INTRODUCTION

Ergodic theory has evolved out of the study of dynamical systems in Physics. The mathematical abstraction of these dynamical systems is a transformation or a flow acting on a measure space. These transformations or the flows are taken to be either measure preserving or at least preserving the measure class. The analysis of Dynamical systems, these days, also involves interaction of differential structure, in smooth dynamics, with measure theory. Due to expansion of interaction of these dynamical systems with other parts of geometry and analysis the study gets even more intricate.

When such complicated dynamical system are witnessing renaissance, is it worthwhile or relevant to study the action of Borel automorphism \( T(T \& T^{-1} \text{ measurable}, T \text{ bijective}) \) on a Borel space \((X,B)\) which has no measure theoretic structure ?. The answer is provided by Benjamin Weiss in [28] where he points out "one would gain new insights by studying the dynamics of such a Borel automorphism without singling out any measure class".

The first basic result in this direction was given by S.Shelah and B.Weiss in [26] where they
characterized the $\sigma$-ideal generated by the Wandering Borel sets (under the action of a Borel automorphism $T$) as the intersection of the $\sigma$-ideals of Borel sets of continuous probability measure quasi invariant and ergodic under $T$. Another result in this direction was M.G.Nadkarni's work on descriptive characterization of ergodic systems (see [18]). In [18] it is also shown that basic concepts such as ergodicity, recurrence etc. are descriptive (set theoretic) in nature. We may call these properties, which are independent of measure and which have set theoretic formulation, descriptive properties. A recent book [22] of M.G.Nadkarni deals extensively with descriptive properties.

Here we raise the question. Is it possible to improve upon some results in measure theoretic ergodic theory using set theoretic methods? The work of W.Krieger [16] and K.Schmidt [25] lends credence to an affirmative answer with regards to the results on orbit equivalence, existence of equivalent invariant measure, and representation of flows. Indeed, V.M.Wagh's work [27], where he gives a descriptive version of Ambrose theorem [1] on flows and M.G.Nadkarni's work on the existence of invariant measure [20] bear testimony to such a belief. These two results, incidently, leave scope of work only in regard to orbit equivalence in the descriptive setting.
The hard solution to the deep questions of isomorphism (Omtstein 1974) and orbit equivalence (Krieger 1969a, 1969b, 1974) are fully measure theoretic. However invariants of orbit equivalence in the descriptive setting needed to be studied. The broad purpose of this work has been to look for the necessary and sufficient set theoretic conditions for the orbit equivalence of Borel automorphism on a Borel space.

In the present work we focus our attention on a basic theorem due to Dye and try to obtain its descriptive version. The context of discussion for the well known Dye’s theorem is a measure theoretic dynamical system \((X, \mathcal{B}, m, T)\) where \((X, \mathcal{B})\) is a standard Borel space, \(m\) is a countably additive measure and \(T\) a Borel automorphism on the measure space \((X, \mathcal{B}, m)\). \(T\) is said to be ergodic if for any invariant set \(A \subset \mathcal{B}\) either \(m(A) = 0\) or \(m(X - A) = 0\). Dye’s theorem says that two ergodic measure preserving transformation are orbit equivalent, where \(m\) is a finite measure (see [7]). The result is also true when \(m\) is a \(\sigma\)-finite measure (see [7]). W. Krieger proved that the result is also true for non singular transformations in special cases [14].

The descriptive setting referred above is a descriptive dynamical system \((X, \mathcal{B}, \mathcal{N}, T)\) where \((X, \mathcal{B})\) is a Borel space, \(\mathcal{N}\) is a \(\sigma\)-ideal, \(T\) a Borel automorphism, \(T^{-1}(\mathcal{N}) = \mathcal{N}\) and \(W \subseteq \mathcal{N}\) where \(W\) is the Shelah-Weiss
ideal generated by the wandering sets. Throughout the following work we take \((X, \mathcal{M})\) to be standard Borel space.

Hopf introduced the concept of compressibility in [12]. We use it to formulate the descriptive version of Dye's theorem. In the last chapter we are able to prove the decomposition theorem which falls short of the version of Dye's theorem we have in mind. This chapter also gives a very deep theorem due to Dougherty, Jackson, and Kechris which provides complete answers to the questions raised by us.

The complete work here can be viewed as contribution to theme of descriptive ergodic theory. We give a brief summary of the work chapterwise.

**Chapter I** sets up the basic framework, namely descriptive dynamical systems and deals it at the elementary level.

**In Chapter II** we discuss classification of dynamical systems with respect to orbit equivalence. We study the prototypes (odometer and von Neumann transformations) and the interrelationship between them.

**Chapter III** provides results necessary for our purpose viz Dye's theorem and Krieger's theorem. These two results give complete classification of measure theoretic dynamical systems in term of orbit
equivalence. Here we also discuss invariants of orbit equivalence for descriptive dynamical systems at the elementary level.

In Chapter IV we recall some of the basic results from descriptive set theory and ergodic theory. Here we obtain Poincarre recurrence theorem in descriptive setting and define induced transformation in the descriptive setting.

We introduce compressibility in chapter V and prove some results on compressibility. We also formulate our problem in terms of compressibility.

Chapter VI contains some results of descriptive ergodic theory due to Chaube & Nadkarni ([4], [5]), B. Weiss [28] and Nadkarni [19].

Chapter VII contains our main result in the form of decomposition theorem. In this chapter we also deal with the major contributions made by Dougherty, Jackson & Kechris.

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