CHAPTER 3
PERFORMANCE OF HYBRID SPACE-TIME CODES

3.1 INTRODUCTION

STBCs make use of both temporal and spatial dimensions in a way to exploit the diversity in MIMO channels. However, their key limitation is that they are not designed to provide significant coding gain. One way of overcoming this limitation is by concatenating STBC with outer channel codes. On the other hand, STTC achieves a full diversity order of $N_T M_R$ and a substantial coding gain.

In this chapter, a novel idea of Combined Hybrid Space-Time Coded (CHSTC) scheme is explained. This hybrid scheme combining STTC and STBC is proposed with the perspective that STTC is equivalent to generalization of TCM for multiple antennas. Further, the combination of STTC with STBC is similar to the serial concatenation of TCM with STBC, both the schemes yielding a significant coding gain.

Before getting into the details of the proposed scheme, a brief review of STTC is presented.

3.2 REVIEW OF SPACE-TIME TRELLIS CODES (STTC)

Space-time trellis codes combine modulation and trellis codes to transmit information over multiple transmit antennas and MIMO channels. They are an extension of convolution trellis codes for multi-antenna systems to achieve correlation in temporal and spatial dimension. STTC provides
improved performance over Delay Diversity scheme, in which the same information is transmitted simultaneously from $N_T$ antennas, but with a delay of one symbol interval. In STTC, the restriction imposed by the delay element in the transmitter is removed. Further, STTC achieves the same diversity as the MRRC technique. In addition, a well-constructed STTC can also achieve a significant coding gain.

### 3.2.1 Encoder Structure for M-PSK STTC

The encoder structure for STTC is given by Tarokh et al (1998). The STTC using M-PSK modulation with $N_T$ transmit antennas is shown in Figure 3.1. The input message stream, denoted by $C_{i/p}$, is given by

$$C_{i/p} = (c_0, c_1, c_2, ..., c_t, ...)$$ (3.1)

where $c_t$ is a group of $m = \log_2 M$ information bits at time $t$ and given by

$$c_t = (c_t^1, c_t^2, ..., c_t^m)$$ (3.2)
The encoder maps the input sequence into an M-PSK modulated signal sequence which is given by

\[ X_{\text{STTC}} = (x_0, x_1, \ldots, x_t, \ldots) \]  

(3.3)

where \(x_t\) is a space-time symbol at time \(t\) and given by

\[ x_t = (x_t^1, x_t^2, \ldots x_t^{N_T})^T \]  

(3.4)

The modulated signals \(x_t^1, x_t^2, \ldots x_t^{N_T}\), are then transmitted simultaneously through \(N_T\) transmit antennas.

Consider a simple space-time trellis coded QPSK with two transmit antennas. The encoder consists of two feed-forward shift registers. The encoder structure for the scheme with memory order of \(v\) is shown in Figure 3.2.
Two binary input streams, $c^1 = (c^1_0, c^1_1, ..., c^1_i, ...)$ and $c^2 = (c^2_0, c^2_1, ..., c^2_i, ...)$, are fed into the upper and lower encoder registers. The memory orders of the upper and lower encoder shift registers are $v_1$ and $v_2$ respectively, where $v = v_1 + v_2$, and the two input streams are delayed and multiplied by the coefficient pairs

\[ g^1 = \left[ (g^1_{0,1}, g^1_{0,2}), (g^1_{1,1}, g^1_{1,2}), ..., (g^1_{v_1,1}, g^1_{v_1,2}) \right] \]  

(3.5)

\[ g^2 = \left[ (g^2_{0,1}, g^2_{0,2}), (g^2_{1,1}, g^2_{1,2}), ..., (g^2_{v_2,1}, g^2_{v_2,2}) \right] \]  

(3.6)

respectively, where $g^k_{j,i} \in \{0,1,2,3\}$, $k = 1,2$; $i = 1,2$; $j = 0,1, ..., v_k$. The multiplier outputs are added modulo 4, giving the output

\[ x^i_t = \sum_{k=1}^{2} \sum_{j=0}^{v_k} g^k_{j,i} c^k_{t-j} \mod 4, i=1,2 \]  

(3.7)

The outputs $x^1_t$ and $x^2_t$ are mapped to the points of complex 4-PSK constellation. The generator sequences of a 4-state space-time trellis coded QPSK scheme with 2 transmit antennas are assumed to be

\[ g^1 = [(02), (20)] \]  

(3.8)

\[ g^2 = [(01), (10)] \]  

(3.9)

and the input sequence is given by

\[ C = (10,01,11,00,01,...) \]  

(3.10)

The output sequence generated by the space-time trellis encoder is given by
The transmitted signal sequences from the two transmit antennas are

\[ X_1 = (0, 2, 1, 3, 0, \ldots) \]
\[ X_2 = (2, 1, 3, 0, 1, \ldots) \] (3.12)

The above example is actually a delay diversity scheme, since the signal sequence transmitted from the first antenna is a delayed version of the signal sequence from the second antenna.

3.2.2 Encoder Structure for 16-QAM STTC

The 16-QAM STTC encoder for two transmit antennas proposed by Tarokh et al (1998) is given in Figure 3.3.

Figure 3.3 16-QAM STTC Encoder for two transmit antenna
In the 16-QAM STTC, the 4 information bits \( b_1^t, b_2^t, b_3^t, b_4^t \) are converted into two components \( u_1^t \) and \( u_2^t \) through natural mapping. These components go through two-branch shift registers with total memory order \( v \). The memory order of the \( k \)-th branch \( v_k \), \( k=1,2 \) is given by

\[
v_k = \left\lfloor \frac{v + k - 1}{m} \right\rfloor \tag{3.13}\]

The two output streams are then multiplied by the coefficient vectors given by

\[
g^1 = \left[ \left( a_{0,1}^{1,I}, a_{0,1}^{1,Q} \right), \ldots, \left( a_{N_1,1}^{1,I}, a_{N_1,1}^{1,Q} \right), \ldots, \left( a_{0,1}^{2,I}, a_{0,1}^{2,Q} \right), \ldots, \left( a_{N_1,1}^{2,I}, a_{N_1,1}^{2,Q} \right) \right] \tag{3.14}\]

\[
g^2 = \left[ \left( a_{0,1}^{2,I}, a_{0,1}^{2,Q} \right), \ldots, \left( a_{N_1,1}^{2,I}, a_{N_1,1}^{2,Q} \right), \ldots, \left( a_{0,1}^{1,I}, a_{0,1}^{1,Q} \right), \ldots, \left( a_{N_1,1}^{1,I}, a_{N_1,1}^{1,Q} \right) \right]
\]

where \( a_{q_k,i}^{k,I}, a_{q_k,i}^{k,Q} \in Z_4 \), \( k = 1,2 \), \( q_k = 0, 1, 2, \ldots, v_k \).

The encoder output represented by \( w_{ti}^k \), \( t = 1, \ldots, l \) and \( i = 1, \ldots, N_T \) is derived as

\[
w_{ti}^k = \sum_{k=1}^{v_k} \sum_{q_k=1}^{v_k} a_{q_k,i}^{k,I} u_{t-q_k}^i + j \sum_{k=1}^{v_k} \sum_{q_k=1}^{v_k} a_{q_k,i}^{k,Q} u_{t-q_k}^i \quad \text{mod } 4 \tag{3.15}\]

where \( w_{ti}^k \in Z_4[j] \).

By linear translation, the coded symbol sequence is mapped as \( x_t^i = w_{ti}^i - (3 + 3j) / 2 \). These modulated symbols enter the STBC encoder which further encodes the modulated symbols and transmits them through the two transmit antennas.
3.2.3 **STTC Decoder**

The decoding of STTC involves Maximum Likelihood Sequence Estimation (MLSE) using Viterbi algorithm (Tarokh et al 1998). Each time the decoder receives a pair of channel symbols, it computes a metric to measure the distance between the received signal and all of the possible channel symbol pairs that could have been transmitted. The metric values computed for the paths between the states at the previous time instant and the states at the current time instant are called “branch metrics”. The path metric of a valid path is the sum of the branch metrics for the branches that form the path. Ideal Channel State Information (CSI) with the path gains $h_{i,j}$ (where $i = 1,2, \ldots N_T$ and $j = 1,2, \ldots M_R$), between the transmit and receive antennas is assumed.

If $r_{jt}^j$ is the signal received at antenna $j$ and time $t$, the branch metric for a transition labeled $x_i^1 x_i^2 \ldots x_i^{N_T}$ is given by

$$
\sum_{j=1}^{M_R} \left| r_{jt}^j - \sum_{i=1}^{N_T} h_{i,j} x_i^j \right|^2
$$

(3.16)

The Viterbi algorithm determines the most likely path as the one with the lowest accumulated path metric.

The trellis code designed by Tarokh et al (1998) given in Table 3.1 is used in the simulation of the proposed hybrid scheme. This table gives the number of states ($2^v$), the minimum rank ($r$), the minimum determinant ($\det$) and the minimum squared Euclidean distance ($d_{\text{min}}^2$). The generator coefficients are given by $a_i^j, b_i^j$ and $c_i^j$. An example of generator matrix for two-transmitter space-time code is:

$$
G_{\text{STTC}} = 
\begin{bmatrix}
    a_0^1 & b_0^1 & a_0^2 & b_0^2 & \ldots & a_n^1 & b_n^1 & b_{n+1}^1 \\
    a_0^2 & b_0^2 & a_0^3 & b_0^3 & \ldots & a_n^2 & b_n^2 & b_{n+1}^2
\end{bmatrix}
$$

(3.17)
Table 3.1 4-PSK Trellis codes for two transmit antennas proposed by Tarokh et al (1998)

<table>
<thead>
<tr>
<th>$2^r$</th>
<th>$a_0^r,a_0^2$</th>
<th>$a_1^r,a_1^2$</th>
<th>$a_2^r,a_2^2$</th>
<th>$a_3^r,a_3^2$</th>
<th>$b_0^r,b_0^2$</th>
<th>$b_1^r,b_1^2$</th>
<th>$b_2^r,b_2^2$</th>
<th>$b_3^r,b_3^2$</th>
<th>$r$</th>
<th>$\text{det}$</th>
<th>$d_{\text{min}}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>(0, 2)</td>
<td>(2, 0)</td>
<td>-</td>
<td>-</td>
<td>(0, 1)</td>
<td>(1, 0)</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>(0, 2)</td>
<td>(2, 0)</td>
<td>-</td>
<td>-</td>
<td>(0, 1)</td>
<td>(1, 0)</td>
<td>(2, 2)</td>
<td>-</td>
<td>2</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>(0, 2)</td>
<td>(2, 0)</td>
<td>(0, 2)</td>
<td>-</td>
<td>(0, 1)</td>
<td>(1, 2)</td>
<td>(2, 0)</td>
<td>-</td>
<td>2</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>32</td>
<td>(0, 2)</td>
<td>(2, 0)</td>
<td>(3, 3)</td>
<td>-</td>
<td>(0, 1)</td>
<td>(1, 1)</td>
<td>(2, 0)</td>
<td>(2, 2)</td>
<td>2</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

3.3 HYBRID COMBINATION SCHEME

The idea of CHSTC has been conceived from the following discussion:

In conventional space-time block coded system, STBCs are concatenated with outer channel codes to provide coding gain and diversity gain. Repetition code is one among such channel codes suggested for outer codes. Delay diversity code which is a special case of space-time trellis code can be viewed as an example of repetition code. For example, consider the following STTC codeword for a frame length of $T = 5$ timeslots:

$$C_{\text{STTC}} = \begin{bmatrix} 0 & 2 & 3 & 1 & 2 \\ 2 & 3 & 1 & 2 & 0 \end{bmatrix} \quad (3.18)$$

where every symbol is repeated at least twice, i.e., when a symbol $c_t$ occurs at any time $t$, the occurrence is repeated even at time $t+1$. Thus, when STTC is used as outer code and STBC as inner code, coding gain benefit from STTC and diversity benefit from STBC can be extracted.

The block diagrams of STBC which is otherwise termed as Space-Time Transmit Diversity (STTD) scheme and the proposed Hybrid Combination of STTC and STBC are given in Figures 3.4 and 3.5,
respectively. It is assumed that STTC encoder is connected to the STBC encoder through two virtual antennas.

![Figure 3.4 STTD transmitter structure](image)

**Figure 3.4 STTD transmitter structure**

![Figure 3.5 Hybrid Combination of STBC and STTC](image)

**Figure 3.5 Hybrid Combination of STBC and STTC**

The information bits are first encoded by the STTC encoder and then fed into the space-time block encoder. At each time slot, output symbols from the STTC encoder are modulated and transmitted simultaneously from the two transmit antennas. The transmitted signal gets faded as it traverses the Rayleigh channel (and corrupted by the addition of noise). At the receiver end, signal combining and ML decoding similar to Alamouti’s two-transmit and two-receive antenna scheme is performed by the space-time block decoder. This is followed by Viterbi decoding in the STTC to decode the transmitted bits.

The Hybrid system description is as follows: The input bits are continuously entered into the 4-PSK STTC encoder, which generates complex constellation symbols. The STTC encoder is described by the following equation (3.19):
\[ x_t^k = \left\{ \sum_{p=0}^{v_1} I_{t-p}^k a_p^k + \sum_{q=0}^{v_2} I_{t-q}^{2^c} b_{t-q}^k \right\} \mod 4, \; k = 1, 2 \]  

(3.19)

where \( v_1 \) is the memory order of the upper branch and \( v_2 \) is the memory order of the lower branch of the STTC encoder (both are 1 for 4-PSK STTC). \( I_t^k \) represents the information bits, \( a_t^k \) and \( b_t^k \) represents the coefficients of the generator matrix for the 4-PSK 2 transmit antenna trellis codes. \( x_t^k \) represents the outputs of STTC encoder that are fed into the space-time block encoders.

The chosen STBC is Alamouti’s \( G_2 \) code which is a unity rate code suitable for combination. For convenience, the transmission matrix of the space-time code \( G_2 \) is reproduced here as

\[
G_2 = \begin{pmatrix}
  x_0 & x_1 \\
  -x_1^* & x_0^*
\end{pmatrix}
\]  

(3.20)

From equation (3.20), the indeterminate \( x_0 \) and \( x_1 \) in the matrix \( G_2 \) is replaced by the modulated symbol \( s_1 \) and \( s_2 \) for the STBC encoder. The transmitted codeword is represented by the following matrix

\[
C_{\text{CHSTC}} = \begin{pmatrix}
  s_1 & s_2 \\
  -s_2^* & s_1^*
\end{pmatrix}
\]  

(3.21)

At time slot \( t \), signals \( s_1 \) and \( s_2 \) are transmitted from the first and second transmit antenna and at the next time slot, \( s_2^* \) and \( s_1^* \) are transmitted from the first and second transmit antenna respectively. This code has unity rate and transmit diversity two.

The channel is assumed to be varying slowly enough that the fading remains constant for two consecutive symbol periods. Let \( h_{ji} \) denote the fading coefficient between the \( j \)th receive antenna and \( i \)th transmit antenna. The
channel coefficients can also be represented as

\[ h_{ji} = |h_{ji}| e^{j\theta_{ji}} \]  \hspace{1cm} (3.22)

where \( |h_{ji}| \) and \( \theta_{ji} \) are the amplitude and phase shifts for the path gains between the transmit and receive antennas which is given in the following Table 3.2 and \( T \) is the symbol duration.

**Table 3.2** The path gains between the transmit (Tx) antenna \( i \) and receive (Rx) antenna \( j \)

<table>
<thead>
<tr>
<th></th>
<th>Rx antenna 1</th>
<th>Rx antenna 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx antenna 1</td>
<td>( h_{11} )</td>
<td>( h_{21} )</td>
</tr>
<tr>
<td>Tx antenna 2</td>
<td>( h_{12} )</td>
<td>( h_{22} )</td>
</tr>
</tbody>
</table>

The received signals are given by the following equations:

\[
\begin{align*}
    r_{1,1} &= r_1(t) = h_{11}s_1 + h_{12}s_2 + n_1 \\
    r_{2,1} &= r_1(t + T) = h_{13}s_1 + h_{12}s_2 + n_2 \\
    r_{1,2} &= r_2(t) = h_{21}s_1 + h_{22}s_2 + n_3 \\
    r_{2,2} &= r_2(t + T) = h_{23}s_1 + h_{22}s_2 + n_4
\end{align*}
\]

(3.23) \hspace{1cm} (3.24) \hspace{1cm} (3.25) \hspace{1cm} (3.26)

In the above equations, \( r_{ij} \) represents the received signal at the \( j^{th} \) receiving antenna at time \( t \). The channel gains \( h_{ij} \) are modeled as independent samples of a complex Gaussian random variable with variance \( \frac{1}{2} \) per dimension. This is equivalent to assuming that the signals transmitted from different antennas undergo independent fading. Likewise \( n_1, n_2, n_3 \) and \( n_4 \) are also modeled as independent samples of a complex random variable with zero mean and variance \( N_0/2 \) per dimension.

The received signals are combined as follows:
\[ \tilde{s}_1 = r_{1,1}^* h_{11} + r_{1,2}^* h_{12} + r_{2,1}^* h_{21} + r_{2,2}^* h_{22} \] (3.27)

\[ \tilde{s}_2 = r_{1,1}^* h_{12} - r_{1,2}^* h_{11} + r_{2,2}^* h_{22} - r_{2,1}^* h_{21} \] (3.28)

The combined signals from the STBC decoder are fed to the STTC decoder along with the channel state information. In the Viterbi algorithm used for ML decoding of STTC, the branch metric of the trellis representing the transmission of symbols \( s_1 \) and \( s_2 \) from antennas one and two, respectively is given by

\[ R_{M t, j 1, j 1 2, j 2} = r_{t,j} \] (3.29)

where \( r_{t,j} \) represents the received signal at time slots \( t = 1, 2, \ldots, T+Q \) and \( Q \) represents the memory of the STTC encoder. \( \alpha_{1,j} \) and \( \alpha_{2,j} \) represents the virtual path gain between the STBC decoder and STTC decoder.

Then, the path metric of a valid path is calculated as the sum of the branch metrics for the branches that forms the path. The most likely path is the one which has the minimum path gain. The ML decoder finds the set of constellation symbols that construct this valid path and solves the following minimization problem.

\[ \min_{c_{1,1}, c_{1,2}, c_{2,1}, c_{2,2}} \sum_{j=1}^{M_k} \sum_{t=1}^{T+Q} \left| r_{t,j} - \alpha_{1,j} c_{1} - \alpha_{2,j} c_{2} \right|^2 \] (3.30)

where \( c_{ij} \) represents the space-time trellis coded symbols and \( T+Q \) represents the frame length of the transmitted codeword. Thus the originally transmitted information bits are decoded at the receiving end.

### 3.4 SIMULATION RESULTS
In this section, the hybrid scheme with two transmit antenna and one/two receive antenna is simulated using MATLAB. The channel is Rayleigh flat slow fading channel. The performance is measured in terms of Frame Error Rate (FER), with a frame length of 130 symbols. In the performance plots, SNR represents the average Signal-to-Noise Ratio per receive antenna.

### 3.4.1 FER Performance Comparison of 4-PSK STTC and 4-PSK CHSTC

Figure 3.6 shows the FER performance of the 4-state 4-PSK CHSTC using two transmit and one receive antenna. The performance result of Tarokh’s 4-state 4-PSK STTC is also shown for comparison. Both the schemes offer the same spectral efficiency 2bps/Hz.

![Figure 3.6 FER Performance of CHSTC with STTC](image)

From Figure 3.6, it can be observed that at a FER of $10^{-2}$, the hybrid scheme offers a coding gain of 0.8 dB when compared to STTC.
3.4.2 FER Performance Comparison of Space-Time Codes

Figure 3.7 shows the FER performance of the 4-state 4PSK CHSTC using two transmit and two receive antenna. The performance results of Tarokh’s 4-state 4PSK STTC, concatenated 4-state TCM-STBC and Alamouti’s STBC are also shown. All the four schemes offer the same spectral efficiency 2bps/Hz.

![Graph showing FER performance comparison](image)

**Figure 3.7** FER Performance Comparison of CHSTC with other Space-time codes with spectral efficiency 2bps/Hz.

From Figure 3.7, it can be derived that at a FER of $10^{-2}$, the hybrid scheme offers a coding gain of 0.6 dB and 1.8 dB compared with STTC scheme and Alamouti scheme, respectively.

Further, compared to the hybrid system using one receive antenna, the system using two receive antenna offers an additional diversity gain which also gets reflected in SNR reduction.
3.4.3 FER Performance Comparison of 16-QAM STTC and 16-QAM CHSTC

Figure 3.8 shows the FER performance comparison between 16-QAM CHSTC and 16-QAM STTC, both using two transmit and two receive antennas. Both the schemes offer the same spectral efficiency 4bps/Hz.

![Figure 3.8 FER Performance Comparison between 16-QAM CHSTC and 16-QAM STTC](image)

From Figure 3.8, it can be inferred that at a FER of $10^{-3}$, the 16-QAM hybrid scheme offers a coding gain of 0.8 dB when compared with 16-QAM STTC. Though the SNR requirement has increased proportionally, the use of higher order modulation has also increased the spectral efficiency of the 4-PSK hybrid scheme.

3.5 CONCLUSION
The Hybrid combination scheme is intended to offer a coding gain advantage. But this advantage is achieved at the cost of increased decoding complexity. Through simulation, it has been shown that the hybrid scheme outperforms the conventional space-time codes like the space-time trellis codes and the space-time block codes in terms of improved coding gain.

Further, the spectral efficiency of the hybrid scheme is increased from 2bps/Hz to 4bps/Hz when higher order modulation is employed. Nevertheless, a trade-off exists between coding gain and spectral efficiency. Higher order modulation achieves less coding gain. The diversity benefit is also obtained with the increase in the number of receive antennas.