CHAPTER – I

INTRODUCTION

The concept of topological spaces grew out of the study of the real line and Euclidean space and the study of continuous functions on these spaces. The study of topological spaces, their continuous mappings and general properties make up one branch of topology known as “General topology”. This thesis is an elaborate study of a new type of generalized closed sets in topological space called $\beta^*$-closed sets, their respective continuous maps, closed maps, homeomorphisms, compactness, connectedness, regular spaces, normal spaces and their extension to bitopological settings.

In this chapter, the author recalls the recent developments of topologies contributed by various authors. Section 1 begins with the discussion of strong and weak forms of open sets and closed sets. Section 2 deals with the strong and weak forms of continuous maps, while Section 3 contains irresolute maps, closed and open maps. Some generalized homeomorphisms and quotient maps are discussed in section 4. Section 5 deals with bitopological spaces while section 6 outlines the contribution of the author. The last section describes various notations used in the thesis.

Throughout the thesis $(X, \tau)$, $(Y, \sigma)$ and $(Z, \eta)$ or simply $X$, $Y$ and $Z$ denote topological spaces on which no separation axioms are assumed unless otherwise mentioned explicitly.
1.1 Strong and Weak forms of Open sets and Closed sets

Stone [54] has introduced and investigated strong form of open sets called regular open sets. Andrijevic [2], Levine [29], Mashour et al. [37] respectively introduced semi-preclosed sets (β-closed sets [1]), semi closed sets, pre-closed sets which are some weak forms of closed sets.

Levine [30], Julian Dontchev [14], P. Bhattacharya and B.K. Lahiri [8], S.P. Arya and T. Nour [6], Sundaram and Sheik John [51] have introduced generalized closed sets, generalized semi-preclosed sets, semi-generalized closed sets, generalized semi-closed set and ω-closed sets respectively which are also weak forms of closed sets.

Veerakumar [59, 63, 60, 61] introduced g*-closed sets, *g- closed sets, g*p-closed sets and pre-semicolon closed sets which are weak forms of closed sets. Recently Antony Rex Rodrigo [3] has introduced *γ*-closed sets. The complements of the various types of closed (resp. open) sets are called by the same name of open (resp. closed) sets. That is, the complement of ω-closed set is called an ω-open set and the complement of generalized closed set is called a generalized open set and so on.

**Definition 1.1.1** A subset $A$ of a topological space $(X, \tau)$ is called

(i) a pre-open set [37] if $A \subseteq \text{int} (\text{cl}(A))$ and a pre closed set if $\text{cl} (\text{int}(A)) \subseteq A,$

(ii) a semi-open set [29] if $A \subseteq \text{cl}(\text{int}(A))$ and a semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A,$

(iii) an $\alpha$-open set [42] if $A \subseteq \text{int} (\text{cl}(\text{int}(A)))$ and an $\alpha$-closed set if $\text{cl} (\text{int}(\text{cl}(A))) \subseteq A,$
(iv) a semi-preopen set [2] (= β-open set [1]) if \( A \subseteq \text{cl}(\text{int}(\text{cl}(A))) \) and a semi-preclosed set (= β closed) if \( \text{int}(\text{cl}(\text{int}(A))) \subseteq A \) and
(v) a regular open set [54] if \( A = \text{int}(\text{cl}(A)) \) and a regular closed set if \( A = \text{cl}(\text{int}(A)) \).

The intersection of all semi-closed subsets of \((X, \tau)\) containing \( A \) is called the semi-closure of \( A \) and is denoted by \( \text{scl}(A) \). Also the intersection of all pre closed (resp. semi-preclosed and α-closed) subsets of \((X, \tau)\) containing \( A \) is called the pre closure (resp. semi-pre closure and α-closure) of \( A \) and is denoted by \( \text{pcl}(A) \) (resp. \( \text{spcl}(A) \) and \( \text{αcl}(A) \)).

The semi-interior (resp.pre interior, α-interior and semi-pre interior) of \( A \), denoted by \( \text{sint}(A) \) (resp.\( \text{pint}(A), \text{α-int}(A), \) and \( \text{spint}(A) \)) is defined by the union of all semi-open (resp. pre-open, α-open and semi-pre open) sets contained in \( A \).

**Definition 1.1.2 [44]** A point \( x \) of \( X \) is called a semi-pre-θ cluster point of \( S \) where \( S \) is a subset of \( X \) if \( \text{spcl}(U) \cap S \neq \emptyset \) for every semi-pre open set \( U \) of \( X \). The set of all semi-pre-θ cluster point of \( S \) is called semi pre θ closure of \( S \) and denoted by \( \text{spcl}_θ(S) \).

**Definition 1.1.3** A subset \( A \) of a topological space \((X, \tau)\) is called

(i) a generalized closed set (briefly g-closed)[30] if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \((X, \tau)\),

(ii) a generalized semi-preclosed set (briefly gsp-closed) [14] if \( \text{spcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \((X, \tau)\),
(iii) a regular generalized closed set (briefly rg-closed) [47] if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is regular open in \((X, \tau)\),

(iv) a generalized pre-regular closed set (briefly gpr-closed)[20] if \( \text{pcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is regular open in \((X, \tau)\),

(v) an \( \omega \)-closed set [58] (=\( \hat{g} \)-closed [64]) if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is semi-open in \((X, \tau)\),

(vi) a *g-closed set[63] if \( \text{cl}(A) \subseteq U \), whenever \( A \subseteq U \) and \( U \) is \( \omega \)-open in \((X, \tau)\),

(vii) a *\( g \)-closed set [59] if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( g \)-open in \((X, \tau)\),

(viii) a *g*-p-closed set [60] if \( \text{pcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( g \)-open in \((X, \tau)\),

(ix) a semi generalized closed (briefly sg-closed) set [8] if \( \text{scl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is semi-open in \((X, \tau)\),

(x) a generalized semi-closed (briefly gs-closed) set [6] if \( \text{scl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \((X, \tau)\),

(xi) an \( \alpha \)-generalized closed [35] (briefly \( \alpha g \)-closed set) if \( \alpha \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \((X, \tau)\),

(xii) a \( g^\# \)-closed set [65] if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \alpha g \)-open in \((X, \tau)\),

(xiii) a \( g^\# \)-semi closed (briefly \( g^\# \)’s closed ) set [66] if \( \text{scl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \alpha g \) open in \((X, \tau)\),

(xiv) a pre-semicolon [61] if \( \text{spcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( g \)-open in \((X, \tau)\),

(xv) an \( \hat{f}^* \)-closed set [3] if \( \text{spcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \omega \)-open in \((X, \tau)\) and
(xvi) a generalized preclosed (briefly gp-closed)[5] set if \( \text{pcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \((X, \tau)\).

The complement of \( g \)-closed (resp-\( g \)-sp-closed, \( r^* \)-closed, \( g^\# \)-closed, \( \omega \)-closed, \( *g \)-closed, \( g^* \)-closed, \( g^p \)-closed, \( s^g \)-closed, \( g^s \)-closed, pre-\( s \)-closed, \( \tilde{\eta}^* \)-closed, \( g^\text{p} \)-closed) set is said to be \( g \) open (resp. \( g \)-sp-open, \( r^* \)-open, \( g^\# \)-open, \( \omega \)-open, \( *g \)-open, \( g^* \)-open, \( g^p \)-open, \( s^g \)-open, \( g^s \)-open, pre-\( s \)-open, \( \tilde{\eta}^* \)-open, \( g^\text{p} \)-open).

**Definition 1.1.4** A topological space \( X \) is

(i) a \( T_\omega \)–space [51] if every \( \omega \)-closed subset of \( X \) is closed in \( X \),

(ii) a \( T_{\tilde{\eta}} \)–space [3] if every \( \tilde{\eta}^* \)-closed subset of \( X \) is closed in \( X \),

(iii) a \( T_b \) space [6] if every \( s^g \)-closed subset of \( X \) is closed in \( X \),

(iv) a \( T_{1/2}^* \)-space [59] if every \( g^* \)-closed subset of \( X \) is closed in \( X \) and

(v) a \( T_{1/2} \)-space [30] if every \( g \)-closed subset of \( X \) is closed in \( X \).

**Lemma 1.1.5** [29, 14] Let \( X \) be a topological space and \( A \subseteq X \). Then

(i) \( A \) is semi-closed (resp. semi-preclosed) if and only if \( A = \text{scl}(A) \) (resp. \( A = \text{spcl}(A) \)).

(ii) \( \text{spcl}(A) \subseteq \text{spcl}(B) \) if \( A \subseteq B \).

**Lemma 1.1.6** [2] Let \( A \) be a subset of a topological space \( X \). Then

(i) \( \text{spcl}(A) = \text{spcl}(\text{spcl}(A)) \).

(ii) if \( F \subseteq A \subseteq X \) and \( A \) is open in \( X \), \( \text{spcl}_A(F) = \text{spcl}(F) \cap A \).
Lemma 1.1.7 [58] Let $X$ be a topological space and $A \subseteq X$. If $A$ is $\omega$–open, then $A$ is $g$-open. (resp. $\alpha g$-open, $sg$-open).

Lemma 1.1.8 [58] Let $X$ be a topological space and $A \subseteq X$. If $A$ is $\omega$-closed then $A$ is pre-closed (semi-preclosed).

Lemma 1.1.9 [14] Let $X$ be a topological space. If $A$ is an open and gsp closed subset of $X$ then $A$ is semi-preclosed.

Lemma 1.1.10 [51] Let $X$ be a topological space and let $A \subseteq Y \subseteq X$ and suppose that $Y$ is closed in $X$ and $A$ is $\omega$-open in $X$. Then $A$ is $\omega$-open relative to $Y$.

Lemma 1.1.11 [51] Let $X$ be a topological space. If $A$ and $B$ are $\omega$-open in $X$ (resp. $\omega$-closed) then $A \cap B$ is $\omega$-open (resp $\omega$-closed).

Lemma 1.1.12 [51] Let $X$ be a topological space. If $A \subseteq B \subseteq X$ where $A$ is $\omega$-open relative to $B$ and $B$ is open in $X$, then $A$ is $\omega$-open in $X$.

Lemma 1.1.13 [1] Let $A$ and $Y$ be subsets of a topological space $X$. If $Y$ is open in $X$ and $A$ is semi-preopen in $X$, then $A \cap Y$ is semi-preopen in $Y$.

Lemma 1.1.14 [14] If $A$ is an open and gsp closed subset of $X$ then $A$ is regular open.

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1.2 Strong and weak forms of continuous maps

In this section, various strong and weak forms of continuous maps contributed by eminent topologists are recalled. Levine[28], Noiri[43] and Jain[24] have respectively introduced and studied strong continuous maps, perfectly continuous maps and totally continuous maps which are strong forms of continuous maps.

Abd El. Monsef et al[1], Sheik John[51], Palaniappan and Rao[47] and J. Antony Rex Rodrigo[3] have introduced β-continuity, ω-continuity, rg-continuity and ñ*-continuity respectively, which are weaker than continuity. Veera kumar[59,60] has introduced g*-continuity and g*p-continuity which are also weak forms of continuity.


We give some definitions of strong and weak forms of continuous maps used in our study.

**Definition 1.2.1** A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) semi-continuous [29] if $f^{-1}(V)$ is a semi-open set of $(X, \tau)$ for each open set $V$ of $(Y, \sigma)$,

(ii) $\beta$-continuous [1] if $f^{-1}(V)$ is a $\beta$-closed set of $(X, \tau)$ for each closed set $V$ of $(Y, \sigma)$,

(iii) $\omega$-continuous [51] if $f^{-1}(V)$ is an $\omega$-closed subset of $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,
(iv) $g^*$-continuous [59] if $f^{-1}(V)$ is a $g^*$-closed subset of $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(v) $g^*p$- continuous [60] if $f^{-1}(V)$ is a $g^*p$ closed subset of $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(vi) gsp-continuous [14] if $f^{-1}(V)$ is a gsp closed subset of $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(vii) $g^# -$continuous[65] if $f^{-1}(V)$ is $g^#$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(viii) $g^#s$-continuous [66] if $f^{-1}(V)$ is $g^#s$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(ix) $\hat{g}^*$- continuous [3] if $f^{-1}(V)$ is $\hat{g}^*$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(x) Pre-semi-continuous [61] if $f^{-1}(V)$ is pre-semi closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(xi) $g^*$- continuous [7] if $f^{-1}(V)$ is $g^*$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(xii) gs- continuous [56] if $f^{-1}(V)$ is gs-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(xiii) gp-continuous [5] if $f^{-1}(V)$ is gp-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(xiv) sg - continuous [8] if $f^{-1}(V)$ is sg-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(xv) rg- continuous [47] if $f^{-1}(V)$ is rg-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$ and

(xvi) gpr-continuous [20] if $f^{-1}(V)$ is gpr-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$. 
**Definition 1.2.2** A map \( f: (X, \tau) \to (Y, \sigma) \) is called 

(i) strongly continuous [28] if \( f^{-1}(V) \) is both open and closed in \( (X, \tau) \) for each subset \( V \) of \( (Y, \sigma) \) 

(ii) perfectly continuous [43] if \( f^{-1}(V) \) is both open and closed in \( (X, \tau) \) for each open set \( V \) of \( (Y, \sigma) \) and 

(ii) contra continuous [15] if \( f^{-1}(V) \) is closed in \( (X, \tau) \) for each open subset \( V \) of \( (Y, \sigma) \) 

**Definition 1.2.3** A map \( f:(X, \tau) \to (Y, \sigma) \) is said to be contra \( \beta \)-continuous [9] if the inverse image of every open set of \( (Y, \sigma) \) is \( \beta \)-closed in \( (X, \tau) \) 

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**1.3 Irresolute maps, closed maps and open maps**

Crossely and Hildebrand [10] introduced and investigated irresolute maps which are stronger than semi-continuous maps but are independent of continuous maps. R.A. Mohmoud and Abd-El-Monsef [38] and Sheik John [51] respectively have investigated \( \beta \)-irresolute maps and \( \omega \)-irresolute maps. Sundaram [56] and Sheik John [51] introduced and studied generalized open maps and \( \omega \)-closed maps respectively. We recall some definitions.

**Definition 1.3.1** Let \( f: (X, \tau) \to (Y, \sigma) \) be a map. Then \( f \) is said to be 

(i) irresolute [10] if \( f^{-1}(V) \) is semi-open in \( (X, \tau) \) for each semi-open set \( V \) of \( (Y, \sigma) \),
(ii) \(\beta\)-irresolute [38] if \(f^{-1}(V)\) is \(\beta\)-closed in \((X, \tau)\) for each \(\beta\)-closed set \(V\) of \((Y, \sigma)\) and

(iii) \(\omega\)-irresolute [51] if \(f^{-1}(V)\) is \(\omega\)-closed in \((X, \tau)\) for each \(\omega\)-closed set \(V\) of \((Y, \sigma)\).

**Definition 1.3.2** A map \(f:(X, \tau) \rightarrow (Y, \sigma)\) is said to be

(i) \(g\)-closed (resp. \(g\)-open) [36] if \(f(V)\) is \(g\)-closed (resp. \(g\)-open) in \((Y, \sigma)\) for every closed (resp. open) set \(V\) of \((X, \tau)\),

(ii) \(pre-\beta\)-closed (resp. \(pre-\beta\)-open) [38] if \(f(V)\) is \(\beta\)-closed (resp. \(\beta\)-open) in \((Y, \sigma)\) for every \(\beta\)-closed (resp. \(\beta\)-open) set \(V\) of \((X, \tau)\),

(iii) \(\omega\)-closed (resp. \(\omega\)-open) [51] if \(f(V)\) is \(\omega\)-closed (resp. \(\omega\)-open) in \((Y, \sigma)\) for every closed (resp. open) set \(V\) of \((X, \tau)\),

(iv) \(\omega^*\)-closed (resp. \(\omega^*\)-open) [51] if \(f(V)\) is \(\omega\)-closed (resp. \(\omega\)-open) in \((Y, \sigma)\) for every \(\omega\)-closed (resp. \(\omega\)-open) set \(V\) of \((X, \tau)\),

(v) \(gsp\)-closed (resp. \(gsp\)-open) [14] if \(f(V)\) is \(gsp\)-closed (resp. \(gsp\)-open) in \((Y, \sigma)\) for every closed (resp. open) set \(V\) of \((X, \tau)\),

(vi) \(\check{\eta}^*\)-closed (resp. \(\check{\eta}^*\)-open) [3] if \(f(V)\) is \(\check{\eta}^*\)-closed (resp. \(\check{\eta}^*\)-open) in \((Y, \sigma)\) for every closed (resp. open) set \(V\) of \((X, \tau)\) and

(vii) \(M\)-preclosed (resp. \(M\)-preopen) [37] if \(f(V)\) is preclosed (resp. preopen) in \((Y, \sigma)\) for every preclosed (resp. preopen) set \(V\) in \((X, \tau)\).

**Definition 1.3.3** A space \(X\) is said to be

(i) almost connected [16] if there does not exist disjoint regular open sets \(A\) and \(B\) such that \(A \cup B = X\),

(ii) nearly compact [53,54] if every regular open cover of \(X\) has a finite subcover,
(iii) nearly Lindelof [16] if every cover of $X$ by regular open set has countable subcover,

(iv) nearly countable compact [22] if every countable cover of $X$ by regular open sets has a finite subcover,

(v) countably s-closed compact [13] if every countable cover of $X$ by regular closed sets has a finite subcover,

(vi) s-Lindelof [53] if every cover of $X$ by regular closed sets has a countable subcover,

(vii) $r$-T$_1$ [16] if for each pair of distinct points $x$ and $y$ of $X$, there exist regular open sets, $U_1$ and $U_2$ such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$,

(viii) $T_2$-space if for each pair of distinct points $x$ and $y$ in $X$, there exist disjoint open sets $U_1$ and $U_2$ in $X$ such that $x \in U_1$ and $y \in U_2$ and

(ix) a locally indiscrete space [14] if every open subset of $X$ is closed.

Result 1.3.4 A space $X$ is s-closed [12] if every regular closed cover of $X$ has a finite subcover.

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1.4 Generalized homeomorphisms

The notion of homeomorphisms in topological spaces has been generalized by many researchers. Maki et al [33] have introduced g-homeomorphisms and gc-homeomorphisms in topological spaces. Using semi-open covers, Di Maio and Noiri[12] have introduced and studied a new
class of compact spaces called s-closed spaces. Maheswari and Prasad [31,32] introduced and studied s-regular and s-normal spaces. Rajamani and Indhumathi [49] have introduced sp-normal spaces which are weaker than s-normal spaces. Sheik John[51] has introduced and studied $\omega$-regular and $\omega$-normal spaces. Here we recall some definitions.

**Definition 1.4.1** A bijective map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) a generalized homeomorphism (briefly g-homeomorphism)[33] if $f$ is both g-continuous and g-open,

(ii) a semi-homeomorphism [11] if $f$ is both semi-continuous and semi-open,

(iii) a gsp-homeomorphism[14] if $f$ is both gsp-continuous and gsp-open,

(iv) an $\eta^*$-homeomorphism [3] if $f$ is both $\eta^*$-continuous and $\eta^*$-open and

(v) a semi-prehomeomorphism [1] if $f$ is both semi-precontinuous ($\beta$-continuous) and semi-preopen ($\beta$-open)

**Definition 1.4.2** [49] A space $(X, \tau)$ is called semi-prenormal (sp normal) if for each pair of disjoint closed sets $A$ and $B$, there exists disjoint semi-pre-open sets $U$ and $V$ such that $A \subseteq U$ and $B \subseteq V$.

**Theorem 1.4.3** Let $X$ be a hausdorff space and $F$ be a compact subset of $X$. Then

(i) for each $x$ in $X-F$, there exist disjoint open sets containing $F$ and $x$ and

(ii) if $F_1$ and $F_2$ are disjoint compact subsets of $X$, then there exist disjoint open sets containing $F_1$ and $F_2$.
1.5 Bitopological spaces

Kelly [25] defined a bitopological space as a set equipped with two topologies on the set and initiated a systematic study of bitopological spaces in 1963. Recently Fukutake[18,19] and Maki et al [34] have extended the notion of generalized closed sets, semi open sets and generalized continuous maps respectively in topological spaces to bitopological spaces and obtained some interesting results. Also Fukutake et al[17], Gnanambal [21], Popa[48] and Sampath kumar[50] have worked on various concepts of topology by considering bitopological spaces instead of topological spaces. Here we recall the following definitions which are used in our later studies.

Definition 1.5.1 Let $i, j \in \{1,2\}$ be fixed integers. In a bitopological space $(X, \tau_i, \tau_j)$ a subset $A$ of $(X, \tau_i, \tau_j)$ is said to

(i) $(\tau_i, \tau_j)$-g*-closed [52] if $\tau_j$-cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_i$-g open,

(ii) $(\tau_i, \tau_j)$-rg closed [47] if $\tau_j$-cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_i$-regular open and

(iii) $(\tau_i, \tau_j)$-H*closed [3] if $\tau_j$-spcl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau_i$-ω open.

Definition 1.5.2 A bitopological space $(X, \tau_i, \tau_j)$ is said to be $(\tau_i, \tau_j)$–$T_{1/2}$* [52] if every $(\tau_i, \tau_j)$–g*-closed set is $\tau_j$-closed.

Definition 1.5.3 A map $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is called

(i) $\tau_j$–$\sigma_k$– continuous [34] if $f^{-1}(V) \in \tau_2$ for every $V \in \sigma_1$

(ii) bi-continuous [34] if $f$ is $\tau_1$–$\sigma_1$– continuous and $\tau_2$–$\sigma_2$– continuous

(iii) strongly bi-continuous (s-bicontinuous) [34] if $f$ is bi-continuous, $\tau_1$–$\sigma_2$–continuous and $\tau_2$–$\sigma_1$– continuous and

(iv) $D^*(\tau_i, \tau_j)$–$\sigma_k$– continuous [52] if the inverse image of every $\sigma_k$ closed set is $(\tau_i, \tau_j)$–g*- closed.
1.6 Contribution of the Author

In the light of the above work, the author has obtained several interesting results on generalizations of closed sets and continuous maps in topological and bitopological spaces on the following topics.

(i) $\beta^*$-closed sets in topological spaces.
(ii) $\beta^*$-continuous maps in topological spaces.
(iii) Topological mappings using $\beta^*$-closed sets.
(iv) $\beta^*$-Homeomorphism in topological spaces
(v) $\beta^*$-closed sets in bitopological spaces.

The rest of the thesis is the detailed study of the above topics.

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1.7 Notations

$\Lambda$ - the index set
$P(x)$ - the power set of $X$
$A^C$ – Complement of $A$
$\text{int}(A)$ – interior of $A$
$\text{cl}(A)$ – closure of $A$
$A - B$ – complement of $B$ with respect to $A$
$\text{Spcl}_A(B)$ – semi preclosure of $B$ with respect to $A$
$\text{SPO}(X)$ – the set of all semi-pre-open subsets of $(X, \tau)$
$\tau_Y$ – the relative topology of $\tau$ to $Y$
$f_{\mid H}$ – the map $f$ is restricted to $H$

In the diagrams, $A \rightarrow B$ we mean, $A$ implies $B$ but not conversely and $A \leftrightarrow B$ we mean, $A$ and $B$ are independent of each other.

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