{mk}{K'd^2}, \quad x^* = \frac{x}{d}, \quad y^* = \frac{y}{d}, \quad z^* = \frac{z}{d}, \quad D = \frac{d}{dz}. \]

Eliminating \( \Theta, \Gamma, Z, X, K \) among equations (4.2.29) – (4.2.34), we have

\[
(D^2 - a^2 - E_1 p_1 \sigma)(D^2 - a^2 - E_2 p_1' \sigma)(D^2 - a^2) \left[ \frac{\sigma}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_1 \sigma} \right) + \frac{1 + F\sigma}{P_1} \right] W + \]

\[ + \left( \frac{G - 1}{G} \right) \frac{M}{1 + \tau_1 \sigma} \left( \frac{H_1 + \tau_1 \sigma}{1 + \tau_1 \sigma} \right) \left[ S a^2 (D^2 - a^2 - E_1 p_1 \sigma) - R a^2 (D^2 - a^2 - E_2 p_1' \sigma) \right] - \]

\[ - \frac{(D^2 - a^2 - E_1 p_1 \sigma)(D^2 - a^2 - E_2 p_1' \sigma)(D^2 - a^2)}{(D^2 - a^2 - p_2 \sigma)} \times \]

\[ \left[ Q (D^2 - a^2 - p_2 \sigma) \left( \frac{\sigma}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_1 \sigma} \right) + \frac{1 + F\sigma}{P_1} \right) D^2 + D^3 Q \right. \]

\[ \left. - \frac{Q}{(D^2 - a^2 - p_2 \sigma) + \frac{Q}{\frac{\sigma}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_1 \sigma} \right) + \frac{1 + F\sigma}{P_1}}} + M_1 (D^2 - a^2)^2 D^2 \right] \]

\[ W = 0 \]

where

\[ Q = \frac{\mu H^2 d^3}{4\pi \rho_0 \varepsilon_1} \] is the Chandrasekhar number,

\[ R = \frac{g_{\alpha} \alpha' \beta d^4}{\kappa \kappa'} \] is the Thermal Rayleigh number,

\[ S = \frac{g_{\alpha'} \beta' d^4}{\kappa \kappa'} \] is the analogous solute Rayleigh number,

\[ M_1 = \left( \frac{cH}{4\pi N'\varepsilon_1} \right)^2 \] is the non-dimensional number accounting for Hall currents.
Consider the case in which both the boundaries are free as well as maintained at constant temperatures while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which the equations (4.2.29) – (4.2.34) must be solved, are

\[ W = D^2W = \Theta = \Gamma = DZ = 0 \quad \text{at} \quad z = 0 \text{ and } 1 \quad (4.2.36) \]

and \( h_x, h_y \) and \( h_z \) are continuous.

On the perfectly conducting boundaries \( DX = 0 \) and \( K = 0 \). The case of two free boundaries, though little artificial, the most appropriate case for stellar atmospheres (Spiegel and Veronis [116]). Using the above boundary conditions, it can be shown that all the even order derivatives of \( W \) must vanish for \( z = 0 \) and \( z = 1 \) and hence proper solution of equation (4.2.35) characterizing the lowest mode is

\[ W = W_0 \sin \pi z, \quad (4.2.37) \]

where \( W_0 \) is constant. Substituting the proper solution (4.2.38) in equation (4.2.35), we obtain the dispersion relation.

\[
R_1 = \frac{i \sigma_1}{\varepsilon} \left( \frac{1 + x + i E_1 p_i \sigma_i}{1 + x + i E_2 p_i' \sigma_i} \right) + \left( \frac{1}{x} \right) \left( \frac{G}{G - 1} \right) \left( 1 + x + i E_1 p_i \sigma_i \right) \left( 1 + x + i E_2 p_i' \sigma_i \right) + \frac{1 + i \pi^2 \tau_1 \sigma_i}{H_1 + i \pi^2 \tau_1 \sigma_i} \]

\[
\left[ \frac{1 + M}{1 + i \pi^2 \tau_1 \sigma_i} + \frac{1 + i \pi^2 F \sigma_i}{P} \right] \]

\[
Q_1 \left[ \frac{1 + x + i \sigma_1 \sigma_i}{1 + x + i \sigma_2 \sigma_i} \left( \frac{i \sigma_1}{\varepsilon} \left( 1 + \frac{M}{1 + i \pi^2 \tau_1 \sigma_i} \right) + \frac{1 + i \pi^2 F \sigma_i}{P} \right) \right] + 1 \]

\[
\left( 1 + x + i \sigma_2 \sigma_i \right) \left( 1 + x + i \sigma_2 \sigma_i \right) \left( i \sigma_1 \varepsilon \left( 1 + \frac{M}{1 + i \pi^2 \tau_1 \sigma_i} \right) + \frac{1 + i \pi^2 F \sigma_i}{P} \right) \]

\[
+ Q_1 + (1 + x) M \]

(4.2.38)

where

\[
R_1 = \frac{g \alpha \beta d^4}{\nu \kappa \pi^4}, \quad S_1 = \frac{g \alpha' \beta' d^4}{\nu' \kappa' \pi^4}, \quad Q_1 = \frac{\mu H^2 d^2}{4 \pi \rho_0 \nu \eta \pi^2}, \quad M = \left( \frac{c H}{4 \pi N \eta} \right)^2, \quad x = \frac{a^2}{\pi^2}
\]

and \( i \sigma_1 = \frac{\sigma_i}{\pi^2} \).
Equation (4.2.38) is the required dispersion relation including the effects of Hall currents and suspended particles on thermosolutal convection of compressible Rivlin-Ericksen fluid in the porous medium. This dispersion relation is identical with that derived by Sunil et al. [185] in the absence of suspended particles and porous medium. This relation is also in agreement with the result derived by Gupta and Sharma [44] in the absence of suspended particle and porous medium.

### 4.2.4 STATIONARY CONVECTION

When the stability sets in the form of stationary convection, the marginal state will be characterized by \( \sigma_1 = 0 \). Putting \( \sigma_1 = 0 \), the dispersion relation (4.2.38) reduces to

\[
R_1 = S_1 + \left( \frac{G}{G-1} \right) (1+x)^2 + \left( \frac{G}{G-1} \right) \left( \frac{1+x}{xPH_1} + \frac{Q_1 (1+x) + PQ_1}{(1+x) + PQ_1 + PM_1 (1+x)} \right),
\]

which expresses the modified Rayleigh number \( R_1 \) as a function of dimensionless wave number \( x \) and the parameters \( S_1, G, P, H, Q_1 \) and \( M_1 \). We thus find that for stationary convection the visco-elastic parameter vanishes with \( \sigma_1 \) and the Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid. This expression is same as given by Gupta and Sharma [44] in the absence of suspended particles and porous medium.

To investigate the effects of solute gradient, compressibility, suspended particles, Hall currents, magnetic field and porous medium, we examine the natures of \( \frac{dR_1}{dS_1}, \frac{dR_1}{dG}, \frac{dR_1}{dH_1}, \frac{dR_1}{dM_1}, \frac{dR_1}{dQ_1} \) and \( \frac{dR_1}{dP} \) analytically.

For analyzing the effect of solute gradient, equation (4.2.39) yields

\[
\frac{dR_1}{dS_1} = 1,
\]

which shows the stabilizing effect of solute gradient on thermal convection. This is in agreement with the result of figures 4.2.2 and 4.2.3.

For analyzing the effect of compressibility, from equation (4.2.39), we obtain

\[
\frac{dR_1}{dG} = -\frac{1}{(G-1)^2} \left( \frac{1+x}{xPH_1} \right) \left( 1+x \right) + \frac{Q_1 (1+x) + PQ_1}{(1+x) + PQ_1 + PM_1 (1+x)}.
\]
The negative sign implies that compressibility has a destabilizing effect on the system. Figures 4.2.4 and 4.2.5 confirm the above result numerically for the permissible range of values of various parameters.

Expression for observing suspended particles is obtained as

$$\frac{dR_1}{dH_1} = -\left(\frac{G}{G-1}\right)\left(1 + \frac{P}{G}xPH_1\right)\left(1 + \frac{Q}{(1+x) + PQ_1 + PM_1(1+x)}\right),$$

(4.2.42)

which reflects the destabilizing influence of suspended particles on thermosolutal stability of Rivlin-Ericksen fluid. Also in figures 4.2.6 and 4.2.7, $R_1$ decreases with increase in $H_1$ which confirms the above result numerically.

The effect of Hall currents can be seen by following expression

$$\frac{dR_1}{dM_1} = -\left(\frac{G}{G-1}\right)\left(1 + \frac{P}{G}xPH_1\right)\left(1 + \frac{Q}{(1+x) + PQ_1 + PM_1(1+x)}\right)^2,$$

(4.2.43)

which shows the destabilizing effect of Hall currents on thermal stability. Figure 4.2.8 and 4.2.9 are in agreement with the above result for different values of various parameters.

For analyzing the effect of magnetic field and porous medium, equation (4.2.39) yield

$$\frac{dR_1}{dQ_1} = \left(\frac{G}{G-1}\right)\left(1 + \frac{P}{G}xPH_1\right)\left[\frac{1}{1 + \frac{P}{(1+x) + PQ_1 + PM_1(1+x)} + \frac{1}{(1+x) + PQ_1 + PM_1(1+x)}^2}\right],$$

(4.2.44)

$$\frac{dR_1}{dP} = -\left(\frac{G}{G-1}\right)\left(1 + \frac{P}{G}xPH_1\right)\left[\frac{Q_1(1+x) + PQ_1 + PM_1(1+x)}{1 + \frac{P}{(1+x) + PQ_1 + PM_1(1+x)} + \frac{1}{(1+x) + PQ_1 + PM_1(1+x)}^2}\right],$$

(4.2.45)

which shows the stabilizing effect of magnetic field and destabilizing effect of porous medium respectively on thermal convection. These are in agreement with figures 4.2.10 and 4.2.11.
Figure 4.2.2: Variation of $R_1$ with $S_1$

Figure 4.2.3: Variation of $R_1$ with $x$

Figure 4.2.4: Variation of $R_1$ with $G$

Figure 4.2.5: Variation of $R_1$ with $x$
Figure 4.2.6: Variation of $R_1$ with $H_1$

Figure 4.2.7: Variation of $R_1$ with $x$  

Figure 4.2.8: Variation of $R_1$ with $M_1$

Figure 4.2.9: Variation of $R_1$ with $x$
**Figure 4.2.10**: Variation of $R_1$ with $Q_1$

**Figure 4.2.11**: Variation of $R_1$ with $x$

**Figure 4.2.12**: Variation of $R_1$ with $P$

**Figure 4.2.13**: Variation of $R_1$ with $x$
**Table 4.2.1:** Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $S_1$.

| $S_1$ | $P = 0.01, H_1 = 0.1,$  
$G = 2, Q_1 = 0.1,$  
$M_1 = 0$ | $P = 0.01, H_1 = 5,$  
$G = 100, Q_1 = 0.1,$  
$M_1 = 0$ | $P = 0.001, H_1 = 10,$  
$G = 100, Q_1 = 0,$  
$M_1 = 0$ |
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<td>$x_c$</td>
<td>$R_c$</td>
</tr>
<tr>
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<td>1.10</td>
<td>890.16</td>
</tr>
<tr>
<td>100</td>
<td>1.11</td>
<td>940.16</td>
</tr>
<tr>
<td>150</td>
<td>1.12</td>
<td>990.16</td>
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<tr>
<td>200</td>
<td>1.13</td>
<td>1040.16</td>
</tr>
<tr>
<td>250</td>
<td>1.14</td>
<td>1090.16</td>
</tr>
</tbody>
</table>

**Table 4.2.2:** Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $G$.

| $G$ | $P = 0.1, H_1 = 1,$  
$S = 0.01, Q_1 = 10,$  
$M_1 = 50$ | $P = 5, H_1 = 1,$  
$S = 15, Q_1 = 5,$  
$M_1 = 0.01$ | $P = 100, H_1 = 2,$  
$S = 2, Q_1 = 0.1,$  
$M_1 = 10$ |
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<td>$x_c$</td>
<td>$R_c$</td>
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<tr>
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<td>1.90</td>
<td>86.85</td>
</tr>
<tr>
<td>10</td>
<td>2.00</td>
<td>77.20</td>
</tr>
<tr>
<td>15</td>
<td>2.10</td>
<td>74.45</td>
</tr>
<tr>
<td>20</td>
<td>2.20</td>
<td>73.14</td>
</tr>
<tr>
<td>25</td>
<td>2.30</td>
<td>72.38</td>
</tr>
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</table>
Table 4.2.3: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $H_1$.

<table>
<thead>
<tr>
<th>$H_1$</th>
<th>$G = 5, M_1 = 500, S = 50, P = 50, Q_1 = 10$</th>
<th>$G = 50, M_1 = 100, S = 15, P = 10, Q_1 = 500$</th>
<th>$G = 100, M_1 = 1, S = 5, P = 5, Q_1 = 1$</th>
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</thead>
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<tr>
<td></td>
<td>$x_c$</td>
<td>$R_c$</td>
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<tr>
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<td>50.02</td>
<td>1.40</td>
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<tr>
<td>10</td>
<td>0.90</td>
<td>50.01</td>
<td>1.41</td>
</tr>
<tr>
<td>15</td>
<td>1.00</td>
<td>50.01</td>
<td>1.42</td>
</tr>
<tr>
<td>20</td>
<td>1.10</td>
<td>50.01</td>
<td>1.43</td>
</tr>
<tr>
<td>25</td>
<td>1.20</td>
<td>50.00</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Table 4.2.4: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $M_1$.

<table>
<thead>
<tr>
<th>$M_1$</th>
<th>$G = 2, H_1 = 300, S = 100, P = 10, Q_1 = 10$</th>
<th>$G = 10, H_1 = 20, S = 20, P = 0.1, Q_1 = 15$</th>
<th>$G = 100, H_1 = 1, S = 200, P = 100, Q_1 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_c$</td>
<td>$R_c$</td>
<td>$x_c$</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>1.80</td>
<td>100.004</td>
<td>2.30</td>
</tr>
<tr>
<td>3</td>
<td>1.90</td>
<td>100.004</td>
<td>2.31</td>
</tr>
<tr>
<td>4</td>
<td>1.91</td>
<td>100.004</td>
<td>2.32</td>
</tr>
<tr>
<td>5</td>
<td>1.92</td>
<td>100.003</td>
<td>2.33</td>
</tr>
</tbody>
</table>
Table 4.2.5: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $Q_1$.

<table>
<thead>
<tr>
<th>$Q_1$</th>
<th>$G = 2, H_1 = 50, S = 1, P = 0.001, M_1 = 1$</th>
<th>$G = 10, H_1 = 5, S = 5, P = 50, M_1 = 10$</th>
<th>$G = 100, H_1 = 1, S = 1, P = 1, M_1 = 200$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_c$</td>
<td>$R_c$</td>
<td>$x_c$</td>
</tr>
<tr>
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</tr>
<tr>
<td>2</td>
<td>1.50</td>
<td>300.81</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>1.90</td>
<td>360.70</td>
<td>1.21</td>
</tr>
<tr>
<td>4</td>
<td>2.00</td>
<td>420.52</td>
<td>1.22</td>
</tr>
<tr>
<td>5</td>
<td>2.50</td>
<td>476.40</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Table 4.2.6: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of stability as stationary convection for various values of $P$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$G = 5, H_1 = 100, S = 10, Q_1 = 0.1, M_1 = 1$</th>
<th>$G = 20, H_1 = 10, S = 0.1, Q_1 = 500, M_1 = 100$</th>
<th>$G = 100, H_1 = 1, S = 200, Q_1 = 100, M_1 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_c$</td>
<td>$R_c$</td>
<td>$x_c$</td>
</tr>
<tr>
<td>2</td>
<td>1.20</td>
<td>10.026</td>
<td>1.49</td>
</tr>
<tr>
<td>4</td>
<td>1.21</td>
<td>10.013</td>
<td>1.50</td>
</tr>
<tr>
<td>6</td>
<td>1.22</td>
<td>10.009</td>
<td>1.51</td>
</tr>
<tr>
<td>8</td>
<td>1.23</td>
<td>10.006</td>
<td>1.52</td>
</tr>
<tr>
<td>10</td>
<td>1.24</td>
<td>10.005</td>
<td>1.53</td>
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</table>
4.2.5 STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Now to determine under what conditions the principle of exchange of stabilities (PES) is satisfied (i.e. $\sigma$ is real and the marginal states are characterized by $\sigma = 0$) and the oscillations come into play, we multiply equation (4.2.29) by $W^*$, complex conjugate of $W$ and integrating over the range of $z$ (i.e. $z = 0$ to $1$) and making use of equation (4.2.30) to (4.2.34) together with the boundary condition (4.2.36), we have

\begin{align*}
- \left[ \frac{\sigma}{\varepsilon} \left(1 + \frac{M}{1 + \tau_1 \sigma}\right) + \frac{1 + F \sigma}{P_i} \right] I_1 + \frac{g a^2 \kappa}{\beta \nu} \left(1 + \tau_1 \sigma^* \right) \left( \frac{G}{H_1 + \tau_1 \sigma^*} \right) (I_2 + E_i p_i \sigma^* I_3) \\
- \frac{g a^2 \kappa^*}{\beta \nu} \left(1 + \tau_1 \sigma^* \right) \left( \frac{G}{H_1 + \tau_1 \sigma^*} \right) (I_4 + E_i p_i \sigma^* I_5) \\
+ \frac{\mu \varepsilon \eta}{4 \pi \rho_0 \nu} \left[ I_6 + p_2 \sigma^* I_7 \right] + \left( \frac{\sigma^*}{\varepsilon} \left(1 + \frac{M}{1 + \tau_1 \sigma^*} + \frac{1 + F \sigma^*}{P} \right) \right] \right] + \frac{\mu_i H^2 d^2}{4 \pi \varepsilon_0 \rho_0 \nu} I_8 = 0
\end{align*}

(4.2.46)

where

\begin{align*}
I_1 &= \int_0^1 \left( |DW|^2 + a^2 |W|^2 \right) dz, \quad I_2 = \int_0^1 \left( |D\Theta|^2 + a^2 |\Theta|^2 \right) dz, \\
I_3 &= \int_0^1 |\Theta|^2 dz, \quad I_4 = \int_0^1 \left( |D\Gamma|^2 + a^2 |\Gamma|^2 \right) dz, \\
I_5 &= \int_0^1 |\Gamma|^2 dz, \quad I_6 = \int_0^1 \left( |DK|^2 + a^2 |K|^2 \right) dz, \\
I_7 &= \int_0^1 |K|^2 dz, \quad I_8 = \int_0^1 |DK|^2 dz
\end{align*}

(4.2.47)

where integrals $I_1, I_2, I_3, \ldots, I_8$ are all positive definite. Putting $\sigma = \sigma_0 + i \sigma_1$, and equating the imaginary parts of equation (4.2.46), we obtain

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\[ \sigma_i \left[ \frac{1}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_i^2 \sigma_i^2} \right) + \frac{F}{P} \right] I_i + \frac{ga^2 \tau_i}{\beta v} \left( \frac{G}{G - 1} \right) \left( \frac{1 - H_i}{H_i^2 + \tau_i^2 \sigma_i^2} \right) (\kappa aI_z - \kappa' \alpha' I_z) \]

\[ = \left[ \frac{\tau_i}{\varepsilon (1 + \tau_i^2 \sigma_i^2)} \left( \frac{1}{\varepsilon} - \frac{\sigma_i^2 F}{P} \right) - \frac{\sigma_i^2 F}{\varepsilon (1 + \tau_i^2 \sigma_i^2)} \right] I_0 + \frac{\mu \eta}{4 \pi \rho_0 \nu (4 \pi N' \eta)^2} \frac{e^2 H^2}{\varepsilon (1 + \tau_i^2 \sigma_i^2)} \left( I_i + \frac{1}{\varepsilon (1 + \tau_i^2 \sigma_i^2)} \right) \]

\[ + \frac{\mu \eta}{4 \pi \rho_0 \nu (4 \pi N' \eta)^2} \frac{e^2 H^2}{\varepsilon (1 + \tau_i^2 \sigma_i^2)} \left( I_i + \frac{1}{\varepsilon (1 + \tau_i^2 \sigma_i^2)} \right) \]

\[ = 0 \]

In the absence of solute gradient and Hall currents, equation (4.2.48) reduces to

\[ \sigma_i \left[ \frac{1}{\varepsilon} \left( 1 + \frac{M}{1 + \tau_i^2 \sigma_i^2} \right) + \frac{F}{P} \right] I_i + \frac{ga^2 \tau_i}{\beta v} \left( \frac{G}{G - 1} \right) \left( \frac{1 - H_i}{H_i^2 + \tau_i^2 \sigma_i^2} \right) \kappa aI_z = 0. \] (4.2.49)

The terms in the bracket are positive definite. Thus \( \sigma_i = 0 \) implies that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied in the absence of solute gradient and Hall currents. It is evident from equation (4.2.48) that presence of solute gradient and Hall currents brings oscillatory modes (as \( \sigma_i \) may not be zero) which were non-existent in their absence for a compressible Rivlin-Ericksen fluid layer with suspended particles and Hall currents heated and soluted from below in the presence of porous medium.

### 4.2.6 CONCLUSIONS

The effect of various parameters such as Hall currents, compressibility, magnetic field, suspended particles and porous medium has been investigated on thermosolutal stability of Rivlin-Ericksen fluid. The principal conclusions from the analysis are as follows.

1. For a stationary convection, Rivlin-Ericksen fluid behaves like Newtonian fluid.
2. Compressibility, suspended particles and Hall currents have destabilizing effect on the system which is evident from the equations (4.2.41) – (4.2.43) and is supported by the figures 4.2.4 – 4.2.9.

3. It is evident from equations (4.2.40) and (4.2.44) that solute gradient and magnetic field have stabilizing effect which is in agreement with figures 4.2.2 and 4.2.10.

4. The permeability has destabilizing effect on the system which is evident from equation (4.2.45) and it is supported by figures 4.2.11 and 4.2.12.

5. The critical Rayleigh numbers and the associated wave numbers are found for stationary convection for various parameters involved and it has been found that it increases with the increase in solute gradient and magnetic field parameters while decreases with the increase in suspended particles, compressibility, Hall currents and permeability parameters thereby confirming the stabilizing role of solute gradient and magnetic field and destabilizing role of suspended particles, compressibility, Hall currents and permeability parameters.

6. The principle of exchange of stabilities is found to hold true in the absence of solute gradient and Hall currents which means that oscillatory modes are introduced due to the presence of solute gradient and Hall currents which were non-existent in their absence.