CHAPTER 2

LITERATURE SURVEY OF SELECTIVE HARMONIC ELIMINATION TECHNIQUES

2.1 HARMONICS IN POWER SYSTEMS

Power system harmonics are power quality issues and harmonic distortion is a representative of power quality problems which receives continuous attention in the recent years and was dealt by Bollen (2003), Choudhury (2001), Heydt (1998), (2001), Mack and Santoso (2001), Narain (1995). Under electric power quality, voltage quality has become one of the most important issues to be considered for electric suppliers to end users.

The usage of power electronic equipments has been increased in recent years in industrial and consumer applications. Such loads draw the non-linear sinusoidal current and voltage from the source (Wagner 2003). These non-linear loads change the sinusoidal nature of the alternating current, thereby resulting in the flow of harmonic currents in the AC power system. The voltage and current waveforms are pure sinusoidal when the electric load is linear. When the load is non-linear, the voltage and current waveforms are quite often distorted. These non-linear loads change the sinusoidal nature of alternating currents and results in flow of harmonic currents in power systems. This deviation from perfect sine wave is to be represented as harmonics.

These harmonics draw the non-linear sinusoidal current and voltage from the source. Harmonics are caused by non-linear operation of devices,
such as power converters, arc-furnaces, and gas discharge lighting devices. These devices have non-linear voltage-current characteristic meaning that the current signal is not proportional to the applied voltage.

In recent years, the power quality issues in the utility grids have received considerable attention to suppress harmonics related problems resulting from a proliferation of non-linear loads. This has led to restricted norms regarding utility power quality standards. As a result, a number of research works are directed to fulfill these requirements by eliminating the power quality degradation problems. The major reasons for power quality degradation are as follows.

1. The modern devices and equipments being used by industrial and commercial customers are more sensitive to power quality variations than equipments used in the past.

2. An increasing number of power electronic devices are being utilised to protect customers from power quality issues or acts as an important part of energy transfer systems. Their non-linear characteristics cause harmonic current which results in additional heat in power system equipment, interference with communication systems, and malfunctioning of controls.

3. There is an increasing emphasis on overall power system efficiency which causes a continuous growth in the application of shunt capacitors for power factor correction. These capacitors change the system impedance vs frequency characteristic, resulting in resonance which can magnify transient disturbances and harmonic distortion levels.
The major power disturbances which frequently appear in power systems are divided into two categories based on the duration of occurrence. They are transient problems and static problems. Transient problems include voltage sags, voltage swells, electrical noise, and momentary interruption. The duration of transient power quality problems varies from few milliseconds to several seconds. Static problems include harmonics, outage, under/over voltage, and impulses. The duration of static power quality problems changes from several seconds to several minutes or even longer.

2.2 SOURCES AND EFFECTS OF HARMONIC DISTORTION

Harmonics are destruction phenomenon which causes enormous amount of energy loss in transmission and distribution systems. The impact of harmonics on the quality of electrical power continues to be a critical concern for industrial and commercial users. According to Bennett et al (1997), Brozek (1990), Henderson and Rose (1994), Purkayastha and Savoie (1990), harmonics have significant impacts on generation units, transmission equipments and customer facilities. The harmonic current flowing through the energy transform/transfer devices generate excess heat, reduce the transmission efficiency and shorten the device lifetime.

2.2.1 Sources of Harmonics

One common source of harmonics is iron core devices like transformer. The magnetic characteristics of iron are almost linear over a certain range of flux density, but quickly saturates as the flux density increases. This non-linear magnetic characteristic is described by a hysteresis curve. The non-linear hysteresis curve produces a non-sinusoidal excitation current.
Core iron is not the only source of harmonics, but the generator also themselves produce some 5\textsuperscript{th} harmonic voltages due to magnetic flux distortions, that occur near the stator slots and non-sinusoidal flux distribution across the air gap. Other producers of harmonics include non-linear loads like rectifiers, inverters, adjustable speed motor drives, welders, arc furnaces, voltage controllers, and frequency converters.

Semiconductor switching devices produce significant harmonic voltages as they abruptly chop voltage waveforms during their transition between conducting and cut-off states. Inverter circuits are notorious for producing harmonics, and are used widely today. An adjustable speed motor drive is one application that makes use of inverter circuits, often using pulse width modulation synthesis to produce the ac output voltage. Various synthesis methods produce different harmonic spectra. Regardless of the method used to produce an AC output voltage from a DC input voltage, harmonics will be present on both sides of the inverter and must often be mitigated.

### 2.2.2 Effects and Negative Consequences

The effects of three-phase harmonics on circuits are similar to the effects of stress and high blood pressure on the human body. High levels of stress or harmonic distortion can lead to problems for the utility's distribution system, plant distribution system and any other equipment serviced by that distribution system. Ambro et al (2003), Irene Yu-Hua and Emmanouil (2003) have enumerated the effects of harmonics and that range from spurious operation of equipment to a shutdown of important plant equipment, such as machines or assembly lines. Harmonics leads to power system inefficiency. Some of the negative consequences of harmonics on plant equipments are listed below:
1. Conductor overheating is a function of the square RMS current per unit volume of the conductor. Harmonic currents on undersized conductors or cables cause a “skin effect”, which increases with frequency and is similar to centrifugal force.

2. Capacitors can be affected by increase in heat rise leads to power loss and reduced life on the capacitors. If a capacitor is tuned to one of the characteristic harmonics such as the 5th or 7th, overvoltage and resonance cause dielectric failure or rupture of capacitor.

3. Harmonics cause false or spurious operations on fuses, circuit breakers and trips, damaging or blowing components for no apparent reason was mentioned by Brozek (1990).

4. Transformers have increased iron and copper losses or eddy currents due to stray flux losses. This causes excessive overheating in the transformer windings was explained by Henderson and Rose (1994).

5. George (2003), Watson and Arrillaga (2003) have discussed problems related to generators. Sizing and coordination is critical to the operation of the voltage regulator and controls. Excessive harmonic voltage distortion will cause multiple zero crossings of the current waveform. Multiple zero crossings affect the timing of the voltage regulator, causing interference and operation instability.

6. Utility meters may record measurements inaccurately, resulting in higher billings.

7. Harmonics cause failure of the commutation circuits, found in DC drives and AC drives with silicon controlled rectifiers.

8. Computers/telephones may experience interference or failures.
2.3 INTERNATIONAL HARMONIC STANDARDS

The study of the effect of the harmonic distortion has lead to the development of standards to limit its magnitude in order to prevent damage on equipment and on the power system itself. After considering the effects and damages due to harmonic distortion, the international standards were introduced to supervise harmonic distortion issues. IEEE standard 519-1992 and IEC standard 61000-4-7 are the main international standards for measurement and analysis of harmonics in power systems. These standards used for specifying harmonic distortion is mainly divided into two types:

1. Load side standards: These are the standards which are used to restrict the harmonic components coming out of the experiments. These standards were mentioned in IEC-555 by Simith (1992).

2. System side standards: These are the standards which are used to limit harmonics in electric power supply or at the point of common coupling. These standards were mentioned in IEEE Standard 519-1992.

In 1969 the harmonic related standards were introduced. IEC (International Electrotechnical Commission) and CENELEC (European Committee for Electrotechnical Standardization) committees were formed to investigate the effects of harmonic distortion in case of home appliances. The CENELEC and IEC came with two harmonic distortion standards namely EN 50006 and IEC 555 in the year 1975 and 1982 respectively. Germany was the first country to use IEC 555 after EN 50006 standard from 1982 because of its more comprehensive nature. Then the IEC 555 was updated as IEC 555-2, which was then agreed by CENELEC to be EN 60555-2 European standard
was described in the year 1987. IEC 555-2 standard was updated and named as IEC 1000-3-2 in the year 1995 by Finlay (1991). IEC 1000-3-2 had great scope of applicability over IEC 555-2 it cover all equipments up to 16 Ampere per phase. According to the updated standards these equipments fall under any one of the four categories.

1. Class A: The equipment which comes under class A category are balanced three phase equipment and equipment not falling into another category.

2. Class B: The equipment which comes under class B category is portable and similar tools.

3. Class C: The equipment which comes under class C category is lighting equipment, including dimmer controls.

4. Class D: The equipment which comes under class D category are equipments having an input current with a “special wave shape”.

This standard also stipulates the maximum allowable harmonic distortion allowed in the voltage and current waveforms on various types of systems. Most of the general equipment comes under class A and the applicable limits for this are given in Table 2.1, all the units are expressed in absolute amps. Similarly the limits for class B, class C and class D equipments are given in the Tables 2.2-2.4 respectively. The limits specified in the table are very important in order to improve power supply design and to reduce problem in equipments.
Table 2.1  Harmonic current limits for class A equipment and certain class C equipment with phase controlled lamp dimmers (IEC 555-2)

<table>
<thead>
<tr>
<th>Harmonic order “n”</th>
<th>Maximum permissible harmonic current (in amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Odd harmonics</td>
</tr>
<tr>
<td>3</td>
<td>2.30</td>
</tr>
<tr>
<td>5</td>
<td>1.14</td>
</tr>
<tr>
<td>7</td>
<td>0.77</td>
</tr>
<tr>
<td>9</td>
<td>0.40</td>
</tr>
<tr>
<td>11</td>
<td>0.33</td>
</tr>
<tr>
<td>13</td>
<td>0.21</td>
</tr>
<tr>
<td>15-39</td>
<td>0.15 $\times$ (15/n)</td>
</tr>
<tr>
<td></td>
<td>Even harmonics</td>
</tr>
<tr>
<td>2</td>
<td>1.08</td>
</tr>
<tr>
<td>4</td>
<td>0.43</td>
</tr>
<tr>
<td>6</td>
<td>0.30</td>
</tr>
<tr>
<td>8-40</td>
<td>0.23 $\times$ (8/n)</td>
</tr>
</tbody>
</table>

Table 2.2 Harmonic current limits for class B equipment

<table>
<thead>
<tr>
<th>Harmonic order “n”</th>
<th>Maximum permissible harmonic current (in amperes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Odd harmonics</td>
</tr>
<tr>
<td>3</td>
<td>3.45</td>
</tr>
<tr>
<td>5</td>
<td>1.71</td>
</tr>
<tr>
<td>7</td>
<td>1.155</td>
</tr>
<tr>
<td>9</td>
<td>0.60</td>
</tr>
<tr>
<td>11</td>
<td>0.495</td>
</tr>
<tr>
<td>13</td>
<td>0.315</td>
</tr>
<tr>
<td>15-39</td>
<td>0.225 $\times$ (15/n)</td>
</tr>
<tr>
<td></td>
<td>Even harmonics</td>
</tr>
<tr>
<td>2</td>
<td>1.62</td>
</tr>
<tr>
<td>4</td>
<td>0.645</td>
</tr>
<tr>
<td>6</td>
<td>0.45</td>
</tr>
<tr>
<td>8-40</td>
<td>0.345 $\times$ (8/n)</td>
</tr>
</tbody>
</table>
Table 2.3  Harmonic limits for class C equipment less than 25 watts (IEC 555-2)

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Maximum Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2%</td>
</tr>
<tr>
<td>3</td>
<td>30%</td>
</tr>
<tr>
<td>5</td>
<td>10%</td>
</tr>
<tr>
<td>7</td>
<td>7%</td>
</tr>
<tr>
<td>9</td>
<td>5%</td>
</tr>
<tr>
<td>11-39</td>
<td>3%</td>
</tr>
</tbody>
</table>

Table 2.4  Harmonic limits for class D and class C equipment larger than 25 watts (The relative limits apply up to 300 watts)

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Maximum permissible harmonic current</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relative (mA/W)</td>
</tr>
<tr>
<td><strong>Odd harmonics</strong></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.6</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
</tr>
<tr>
<td>7</td>
<td>1.5</td>
</tr>
<tr>
<td>9</td>
<td>1.0</td>
</tr>
<tr>
<td>11-39</td>
<td>0.6× (11/n)</td>
</tr>
<tr>
<td><strong>Even harmonics</strong></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

To evaluate the harmonic level in power systems guide IEEE-519 was issued by IEEE-IAS (Institute of Electrical and Electronics Engineering-Industry Application Society) and it is used for harmonic inspection and control since 1992. This IEEE-519 is found to be more comprehensive than other standards like IEC 555-2 and this is mainly concerned with the Point of
Common Coupling (PCC) in a power system. This IEEE-519 shows the proposed harmonic injection limit and proposed harmonic voltage distortion limit for general power systems which are given in Table 2.5 and Table 2.6 respectively. The main difference between the IEEE 519 and IEC 555-2 are given below.

1. IEEE 519 restricts the levels of supply voltage and load current at the same time where as IEC 555-2 aims at load devices.

2. IEEE 519 is just an industrial suggestion where as IEC 555-2 is compulsory.

**Table 2.5 IEEE-519 voltage distortion limits**

<table>
<thead>
<tr>
<th>Bus voltage at PCC</th>
<th>Individual voltage distortion (%)</th>
<th>Total voltage distortion (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 69 kV</td>
<td>3.0</td>
<td>5.0</td>
</tr>
<tr>
<td>69 kV to 138 kV</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>138 kV and above</td>
<td>1.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Table 2.6 Proposed harmonic current injection limits for individual customers on the power systems-IEEE 519 (1991)**

<table>
<thead>
<tr>
<th>$\frac{I_s}{I_l}$</th>
<th>Individual frequency limits (%)</th>
<th>TCD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n&lt;11 11&lt;n&lt;17 18&lt;n&lt;23 24&lt;n&lt;35 n&gt;34</td>
<td></td>
</tr>
<tr>
<td>&lt;20</td>
<td>4.0 2.0 1.5 0.6 0.3</td>
<td>5.0</td>
</tr>
<tr>
<td>20-50</td>
<td>7.0 3.5 2.5 1.0 0.5</td>
<td>8.0</td>
</tr>
<tr>
<td>50-100</td>
<td>10.0 4.5 4.0 1.5 0.7</td>
<td>12.0</td>
</tr>
<tr>
<td>100-1000</td>
<td>12.0 5.5 5.0 2.0 1.0</td>
<td>15.0</td>
</tr>
<tr>
<td>&gt;1000</td>
<td>15.0 7.0 6.0 2.5 1.4</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Notes: $I_s$ is the maximum short circuit current at PCC. $I_l$ is the rated or maximum customer load current. ‘n’ is the harmonic order and TCD means Total Current Distortion which is calculated as a square root of the sum of the squares.
2.4 HARMONIC ELIMINATION SCHEMES

Arun Arota et al (1998), Das (2004), Key and Lai (1998) have widely suggested few methods to mitigate the harmonics. Mohamed et al (2007), Tihamer Adam et al (2002), Yaow-Ming (2003), Zobaa (2004), and Zacharia et al (2007) have discussed the application of LC filters to control harmonic component in the power electronic devices. The usage of active power filter to mitigate the harmonics was explained by Bhim Singh et al (1998), Bor-Ren Lin et al (2002), Domijan and Embriz-Santander (1990), Mahanty and Kapoor (2008), and Singh et al (1999). The multi-pulse technique and pulse width modulation has been described in length by Bowes and Grewal (1999), Joong-Ho Sung et al (1997), and Trzynadlowsk (1996). One of the important methods used to mitigate harmonics are passive LC filters that are used to compensate reactive power and control harmonics. The principle behind this is to mitigate higher order harmonics by bypassing the higher order harmonic currents using a capacitor as low impedance. The main limitation of a passive filter is that its performance is easily influenced by impedance and operation status of the power grid. This also creates parallel resonance with system impedance results in magnification of harmonic current, overload and some times break LC filters. The other simple mitigation method is using single bandwidth filter, but it is used to control certain order of harmonics. The principle for the single bandwidth filter is to create a series resonance for particular frequency and will not allow the back flow of frequency harmonic current to the system.

In order to overcome the limitations of the passive filter Bhattacharya et al (1998) proposed the other method to mitigate harmonics by active power filter which extracts the harmonic current components from compensated load which in turn create compensation current of opposite polarity. This kind of filter follows the change of harmonic amplitude and
frequency. Its performance is not influenced by the system impedance. The converter part APF is of two types based on the source type. They are voltage source type and current source type. 90% of the active power filter equipments are voltage source type. Depending upon the mode of connection with the load, active power filters are divided into 2 types namely:

1. Series connected APF

2. Parallel connected APF

Most of the APFs are operated under parallel connection mode and these are used individually, or along with passive LC Filters. The harmonic mitigation is carried out effectively by creating a unity power factor converter, which do not create harmonic current and keep the power factor value nearer to 1. In some cases, high power factor converters are considered as unity power factor converter.

In high power factor converter, multi pulse technique and PWM control technique are effectively used to eliminate lower order harmonics, and to create waveform similar to sine waveforms. Similar output waveforms are obtained for pure sine wave if higher levels are used. Multi pulse technique is mainly used in high power range.

2.5 PULSE WIDTH MODULATION

One of the most widely used strategies for controlling the AC output of power electronic converters is the technique known as pulse width modulation, which varies the duty cycle of the converter switches at a high switching frequency to achieve a target of average or low frequency output voltage or current.
Pulse width modulation techniques has been discussed in detail by Joachim Holtz (1992) and are used to design the width of pulse sequences so that a fundamental component voltage with specified magnitude and phase emerges, and harmonics are shifted towards higher frequency bands. PWM allows the freedom of controlling the harmonic spectra of the converter voltage or current. Pulse width modulation does not reduce the total distortion factor of the current or voltage, but filtering becomes easier (also reduces filter size) due to the fact that the first present harmonics are of higher order.

A PWM waveform consists of a series of positive and negative pulses of constant amplitude but with variable switching instances. The typical goal is to generate a train of pulses such that the fundamental component of the resulting waveform has a specified frequency and amplitude. According to Sidney and Paul (1988), the converter switches are turned on and off several times during each half cycle and the output voltage is controlled by varying the width of the pulses.

PWM technique has been widely used in DC-AC inverter control. It effectively reduces the power loss and heat dissipated in the output stage when delivering power to a load. PWM control strategy results in a pulse train of fixed amplitude and frequency, only the width of pulse is varied in proportion to a reference voltage. The end result is that the effective voltage at the load is proportional to the reference voltage. Due to the rectangular shape of the output signal, little power is wasted in the output stage. In this way, PWM techniques offer opportunity to build efficient power delivery systems. PWM approach is applied when the pulse period of a PWM waveform is much shorter than the time constant of the load.

Elimination of lower order harmonics from the output of voltage source PWM inverters brings two major benefits, which were explained by Sun et al (1994).
1. If the inverter is used to supply constant frequency AC power to general AC loads, a filter is usually installed at its output. In this case, when lower order harmonics are eliminated through proper modulation of the inverter, only higher order harmonics appear at the output and need to be attenuated by the filter. The cut-off frequency of the filter can thus be increased, leading to the reduction of the filter size, and increase in system efficiency.

2. When used in an AC drive system, elimination of lower order harmonics from the inverter output leads to great reduction of lower order harmonic torques generated by the motors. Although harmonic torque is the interaction result between stator and rotor harmonic currents of different order, higher order harmonic currents have smaller magnitudes due to the larger impedance, which the motor presents to higher order harmonic voltages. Their contributions to lower order harmonic torques are thus less significant. Therefore, lower order harmonic torques generated by the motor is greatly reduced. Consequently, lower frequency resonance of the mechanical system driven by the motor is avoided.

Various PWM techniques have been designed to minimize harmonics in converters. They are:

1. Carrier based pulse width modulation
2. Space vector modulation pulse width modulation
3. Third harmonic injection pulse width modulation
4. Selective harmonic elimination pulse width modulation
2.5.1 Carrier Based Pulse Width Modulation

According to Antonio Cataliotti et al (2007), Carrier Based PWM (CBPWM) methods compare a reference waveform with a triangular or saw-tooth carrier at a higher frequency $f_s$ (sampling frequency) and decide whether to turn a switch on or off. Three significantly different PWM methods for determining the converter switching on time have been proposed for fixed frequency modulation systems by Azli and Baskar (2004), Holmes and Lipo (2003), Sidney and Sukhminder (2000) are as follows,

1. Analog or naturally sampled PWM: Switching at the intersection of a target reference waveform and a high frequency carrier.

2. Digital or regularly sampled PWM: Switching at the intersection between a regularly sampled reference waveform and a high frequency carrier.

3. Direct PWM: Switching so that the integrated area of the target reference waveform over the carrier interval is the same as the integrated area of the converter switched output.

Further, if one sample is used per carrier period, the regular method is symmetric, while in case of two samples it is asymmetric.

The conventional carrier based method was explained by Halasz et al (1995), Leon and Thomas (1998) to control a voltage source converter by Sinusoidal PWM (SPWM), for which the reference waveform is a sinusoidal waveform. In SPWM the pulse width is not constant but varied by changing the amplitude of the sinusoidal waveform. In this method a triangular waveform of a particular amplitude and frequency is compared to a sinusoidal waveform in phase with the input voltage of an AC/DC converter. Lower
order harmonics are eliminated using this technique. A three-phase SPWM controller essentially consists of three separate SPWM controllers with reference waveforms that are $120^\circ$ out of phase. SPWM is used to control a Voltage Source Converter (VSC) with two, three, or higher number of levels. Using the same carrier frequency, the larger the number of voltage levels, the higher the quality of the output waveform. The number of used carrier waveforms and the number of switches in each leg, depend on the number of VSC voltage levels. In SPWM, only the value of the reference waveform at its intersection with the carrier is used to determine voltage pulses. This method essentially uses an approximation of the reference waveform. Figure 2.1 show an example of pulse signal generation for sinusoidal pulse width modulation reconstruction.

![Figure 2.1 Pulse signal generation of sinusoidal pulse width modulation](image)
The other types of carrier based pulse width modulation are: Uniform Pulse Width Modulation (UPWM), and Modified Sinusoidal Pulse Width Modulation (MSPWM). In UPWM switching is symmetrical causing all pulses to have the same width. In this method a rectangular waveform of certain frequency is compared to a rectangular waveform in phase with the supply voltage. A triangular waveform determines the switching frequency, and its amplitude determines the width of the pulses. Elimination of certain harmonics depends on the pulse width chosen. In MSPWM the carrier wave is applied during the first and last $60^\circ$ intervals per half cycle. A higher fundamental waveform component is achieved using the MSPWM. Elimination of harmonics depends on the switching frequency and is limited by device switching speed, switching loss and device power ratings.

### 2.5.2 Space Vector Modulation Pulse Width Modulation

The direct digital technique or the space vector modulation technique was proposed by Pfaff et al in 1984. This scheme was further developed by Vander et al in the year 1988 and explained in detail by Narayanan and Ranganathan (2009), Sidney and Sukhminder (2000), Thomas and Donald (2005).

Space Vector Modulation (SVM) is one kind of pulse width modulation strategy, which is quite different from CBPWM methods. CBPWM methods are based on comparison of a reference waveform with a high frequency carrier, whereas in the SVM strategy switching instants and duration of each switching state are calculated from simple equations. Moreover, there is only one reference vector, in contrast to the three individual reference waveforms for three phases of the system. It offers more flexibility as well as a maximum modulation index, $M$ of 1.15.
In conventional pulse width modulation strategies, each leg of the VSC is controlled independently. That is, SPWM uses one sinusoidal waveform for controlling each of the three legs of the converter. Likewise, placement of the switching pulses for each leg of a converter under SHE control is determined separately. In contrast, space vector modulation is intrinsically designed for three-phase converters. It combines the three reference waveforms used in CBPWM methods into one single vector, called the reference space vector. In SVM, instead of modulating waveforms, a modulating reference vector is employed.

2.5.3 Third Harmonic Injection Pulse Width Modulation

Many other techniques were developed for harmonic elimination in order to suppress the lower ordered harmonics. The Third Harmonic Injection PWM (THIPWM) technique was described by Ali (2007), (2008), Boglietti et al (1995), Kaili Xu et al (2007), and Lawrance et al (1996). According to them, adding a measure of third harmonic to the output of each phase of a three-phase inverter, it is possible to obtain a line-to-line output voltage that is 15 percent greater than that obtainable when pure sinusoidal modulation is employed. The line-to-line voltage is undistorted. This method permits the inverter to deliver an output voltage approximately equal to the voltage of the AC supply to the inverter. This method is still being used in dedicated applications, which describes a technique of injecting third harmonic zero sequence current components in the phase currents, which greatly improves the machine torque density. Among all the PWM techniques only a few PWM strategies have been accepted and used mainly due to the simplicity of implementation.
2.5.4 Selective Harmonic Elimination Pulse Width Modulation

Selective harmonic elimination PWM technique was introduced by Husmukh and Richard in 1973. The idea of this method is the basic square wave output is “chopped” a number of times. The chopping time is related to a set of switching angles of inverter switches, which are obtained by proper off-line calculations. By appropriate distribution of the switching angles to turn the inverter bridge switches on and off, the output waveform of the inverter is controlled and reaches the goal of eliminating lower order harmonics.

In contrast to carrier based PWM methods, in which switching instants are determined by direct comparison of reference and carrier waveforms, in selective harmonic elimination method the exact moments of switching instants are calculated according to the desired fundamental component and the harmonic components to be eliminated. Because of complexity of equations to be solved to find switching instants, the number of switching angles is normally kept low to make the calculations simple. This also has the advantage of lowering the converter switching losses and switching instants are found by offline calculations.

According to Hasmukh and Richard (1974), Chiasson (2004) and Sirisukprasert et al (2002), the selective harmonic elimination PWM technique is one of the optimal PWM techniques. It effectively reduces the harmonics content of inverter output waveform and generate higher quality spectrum through elimination of specific lower order harmonics. Therefore, it has been applied in power electronic controllers extensively and many related techniques have been proposed in recent year. The basic idea is to set up the notches at the specially designated sites of PWM waveform and the inverter alters directions many times per half cycle to control the inverter’s output waveform appropriately.
2.6 SHE-PWM WAVEFORM SYNTHESIS

Selective harmonic elimination is used to control both two-level and higher level voltage source converters. In two-level SHE, for each half period both $+V_{dc}$ and $-V_{dc}$ voltage pulses are used, which is called as bipolar SHE, while in three-level, for each half cycle only one of $+V_{dc}$ or $-V_{dc}$ is used, and is known as unipolar SHE. The output waveform of a SHE modulated VSC is normally constructed in a way that it possesses Quarter Wave Symmetry (QWS). The output waveform of one phase is expressed in Fourier series expansion and it is given in equation (2.1).

\[ v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi f_0 nt) + b_n \sin(2\pi f_0 nt) \]  

(2.1)

where, $f_0$ is the fundamental frequency and ‘n’ is the order of harmonic.

$a_0$, $a_n$ and $b_n$ are the Fourier coefficients and is obtained from $v(t)$,

\[ \omega = 2\pi f_0 \]

Then the expression is written as equation (2.2),

\[ v(\omega t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \]  

(2.2)

The coefficients $a_0$, $a_n$ and $b_n$ are found from the canonical form and expressed as equations (2.3)-(2.5).

\[ a_0 = \frac{1}{2\pi} \int_0^{2\pi} V_{dc} d\omega t \]  

(2.3)

\[ a_n = \frac{1}{\pi} \int_0^{2\pi} V_{dc} \cos(n\omega t) d\omega t \]  

(2.4)

and \[ b_n = \frac{1}{\pi} \int_0^{2\pi} V_{dc} \sin(n\omega t) d\omega t \]  

(2.5)
The equation (2.2) is written as equation (2.6).

\[ v(\omega t) = a_0 + \sum_{n=1}^{\infty} c_n \sin(n \omega t + \varphi_n) \]  

(2.6)

where, 
\[ a_n = c_n \sin \varphi_n \]
\[ b_n = c_n \cos \varphi_n \]
\[ c_n = \sqrt{a_n^2 + b_n^2} \]
\[ \varphi_n = \tan^{-1} \frac{a_n}{b_n} \quad \text{where} \quad b_n > 0 \]
\[ \varphi_n = \tan^{-1} \frac{a_n}{b_n} + 180^\circ \quad \text{where} \quad b_n < 0 \]

2.6.1 Periodicity

Periodicity of waveforms ensures that they have a discrete spectrum. This is guaranteed if the sampling frequency \( f_s \) is an integer multiple of fundamental frequency \( f_0 \) of the reference waveform. Therefore, the switching pattern of the inverter remains identical in all periods as such and constructed voltages are also periodic. In order to keep harmonics minimal, to improve harmonic indices, and to meet the power system requirements, in addition to periodicity (or synchronization), it is desired that synthesized waveforms possess three-Phase Symmetry (3PS), Half Wave Symmetry (HWS), and quarter wave symmetry.

2.6.2 Three Phase Symmetry

For balanced operation of the load, it is required that the converter output waveforms possess three-phase symmetry and all harmonics are balanced. In the particular case of triplen harmonics in a three-phase system, this leads to their elimination from the line voltages. The necessary and sufficient condition for 3PS of three-phase voltages is that they are displaced
by 120°. For positive sequence, the phase voltages are displaced by 120° and they are written in equation (2.7).

\[ v_a(\omega t + \frac{2\pi}{3}) = v_c(\omega t) \]
\[ v_b(\omega t - \frac{2\pi}{3}) = v_a(\omega t) \]
\[ v_c(\omega t + \frac{2\pi}{3}) = v_b(\omega t) \] (2.7)

### 2.6.3 Odd Symmetry

If the voltage function \( v(\omega t) \) is periodic function and also contains symmetries, it satisfies the following periodicity property equation (2.8).

\[ v(\omega t) = -v(-\omega t) \] (2.8)

For periodic functions with odd symmetry, the Fourier coefficients are given in equation (2.9).

\[ a_0 = 0 \]
\[ a_n = 0 \text{ for all } n. \] (2.9)

\[ b_n = 4 \frac{n}{\pi} V_{dc} \sin(n\omega t) d(\omega t) \]

A periodic function possessing odd symmetry is written in terms of an infinite series of only sine functions.

### 2.6.4 Half Wave Symmetry

While 3PS is targeted at eliminating triplen harmonics, half wave symmetry eliminates even harmonics. Absence of even harmonics is particularly important or otherwise they lead to resonance in power networks.
Moreover, the DC component, because of changing the operating point of electrical apparatus, is harmful. A waveform possesses HWS if its mirror in x-axis shifted by half of the period is identical to itself. If the function \( v(\omega t) \) is half wave symmetry function and then it satisfies the half wave symmetry property equation (2.10).

\[
v(\omega t) = -v(\omega t + \pi)
\]  

(2.10)

For periodic function with half wave symmetry, the Fourier coefficients are given in equation (2.11).

\[
a_0 = 0 \\
a_n = 0 \text{ for even } n. \\
a_n = \frac{4}{\pi} \int_{0}^{\pi} V_{dc} \cos(n\omega t) d(\omega t) \text{ for odd } n. \\
b_n = 0 \text{ for even } n. \\
b_n = \frac{4}{\pi} \int_{0}^{\pi} V_{dc} \sin(n\omega t) d(\omega t) \text{ for odd } n.
\]

(2.11)

A periodic function possessing half wave symmetry has an average value of zero and its even harmonic components are zero.

### 2.6.5 Quarter Wave Symmetry

Quarter wave symmetry, which is a subset of HWS, guarantees not only the even harmonics are zeros, but all harmonics are either in phase or anti-phase with the fundamental component. Since there are only two phase angles (0° and 180°) involved for a waveform with such symmetry, elimination of harmonics by injection requires less effort. This is a desired feature, although a restrictive one. QWS is obtained if the waveform has
symmetry around the midpoints of positive and negative half cycles. This means that the waveform repeats the same pattern every quarter cycle. Such a waveform is expressed in equation (2.12).

\[
v(\omega t) = \begin{cases} 
  v(\omega t); & 0 \leq \omega t \leq \frac{\pi}{2} \\
  v(-\omega t + \pi); & \frac{\pi}{2} \leq \omega t \leq \pi \\
  -v(\omega t - \pi); & \pi \leq \omega t \leq \frac{3\pi}{2} \\
  -v(-\omega t + \pi); & \frac{3\pi}{2} \leq \omega t \leq 2\pi 
\end{cases}
\]  

(2.12)

If the function \( v(\omega t) \) is half wave symmetry and periodic function, then it is called as odd quarter wave symmetry. For periodic function with odd quarter wave symmetry, the Fourier coefficients are given by equation (2.13).

\[
a_0 = 0 \\
a_n = 0 \text{ for all } n. \\
b_n = 0 \text{ for even } n. \\
b_n = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} V_{dc} \sin(n\omega t) d(\omega t) \text{ for odd } n.
\]  

(2.13)

A periodic function possessing odd quarter wave symmetry has zero average value. The reason is due to the fact that the function is odd. Also, odd symmetry results in all of the cosine harmonics being zero. Because of the half wave symmetry of the waveform, all \( a_n \) and even-numbered \( b_n \) coefficients are zero. The \( n^{th} \) harmonic is eliminated if the respective \( b_n \) coefficient is set equal to zero. Also, for a three phase system, triplen harmonics in the phase voltage are cancelled out in the line voltage and hence, are not important. Therefore, the lower order harmonics to be removed are odd, non-triplen components starting as 5, 7, 11, 13, 17…. Selective harmonic
elimination pulse width modulation techniques have been mainly developed for two level (Bipolar) and three-level (Unipolar) converter schemes and Fourier series expansion of waveforms are explained below.

2.7 UNIPOLAR SELECTIVE HARMONIC ELIMINATION

In unipolar SHE-PWM, the output voltage can be $+V_{dc}$, $-V_{dc}$ or 0. Figure 2.2 illustrates a unipolar SHE-PWM switching scheme using three switching angles. Unipolar SHE-PWM uses predetermined switching angles to produce an output consisting of multiple pulses of varying widths was discussed in detail by Prasad et al (1990). The number of pulses per fundamental cycle is equal to twice the number of switching angles used.

![Unipolar PWM switching scheme](image)

**Figure 2.2 Unipolar PWM switching scheme**

The mathematical models of unipolar programmed PWM scheme includes single phase application and three phase application were discussed

\[ v(\omega t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t) \]

\[ b_n = \frac{4}{\pi} \int_{0}^{\pi} V_{dc} \sin(n\omega t) d(\omega t) \text{ for odd } n. \]

\[ a_n = 0 \text{ for all.} \]

The expression for Fourier coefficients of a waveform with N switching angles per cycle is given in equation (2.14).

\[ a_n = 0 \]

\[ b_n = \frac{4V_{dc}}{n\pi} \sum_{i=1}^{N} (-1)^{i-1} \cos(n\alpha_i) \quad (2.14) \]

The equation (2.15) gives the non-zero \( b_n \) coefficients for an odd \( n \).

\[ b_n = \frac{4V_{dc}}{n\pi} \left( \cos n\alpha_1 - \cos n\alpha_2 + \cos n\alpha_3 - ..... \right) \quad (2.15) \]

Fourier series expansion of the waveform is given in equation (2.16) and its summarized form is given in equation (2.17).

\[ v(\omega t) = \frac{4V_{dc}}{\pi} \left\{ \left( \cos \alpha_1 - \cos \alpha_2 + \cos \alpha_3 - .... \right) \sin \omega t \right. \]
\[ + \left( \cos 3\alpha_1 - \cos 3\alpha_2 + \cos 3\alpha_3 - ..... \right) \frac{\sin 3\omega t}{3} \]
\[ + \left( \cos 5\alpha_1 - \cos 5\alpha_2 + \cos 5\alpha_3 - ..... \right) \frac{\sin 5\omega t}{5} + .... \left\} \quad (2.16) \]
\[ v(\omega t) = \frac{4V_{dc}}{\pi} \left\{ \sum_{n=1,3,5,...}^{N} \frac{\sin n\omega t}{n} \left[ \sum_{i=1}^{N} (-1)^{i+1} \cos n\alpha_i \right] \right\} \] (2.17)

where, \( V_{dc} \) is the available DC bus voltage and \( \alpha_1 < \alpha_2 < ... < \alpha_N < \frac{\pi}{2} \).

When there are three switching in each quarter cycle as depicted in Figure 2.2, three unknowns of \( \alpha_1, \alpha_2, \) and \( \alpha_3 \) lead to three equations. Again, one of these equations is used to satisfy the condition on the magnitude of the fundamental component, and the remaining two equations are used to eliminate two lowest harmonics (5 and 7). The final set of non-linear equations for three switching angles is given in the equation (2.18).

\[
\frac{4V_{dc}}{\pi} \left( \cos \alpha_1 - \cos \alpha_2 + \cos \alpha_3 \right) = v_i
\]
\[
(\cos 5\alpha_1 - \cos 5\alpha_2 + \cos 5\alpha_3) = 0
\]
\[
(\cos 7\alpha_1 - \cos 7\alpha_2 - \cos 7\alpha_3) = 0
\] (2.18)

Because of the three phase balanced circuit characteristics, the harmonics whose order is an integer multiple of three, will be cancelled automatically. The mathematical models of single phase application and three phase application are the same except that triplen harmonics must also be eliminated in single phase application.

Unipolar SHE-PWM shares many of the advantages of bipolar SHE-PWM. Unipolar SHE-PWM is still used with low modulation indices as well. Like bipolar SHE-PWM, one disadvantage of unipolar SHE-PWM lies in harmonic distortion. For low modulation indices, unipolar SHE-PWM leads to an output with higher total harmonic distortion. However, unipolar SHE-PWM tends to produce a lower THD than bipolar SHE-PWM. It provides a more natural approximation to a sinusoidal waveform. Unipolar SHE-PWM also tends to produce less EMI than bipolar SHE-PWM. Bipolar
SHE-PWM produces voltage changes equal to $2V_{dc}$. However, unipolar SHE-PWM produces voltage changes equal to $V_{dc}$. Furthermore, unipolar SHE-PWM increases the effective switching frequency by a smaller factor than bipolar SHE-PWM.

2.8 **BIPOLAR SELECTIVE HARMONIC ELIMINATION**

Bipolar SHE-PWM is another switching scheme, which involves harmonic elimination and was described in detail by Jose et al (2001), Guzman et al (2004), Maswood and Wei (2005), Salam et al (2003). One switching scheme involving harmonic elimination that has been widely used for many years is bipolar SHE-PWM. In bipolar SHE-PWM, the line to neural output voltage is either $+V_{dc}$ or $-V_{dc}$. The mathematical models of bipolar programmed PWM scheme includes single phase applications (SLN1: quarter wave symmetric PWM, switching angle spread $0^0$ to $90^0$ and SLN2: same as SLN1 with the phase shift, to suppress the first significant harmonic) and three phase applications (TLN1: quarter wave symmetric PWM, switching angle spread $0^0$ to $90^0$ and TLN2: quarter wave symmetric PWM, switching angle spread $0^0$ to $60^0$). Figure 2.3 illustrates the bipolar SHE-PWM switching scheme using three switching angles for TLN2. Though many different type of quarter wave symmetric SHE-PWM methods are available for three phase Voltage Source Inverter (VSI), this thesis deals with TLN1 type of SHE-PWM technique. The main reason for choosing this type of SHE-PWM is that, the TLN1 SHE-PWM results in lower harmonic losses and therefore contributes to lower harmonic heating and consequently lowers derating of the AC motor drive.

TLN1 waveform has quarter wave symmetry with switching angle spread $0^0$ to $90^0$, TLN1 waveform is considered for the calculation. The Fourier series expression for single phase output waveform is expressed in equation (2.2).
The output waveform of a SHE modulated voltage source converter is normally constructed in a way that it possesses quarter wave symmetry. The SHE-PWM output waveform of three phase application which has QWS is mathematically obtained by Fourier series for TLN1 is given in equation (2.19).

\[ v(\omega t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t) \]

\[ a_n = 0 \text{ for all } n. \] (2.19)

\[ b_n = \frac{4 \sqrt{3}}{\pi} V_{dc} \sin(n\omega t) d(\omega t) \text{ for odd } n. \]

The expression for Fourier coefficients of a waveform with N switching angles per cycle is given in equation (2.20).
\( a_n = 0 \) for all \( n \).

\[
b_n = \frac{4}{n\pi} \left[ -1 - 2 \sum_{i=1}^{N} (-1)^i \cos(n\alpha_i) \right] \tag{2.20}
\]

Non-zero \( b_n \) coefficients are calculated from the equation (2.21).

\[
b_n = \frac{4V_{dc}}{n\pi} \left\{ (-1 + 2\cos n\alpha_1 - 2\cos n\alpha_2 + 2\cos n\alpha_3 - \ldots) \right\} \tag{2.21}
\]

Fourier series expansions of a waveform with \( N \) switching per quarter cycle are given in equation (2.22) and summarised form is given in equation (2.23).

\[
v(\omega t) = \frac{4V_{dc}}{\pi} \left\{ (-1 + 2\cos \alpha_1 - 2\cos \alpha_2 + 2\cos \alpha_3 - \ldots) \sin \omega t \\
- (-1 + 2\cos 3\alpha_1 - 2\cos 3\alpha_2 + 2\cos 3\alpha_3 - \ldots) \frac{\sin 3\omega t}{3} \\
- (-1 + 2\cos 5\alpha_1 - 2\cos 5\alpha_2 - 2\cos 5\alpha_3 - \ldots) \frac{\sin 5\omega t}{5} - \ldots \right\} \tag{2.22}
\]

\[
v(\omega t) = \frac{4V_{dc}}{\pi} \left\{ \sum_{n=1}^{\infty} \frac{\sin n\omega t}{n} \left[ -1 - 2 \sum_{i=1}^{N} (-1)^i \cos n\alpha_i \right] \right\} \tag{2.23}
\]

where, \( V_{dc} \) is the available DC bus voltage and \( \alpha_1 < \alpha_2 < \ldots < \alpha_N < \frac{\pi}{2} \).

When there are three switching in each quarter cycle, three unknowns of \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) lead to three equations. Figure 2.3 shows the output waveform of a two-level SHE controlled VSC with three switching angles. One equation is used to satisfy the condition of the magnitude of the fundamental component, and the remaining two equations are used to eliminate the 5\(^{th}\) and 7\(^{th}\) harmonic components. This is shown in the following equation (2.24).
\[
\frac{4V_{dc}}{\pi} \left( -1 + 2 \cos \alpha_1 - 2 \cos \alpha_2 + 2 \cos \alpha_3 \right) = v_i \\
\left( -1 + 2 \cos 5\alpha_1 - 2 \cos 5\alpha_2 + 2 \cos 5\alpha_3 \right) = 0 \tag{2.24} \\
\left( -1 + 2 \cos 7\alpha_1 - 2 \cos 7\alpha_2 + 2 \cos 7\alpha_3 \right) = 0
\]

The most important advantage of bipolar SHE-PWM is that the control is not as complicated as in other switching schemes. One of the main disadvantages of using bipolar SHE-PWM concerns its applicability when low modulation indices are used. When low modulation indices are used, one may not be able to use the fundamental switching scheme to perform the desired harmonic elimination process. Both methods are designed based on the frequency domain, in contrast to space vector PWM and bipolar modulation which are based on the time domain. So, these two methods have greatly reduces harmonic content and are highly recommended from the medium to the high modulation region. The three-level inverter usually uses high voltages and is made using Gate Turn Off (GTO) switches which require a low switching frequency. This fact strongly supports the need to use an efficient strategy from the medium to the high modulation region.

Some of the methods proposed in the literature for PWM waveform design are: modulation function techniques, space vector techniques, and feedback methods. These methods suffer from high residual harmonics that are difficult to control. A method that theoretically offers the highest quality of the output waveform is the so-called programmed or optimal PWM. A sizable amount of work has been done on the optimal solution for the transcendental equations describing the SHE-PWM switching patterns.
2.9 SOLUTION METHODOLOGY FOR HARMONIC ELIMINATION

Many methods are available presently for the optimal solution of non-linear equations explained by Ali et al (2001), Dariusz et al (2002), Hyo et al (1995), Jurgen (1992) and these methods are based on mathematical programming techniques involving gradient search discussed by Maswood et al (1998) and direct search assuming that, the design variables are continuous. The main challenge associated with SHE-PWM techniques is to obtain the analytical solution for the resultant system of non-linear transcendental equations that contain trigonometric terms which in turn provide multiple sets of solutions, which was enumerated by Vassilios et al (2004). Several algorithms have been reported in the technical literature concerning methods of solving the resultant non-linear transcendental equations, which describes the SHE-PWM problem. For SHE-PWM, the switching instants are determined by solving a set of non-linear equations. Due to non-linear and transcendental characteristics, such equation can only be solved numerically.

To obtain fast convergence, the initial values must be selected close to the exact solutions. This is one of the most difficult tasks associated with programmed PWM techniques. However, it is difficult to derive the solutions for simultaneous transcendental equations for eliminating selected harmonics in real time applications. It is interesting to observe that the applied optimization technique, which finds the solution for higher values of modulation indices without any failure in convergence.

The problem is formulated around the desired value of the fundamental component to be generated. This method then seeks to find the angles that would provide fundamental amplitude and, would result in the elimination of a number of selected harmonics. The generalized case is to find
appropriate angles $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_i$ where $i=N$ so that $N-1$ non-triplen odd harmonics (i.e., $5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$, \ldots, $n^{th}$) are eliminated and control of the fundamental is also achieved. As stated in expression (2.21), in order to eliminate $N-1$ non-triplen odd harmonics, $N$ switching angles need to be found, and the following system of equations (2.25) must be solved.

\[-1 - 2 \sum_{i=1,2,3,\ldots}^{N} (-1)^{i} \cos (\alpha_i) - M = 0\]

\[-1 - 2 \sum_{i=1,2,3,\ldots}^{N} (-1)^{i} \cos (5\alpha_i) = 0\]

\[\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\]

\[\text{where, } 0 \leq M \leq 1.3\]

The equation (2.25) is the generalised equation to be solved to determine the switching angles. To eliminate $5^{th}$ and $7^{th}$ harmonic contents of the output waveform, the equation (2.25) is written as equation (2.26) with three unknown switching angles $\alpha_1, \alpha_2$ and $\alpha_3$.

\[(-1 + 2 \cos \alpha_1 - 2 \cos \alpha_2 + 2 \cos \alpha_3) = M\]

\[(-1 + 2 \cos 5\alpha_1 - 2 \cos 5\alpha_2 + 2 \cos 5\alpha_3) = 0\]

\[(-1 + 2 \cos 7\alpha_1 - 2 \cos 7\alpha_2 + 2 \cos 7\alpha_3) = 0\]

\[n=7,13, \ldots, 3N+1, \text{ when } N=\text{even for three phase systems}\]

\[n=5,11, \ldots, 3N+2, \text{ when } N=\text{odd for three phase systems}\]

and $0 \leq M \leq 1$. If $v_f$ is the amplitude of the fundamental component to be generated, then the equation (2.23) yields, $|v_f| = \frac{4|M|}{\pi}$
Choosing the switching angles such that a desired fundamental output is generated and specifically selected harmonics of the fundamental are suppressed. This is referred as harmonic elimination or programmed harmonic elimination as the switching angles are selected to eliminate a specific harmonics. The harmonic elimination problem was formulated as a set of transcendental equations that must be solved to determine the switching angles in an electrical cycle for turning the switches on and off in a full bridge inverter so as to produce a desired fundamental amplitude while eliminating the specific order of harmonics. These transcendental equations are then solved using iterative numerical techniques mentioned by Chunhui et al (2005) to compute the switching angles.

In order to proceed with the optimization/minimization, an objective function describing a measure of effectiveness for eliminating selected order of harmonics while maintaining the fundamental at a pre-specified value must be defined. This is converted to an optimization problem subject to constraints. The task is to determine the firing instants such that objective function $F(\alpha)$ (2.27) is minimized. Therefore, the output voltage is regulated ideally over the full range $[0, V_{dc}]$ by changing the modulation index $M$ and has no harmonics within that range, to obtain the switching instants.

The non-linear transcendental equations must be solved in order to get the desired values of the switching angles for any value of $M$. The following objective function (2.27) is proposed in this thesis as a minimisation function to determine the set of solutions for one value of $M$.

$$F(\alpha) = \text{Min} \left[ -1 - 2 \sum_{i=1,3,5,...}^{N} (-1)^i \cos(\alpha_i) - M \right] - \left[ -1 - 2 \sum_{i=1,3,5,...}^{N} (-1)^i \cos(5\alpha_i) \right]^2$$

$$+ \ldots \ldots + \left[ -1 - 2 \sum_{i=1,3,5,...}^{N} (-1)^i \cos(n\alpha_i) \right]^2$$

(2.27)
with the constrain that \( 0 < \alpha_1 < \alpha_2 < \ldots < \alpha_{i=N} \leq \frac{\pi}{2} \).

According to Vassilios et al (2004) and (2006), minimization technique combined with a random search method is applied directly to the set of the transcendental equations which results in all solutions for the specified harmonic elimination problem. The set of all equations (2.26) are derived from equations (2.23) and (2.25) have multiple solutions, which has QWS to create the desired harmonic elimination waveform and can be obtained using different iterative methods which are described in the forthcoming sessions.

Despite these difficulties, programmed PWM exhibit several distinct advantages in comparison to the conventional carrier based sinusoidal PWM schemes that are listed below.

1. About 50% reduction in the inverter switching frequency is achieved when comparing with the conventional carrier modulated sinusoidal PWM scheme.

2. Higher voltage gain due to over modulation contributes to higher utilization of the power conversion process.

3. Due to the high quality of the output voltage and current, the ripple in the DC link current is also small. Thus, a reduction in the size of the dc link filter components is achieved.

4. The reduction in switching frequency contributes to the reduction in switching losses of the inverter and permits the use of Gate Turn Off switches for high power converters.

5. Elimination of lower order harmonics causes no harmonic interference such as resonance with external line filtering networks typically employed in inverter power supplies.
A fundamental issue in the control of a voltage source inverter is to determine the switching angles, so that the inverter produces the required fundamental voltage and does not generate specific lower dominant harmonics. Due to its inherent non-linear nature, the system for harmonic elimination equations has to be solved numerically and for this purpose iterative technique such as Newton Raphson iterative algorithm is used to obtain the desired solutions for the angles. The various traditional methods used to solve the transcendental equations in order to eliminate the lower order harmonics are listed below.

### 2.10 Newton Raphson Iterative Method

At this particular phase, an iterative algorithm such as the Newton–Raphson is used to obtain the desired solutions for the angles. The iterative method is implemented using a software package such as Mathematica.

Due to its inherent non-linear nature, the system of harmonic elimination equations (2.25) has to be solved numerically and for this purpose Newton Raphson iterative Algorithm which was described by Benghanem and Draou (2005), Sahali and Fellah (2003), Sun and Grotstollen (1994a) is found to be very effective. For three phase inverter, the system of harmonic elimination equations is given in equation (2.25). For given M, this algorithm solves harmonic elimination equations iteratively in the following sequence:

1. Initial guess of a string point $\alpha^{(k)}$ for $k=0$.
2. Formulation of a local linear model using Newton Raphson method is given by the equation (2.28).

$$J(\alpha^{(k)}) \Delta \alpha^{(k)} - f(\alpha^{(k)}) = 0 \quad (2.28)$$

3. Solving of local linear model (2.28) for $\Delta \alpha^{(k)}$.
4. Updating: $\alpha^{(k+1)} = \alpha^{(k)} + \Delta \alpha^{(k)}$; $k=k+1$, return to 2
Here \( f(\alpha) \), the vector represents the left hand sides of harmonic elimination equations (2.26) and \( J(\alpha) \) is the Jacobian matrix of \( f(\alpha) \). The Jacobian matrix for the harmonic elimination equations (2.26) is written by the equation (2.29).

\[
J(\alpha) = \frac{\partial f}{\partial \alpha} = \begin{bmatrix}
-2 \sin \alpha_1 & 2 \sin \alpha_2 & -2 \sin \alpha_3 \\
-10 \sin 5 \alpha_1 & 10 \sin 5 \alpha_2 & -10 \sin 5 \alpha_3 \\
-14 \sin 7 \alpha_1 & 14 \sin 7 \alpha_2 & -14 \sin 7 \alpha_3 
\end{bmatrix}
\]

(2.29)

It is formally proved that the Jacobian matrix \( J(\alpha) \) of three-phase harmonic elimination equation is non-singular, regardless of the waveform structure and the number of switching angles. The only condition is that the \( N \) switching angles should be distinct from each other and not equal to 0, that is \( 0 < \alpha_1 < \alpha_2 < \ldots \ldots < \alpha_N \leq \frac{\pi}{2} \). This guarantees that the local linear model (2.28) is solved without any numerical difficulty.

However, providing a suitable initial guess \( \alpha^{(0)} \) which must be close enough to the exact solution so as to ensure the convergence of Newton’s algorithm is not a trivial task and deserves special attention. The traditional method for solving SHE-PWM problem using Newton Raphson algorithm, whose main shortcoming is that the results deeply depend on the selection of initial values.

Commonly, for different numerical algorithms used for solving SHE-PWM switching pattern, some specific analyses must be taken to preset the initial values and predict the trend of these values over whole range of modulation index.

Traditional optimization methods suffer from various drawbacks, such as prolonged period, tedious computational steps and convergence to local optima; thus, the more the number of harmonics to be eliminated, the
larger the computational complexity and time required. The traditional PWM method cannot completely eliminate the specified lower order harmonics. This programmed method is also called as computed PWM method, since their pulses are computed.

2.11 RESULTANT THEORY

A fundamental issue in the control of a voltage source inverter is to determine the switching angles so that the inverter produces the required fundamental voltage and does not generate specific lower dominant harmonics. The approach demonstrated here is accomplished by transforming the non-linear transcendental harmonic elimination equations for all possible switching schemes into a single set of symmetric polynomial equations in the first step. Then it is shown that a particular switching scheme is simply characterized by the location of the roots of these polynomial equations. For each value of $M$, the complete set of solutions to the equations is found using the method of resultants from elimination theory was discussed in detail by John et al (2002), (2003), (2004), (2005), Leon et al (2005) and Zhong et al (2004), (2004a).

The Fourier series expansion of the output voltage waveform is given in equation (2.24). The main objective of this method is to determine the switching angles $\alpha_1, \alpha_2$ and $\alpha_3$ to obtain the desired fundamental voltage $v_f(t)$. To use the resultant theory method, the harmonic elimination equations (2.26) are first converted to an equivalent polynomial system. Specifically, one defines $x_1 = \cos(\alpha_1), x_2 = \cos(\alpha_2), x_3 = \cos(\alpha_3)$ and uses the trigonometric identities.

$$\cos(5\alpha) = 5\cos(\alpha) - 20\cos^3(\alpha) + 16\cos^5(\alpha)$$
$$\cos(7\alpha) = -7\cos(\alpha) + 56\cos^3(\alpha) - 112\cos^5(\alpha) + 64\cos^7(\alpha)$$
To transform the conditions (2.26) into the equivalent conditions.

\[ p_1(x) \triangleq 1 - M - 2x_1 - 2x_2 - 2x_3 = 0 \]
\[ p_2(x) \triangleq 1 + 2 \sum_{i=1}^{3} (-1)^i (5x_i - 20x^3_i - 16x^5_i) = 0 \]  \hspace{1cm} (2.30)
\[ p_3(x) \triangleq 1 + 2 \sum_{i=1}^{3} (-1)^i (-7x_i + 56x^3_i - 112x^5_i + 64x^7_i) = 0 \]

where, \( x = (x_1, x_2, x_3) \) and \( M \triangleq V / (4V_d / \pi) \). Equation (2.30) is a set of the polynomial equations with three unknowns \( x_1, x_2, x_3 \). Further, the solutions must satisfy the condition \( 0 < x_3 < x_2 < x_1 \leq 1 \). Such a transformation to polynomial equations was used by Sahali and Fellah (2003), where the polynomials were then solved using iterative numerical techniques. In contrast, it is shown here how the polynomial equations are solved directly for all solutions.

### 2.11.1 Elimination using Resultants

In order to explain how one computes the zero sets of polynomial systems, a brief procedure for solving such systems is given. A systematic procedure to do this is to apply the elimination theory and use the notion of resultants. Briefly, one considers \( a(x_1, x_2) \) and \( b(x_1, x_2) \) as polynomials in \( x_2 \) whose coefficients are polynomials in \( x_1 \). Then, for example, letting \( a(x_1, x_2) \) and \( b(x_1, x_2) \) have degrees 3 and 2, respectively in \( x_2 \), they are written as the equation (2.31).

\[
\begin{align*}
  a(x_1, x_2) &= a_3(x_1)x_2^3 - a_2(x_1)x_2^2 - a_1(x_1)x_2 - a_0(x_1) \\
  b(x_1, x_2) &= b_2(x_1)x_2^2 + b_1(x_1)x_2 + b_0(x_1)
\end{align*}
\]  \hspace{1cm} (2.31)

The \( p \times p \) Sylvester matrix,
where \( p = \deg_x \{a(x_1, x_2)\} + \deg_x \{b(x_1, x_2)\} = 3 + 2 = 5 \), is defined by the equation (2.32).

\[
S_{u,b}(x_i) = \begin{bmatrix}
  a_0(x_i) & 0 & b_0(x_i) & 0 & 0 \\
  a_1(x_i) & a_0(x_i) & b_1(x_i) & b_0(x_i) & 0 \\
  a_2(x_i) & a_1(x_i) & b_2(x_i) & b_1(x_i) & b_0(x_i) \\
  a_3(x_i) & a_2(x_i) & 0 & b_2(x_i) & b_1(x_i) \\
  0 & a_3(x_i) & 0 & 0 & b_3(x_i)
\end{bmatrix}
\]  

(2.32)

The resultant polynomial is then defined by the equation (2.33). The equation (2.33) is the result of solving \( a(x_1, x_2) = 0 \) and \( b(x_1, x_2) = 0 \) simultaneously for \( x_1 \), that is for eliminating \( x_2 \).

\[
r(x_i) \triangleq \det S_{u,b}(x_i)
\]  

(2.33)

### 2.11.2 Solving the Bipolar Equations

The resultant methodology is used to solve all possible switching angles. That is \( x_3 = M - (x_1 + x_2) \) is used to eliminate \( x_3 \) from \( p_3 \) and \( p_7 \) in (2.30) to get the two polynomial equations \( p_3(x_1, x_2) = 0 \), \( p_7(x_1, x_2) = 0 \) with two unknowns which must be solved simultaneously. This is reduced to one polynomial given in equation (2.34) with one unknown by computing the resultant polynomials \( r_{p_3/p_7}(x_1) \) of the polynomial pair \( \{p_3(x_1, x_2), p_7(x_1, x_2)\} \).

\[
r_{p_3/p_7}(x_1) = 16777216M^2(1 + M - 2x_i)^4r_{u}^2(x_i)
\]  

(2.34)

where \( r_{u}(x_i) \) is a polynomial of 9th degree which is described by Chiasson et al (2004). As the parameter \( M \) is incremented in steps of 0.01, the roots of \( r_{u}(x_i) \) are found and used to back solve for \( x_2 \) and \( x_1 \). The set of all three unknowns
\((x_1, x_2, x_i)\) which satisfies \(0 \leq x_j \leq x_i \leq 1\) are used to calculate the switching angles for the harmonic elimination equations.

\[
\{(\alpha_3, \alpha_2, \alpha_1)\} = \{\cos^{-1}(x_1), \cos^{-1}(x_2), \cos^{-1}(x_3)\} \tag{2.35}
\]

The set of all possible solutions to (2.25) for the particular value of \(M\) is given in the equation (2.35). This computation was done for the various value of \(M\) by incrementing the value of \(M\) between 0 and 1.

According to John et al (2002), (2004), and Leon et al (2005), the difficulty of this approach is that when there are several DC sources, the degrees of the polynomials are quite large, thus making the computational burden of resultant polynomials quite high. However, the method introduces another step into the problem through the manipulation of higher order polynomials whose order increases as the number of harmonics to be eliminated also increases. Furthermore, it has limited chance to work for a higher order of harmonics and it is easy to apply only when such numbers are low.

2.12 CURVE FITTING TECHNIQUE

One of the most popular real world applications of mathematics is curve fitting. Curve fitting involves examining what might seem like a random data set and deriving an equation that strongly describes that set. Functions commonly used for the purpose of curve fitting include exponential and logarithmic functions, but polynomial functions probably hold the most important role. Development of equations based on a curve fitting technique was discussed by Ahmad and Yatim (2001), (2001a), (2002), Salam and Lynn (2002), Salam et al (2003), Salam (2004), Tan and Bian (1991), that can be used to calculate the optimal switching angles.
The equations (2.26) that need to be solved are transcendental in nature and incorporate periodic trigonometric terms, which means more than one set of solutions usually exists. To calculate the optimal PWM switching angles for the VSI, a Curve Fitting Technique (CFT) is adopted due to the curvilinear or non-linear feature of the switching angles solutions trajectories. This method is based on quadratic approximation approach which is derived from the computed trajectories of angles. The CFT involves accurate representation of each of these trajectories by the equations in terms of switching angle and modulation index. The coefficients of these polynomials are computed by simple minimization technique. MATLAB curve fitting function, based on polynomial regression is found to be adequate tool to approximate the trajectories. The algorithm results in quadratic equations which require only the multiplication process and therefore this technique is implemented efficiently.

2.13 NELDER-MEAD SIMPLEX ALGORITHM

The minimization problem to find the set of solutions for one value of \( M \) was dealt with using the Nelder–Mead simplex algorithm by Quintana et al (1989). The Nelder-Mead algorithm technique in combination with a random search finds all the sets of possible solutions for one value of \( M \), that is \( M = 0.1 \) and then such information is used as initial value to find all possible sets of solutions for all values of \( M \). Specifically, for the next value of \( M \), the solutions from the previous value of \( M \) are used as an initial point. According to Nelder and Mead (1965), the algorithm is used to find the first or initial set of solutions for the given non-linear equations. An iterative algorithm such as the Newton–Raphson algorithm can be used to obtain the desired solutions for the angles. This is done to further improve the speed of the method.
2.14 CHEBYSHEV POLYNOMIALS

Chebyshev’s Polynomials are of great importance in many area of mathematics, particularly approximation theory. The Chebyshev polynomials have many properties and applications, arising in a variety of continuous settings. They are a sequence of orthogonal polynomials appearing in approximation theory, numerical integration and differential equations. The trigonometric identity of Chebyshev property which was used in this method is given in the equation (2.36).

\[
\cos(n\alpha) = T_n \cos(\alpha)
\]  

(2.36)

where, \( T_n \) is the Chebyshev polynomial of the first kind.

2.14.1 Formulation of Harmonic Elimination Equations

The Fourier series expansion of the output voltage waveform was given in the equation (2.22). Classical formulation yields to the equation (2.24) to eliminate N-1 lower order harmonics such as 5\(^{th}\) and 7\(^{th}\). While formulating the equation (2.24), fundamental amplitude at a specified value per unit of supply voltage was maintained and is given by the equation (2.37).

\[
\frac{4}{\pi} \left(1 + 2 \cos \alpha_1 - 2 \cos \alpha_2 + 2 \cos \alpha_3 - \ldots\right) = M
\]

The generalized form of the above mentioned equation is written as,

\[
\frac{4}{\pi} \left[-1 - 2 \sum_{i=1}^{N} (-1)^i \cos(\alpha_i)\right] = M
\]

\[
f_i(\alpha) = \frac{4}{\pi} \left[-1 + 2 \cos \alpha_1 - 2 \cos \alpha_2 + 2 \cos \alpha_3\right] = M
\]  

(2.37)
The modulation index, \( M \) indicates the fundamental component of output voltage. Therefore, SHE-PWM comprises of such a degree of freedom in order to control the amplitude of fundamental component, permits the elimination of \( N-1 \) odd harmonics. Equation (2.37) is rewritten as equations (2.38) and (2.39).

\[
f_i(\alpha) = M
\]

\[
f_n(\alpha) = 0
\]  

(2.38)

\[
f_i(\alpha) = \sum_{i=1}^{N} (-1)^i \cos(\alpha_i) + \frac{1}{2} \left[ 1 + \frac{\pi M}{4} \right] = 0
\]

\[
f_n(\alpha) = \sum_{n=3,5, \ldots, 2N-1} (-1)^i \cos(n\alpha_i) + \frac{1}{2} = 0
\]  

(2.39)

The equations (2.39) is written as equation (2.40)

\[
F(\alpha) = \left\{ f_n(\alpha) \right\} = 0
\]

\[
\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T
\]  

(2.40)

2.14.2 Transformation in to an Algebraic System

The solutions of these sets are tedious, because they yield to a trial and error process with additional difficulties due to convergence problems. The Chebyshev polynomials of the first kind can be defined by the trigonometric theory is given by the equation (2.41).

\[
T_n(x) = \cos n(\cos^{-1} x)
\]

\[
F(x) = \left\{ f_n(x) \right\} = 0
\]

\[
x = [x_1, x_2, \ldots, x_N]^T
\]  

(2.41)

where, \( 0 \leq \cos^{-1} x \leq 1 \)

\( x \) is the solution vector.
The roots of the $T_n(x)$ are $\cos \frac{(2i-1)\pi}{2n}$ \quad i=1,2,\ldots,n

Using Chebyshev polynomial theory, the problem is alternatively formulated by transforming the trigonometric equations into algebraic equations is given by the equation (2.42).

\[
\begin{align*}
  f_1(x) &= \sum_{i=1}^{N} (-1)^i T_i(x) - \frac{1}{2} \left[ 1 + \frac{\pi M}{4} \right] = 0 \\
  f_n(x) &= \sum_{i=1 \atop n=3,5,\ldots,2N-1}^{N} (-1)^i T_i(x) + \frac{1}{2} = 0 
\end{align*}
\] (2.42)

The most useful process for solving systems of non-linear equations shown above (2.41) is based on Newton method. The Newton-Raphson method tries to solve the non-linear set of equations (2.41) iteratively and solution vector is updated using the equation (2.44) until the stopping criteria are met. These linear system of equation (2.42) are satisfactorily solved by means of an iterative process equations (2.43) and (2.44) by the application of Newton-Raphson or Gauss reduction methods.

\[
J(x^{(i)}) \Delta x^{(i)} - Fx^{(i)} = 0 \\
\Delta x^{(i-1)} = x^{(i)} + \Delta x^{(i)}
\] (2.43) (2.44)

where, \[ J(x) = \begin{bmatrix}
\frac{dF_1}{dx_1} & \cdots & \frac{dF_1}{dx_n} \\
\vdots & \ddots & \vdots \\
\frac{dF_n}{dx_1} & \cdots & \frac{dF_n}{dx_n}
\end{bmatrix} \]
is the Jacobian matrix of $F(x)$.

\[ \Delta x^{(i)} \] is the derivative of the solution vector.
The fundamental amplitude of the output voltage was obtained by initializing the value of modulation index, \( M=0 \) and the solution vector as initial vector \( x^{(0)} \). The successive values of the switching angles have been calculated according to an increment \( \Delta M \) and using the solution vector \( x \) as an initial value for the next. By keeping \( \Delta M=0.01 \), the calculation is repeated for the range of modulation index \( 0 \leq M \leq 1.0 \).

The main advantages of Chebyshev function method are as follows:

1) The use of algebraic variables avoids the restrictive margin \((1,-1)\) of the trigonometric ones and provides an excellent convergence ratio.

2) It improves the processing time due to minimization of the trigonometric functions application.

### 2.15 Walsh Harmonic Elimination Method

The Walsh function harmonic elimination method was first introduced by Asumadu and Hoft (1989). This function forms an ordered set of rectangular waveforms taking only two amplitude values +1 and -1, over one normalized frequency period. The walsh function forms a complete orthogonal set; hence walsh functions can be used to represent signals in the same way as the Fourier series. By using the walsh function analytic technique, the harmonic amplitude is expressed directly as a function of switching angles was described by Chunfang et al (2005), Liang and Hoft (1993), Nazarzadeh et al (1997), Swift and Kamberis (1993), Tsorng-Juu et al (1997). Then, linear algebraic equations are solved to obtain the switching angles resulting in elimination of unwanted harmonics. The global solution is found by searching all possible switching patterns, since the local solutions are obtained only under an appropriate initial condition. This method is not
very efficient when a large number of lower order harmonics need to be eliminated.

2.16 HOMOTOPY BASED COMPUTATION

A systematic homotopy based computation method is used to solve the SHE problem, which has been proposed by Kato (1999). This method finds multiple solutions for a specific N switching angles reducing N-1 harmonic content from the output of the circuit by varying a fundamental component value as the homotopy parameter. However, the method is long and cumbersome and the method does not make any contribution towards the set of solutions from the multiple available ones is optimum against overall harmonic performance and presents no experimental results to confirm the analysis.

2.17 EIGEN SOLVE ALGORITHM

Though computing the roots for a non-linear polynomial, which has received much discussion in the literature so far, it is still a tedious problem for numerical computation. Recently, the eigen solve algorithm is proposed by Fortune (2002), which computes the zeros of polynomials. The method is based on iterative conditioning technique. That is, given an unconditioned instance, the problem is first solved by a standard algorithm and the result is then used to compute a new instance which is better conditioned or at some times has the same solution as the original one. The iteration is repeated until a well conditioned instance (which can be easily solved) is obtained. It is shown by Han et al (2004) that the eigen solve algorithm can effectively compute all zeros for highly ill-conditioned polynomial, and is faster than the best alternative in an order of magnitude. An eigen solve algorithm is introduced to non-linear equations and it is especially good for solving the highly ill-conditioned polynomial.
2.18 NEURAL NETWORK METHOD

Application of neural networks to the optimal control of three phase voltage controlled inverters was described by Andrzej and stanislaw (1992), Mohaddes et al (1997). Pre-calculated switching angles for the elimination of lower order harmonics of the output voltage are used to train a software emulated network. The trained network generates approximate switching angles in response to the required value of the modulation index applied to its output. A neural network to be used for the generation of optimal switching angles have single input accepting desired values of the modulation index and time of the first quarter switching angles. A hidden layer is necessary to provide the required number of degrees of freedom for accurate mapping.

2.19 EQUAL AREA ALGORITHM METHOD

The pulse width in each sampling interval is determined by making the area of the inverter output equal to the area of sinusoidal reference voltage, which is shown in Figure 2.4.

Figure 2.4 Equal area PWM algorithm
The pulse width in each sampling interval is given by the equation (2.45).

\[ dt(i) = \frac{V_m}{wV_{dc}} \left[ \cos \omega t_i - \cos \omega t_{i+1} \right] \quad (2.45) \]

The switching angles in a sampling interval are given in equation (2.46).

\[
\begin{align*}
\alpha_{s,1} &= T(i - 0.5) - dt(i)/2 \\
\alpha_{s,2} &= T(i - 0.5) + dt(i)/2, \\
\text{where, } i &= 0, 1, 2, \ldots,
\end{align*}
\]

(2.46)

where,

- \( \alpha_{s,i} \) is the \((i+1)\)th switching angle
- \( t_i \) is the \(i\)th sampling time
- \( dt(i) \) is the \(i\)th sampling pulse width
- \( V_{dc} \) is the input DC voltage
- \( V_m \) is the maximum sinusoidal voltage
- \( w \) is the sampling frequency
- \( T \) is the sampling period

The switching angles obtained by equal area algorithm described by Chen and Liang (1997), Jyh-Wei (2005) are the starting values for numerical analysis. This method is also one of the methods used to find the starting solution values for first set of solution values or starting values of the non-linear transcendental equations.

The global solution is found by searching all possible switching patterns since the local solutions are obtained only under an appropriate initial condition. These traditional methods are not very efficient when a large number of lower order harmonics need to be eliminated.
2.20 NON-TRADITIONAL METHODS

The main advantages of the non-traditional optimization technique, when compared to traditional methods are the following. No preliminary calculations are necessary to find the solutions of non-linear transcendental equations. In fact, these values are not necessary for the division of the entire solution space into small regions to ensure convexity.

Many non-traditional techniques have become admired in engineering optimization problem and recently, stochastic approach has achieved increasing popularity among researchers. The efficiency of these iterative solution methods depends mostly on the modelling. A fine tuning of parameters will never balance a bad choice of the neighbourhood structure or of the objective function. An effective modelling should lead to robust techniques that are not too sensitive for different parameter settings. There have been many approaches to this problem reported in the technical literature including: Genetic algorithm described by Al-Othman et al (2007), Burak et al (2005), Han et al (2004a), Hasanzadeh et al (2003), John et al (2004), Maswood et al (2001), Mohamed et al (2006), (2008), Shen and Ali (2000), Shi and Hui Li (2005), Sundareswaran and Mullangi (2002), Yaow-Ming (2003), Ant colony search algorithm applied by Kinattingal et al (2007), Simulated Annealing explained by Leopoldo et al (2007) and particle swarm optimization mentioned by Said et al (2008). The bipolar waveform has been treated in detail by Hasanzadeh et al (2003) where a minimization technique is employed along with a biased optimization search method to get the multiple sets of solution.

The voltage source inverter with non-traditional methods optimization solution significantly enhances the handling capacity of power electronics equipments using currently available switching devices without bulky transformer connections and problematic series connections. The use of
optimization techniques including genetic algorithm has been shown to overcome all known obstacles of the previous approaches, which was explained by Dahidah and Agelidis (2005).

There are many advantages that make genetic algorithm attractive was mentioned by Frenzel (1993). Genetic algorithm does not require the use of derivatives. They offer a parallel searching of the solution space rather than the point-by-point searching in a small region. Since the location of the optimal solution is unknown before the search, it is very likely that the point-by-point search needs to search through the entire solution space in order to find the optimal solution, not to mention the traps of the local optima. Hence, genetic algorithm can find a near optimal solution for a complex problem very quickly and efficiently.

2.21 CONCLUSION

In this chapter, various pulse width modulation techniques for inverter/converter circuits have been presented. Among various methods, SHE-PWM has its own advantages in modern power electronics applications. Selective harmonic elimination PWM technique, is a kind of popular control signal generation technique, is widely used in UPS, reactive power compensators and power filters. It has a clear physical model and is easy to realize. However, its complex multi root transcendental equations provide difficulties for root finding. Consideration of special characteristics of SHE-PWM allowed for obtaining results which offer ideas for optimization of output spectrum by employing advanced algorithms and analog circuits. Many methods are there to find optimum solution for non-linear transcendental equations.

Due to the special characteristics of SHE-PWM output spectrum, some minimization techniques have been used to solve the non-linear
transcendental equations. The details of solving the non-linear equations using the traditional algorithms are explained in this chapter. These methods have their own shortcomings to find the initial values for the solution of the equations. In the proceeding chapters, the usage of non-traditional methods to solve the transcendental equations is given in detail by overcoming the disadvantages of traditional minimization techniques. By optimizing the switching angles distribution, the harmonic content can be flexibly redistributed and high quality output waveforms are obtained.