Chapter VII

STIMULATED BRILLOUIN SCATTERING OF AN ALFVEN WAVE IN A MULTI-ION SPECIES PLASMA

7.1 Introduction

The stimulated Brillouin scattering (SBS) process in a multi-ion species plasma is of prime importance in the heating of the plasma for the controlled thermonuclear fusion. A multi-ion component plasma exists in planetary ionospheres which exhibit some phenomena which do not occur in a single ion-plasma and depend on the electron to ion temperature ratio and ion mass ratio.

Lashmore-Davies and Ong 1974 have studied the Alfven wave instability threshold and have shown that by taking the plasma pressure into account not only Alfven waves may be excited directly but also ion-acoustic waves. Cramer and Donnelly 1981 have studied the parametric excitation of kinetic Alfven waves by a magnetic pump wave employing the mode conversion of magneto-acoustic waves at the Alfven resonance where the excited Alfven wave plays an important role in linear heating schemes.
Winglee 1984 has examined the heating of Tokamaks by Alfven waves in a hot multiple ion-component plasma where the majority ions determine the dispersive properties of the waves while the minority ions and the electrons determine the damping. He has shown that heavy ion minority in light ion plasma causes roughly equal ion and electron heating which is in conformity with our results. Jain et al. 1986 have analytically investigated stimulated Brillouin scattering of an Alfven wave in a collisionless low density plasma in the presence of negative ions. They have found that the growth rate of the ion-acoustic wave decreases significantly due to the formation of the negative ions.

In this chapter, we have analytically investigated the stimulated Brillouin scattering of an Alfven wave in a collisionless homogeneous multi-ion species plasma by including the effect of the ponderomotive force where the high-and low-frequency nonlinearities arise due to the nonlinear current density and the parallel ponderomotive force on electrons respectively. In Section 7.2, the derivation of linear dispersion relation of an Alfven wave has been shown. In Section 7.4,
we have derived the nonlinear dispersion relation of an Alfven wave. Section 7.5 contains the ratio of growth rates of the excited modes in the presence of both types of ions and in the absence of second ions. The results are discussed in Section 7.6.

7.2 Basic Equations

The basic equations used are as follows:

\[ \frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) \mathbf{v}_\alpha = \frac{e}{m_\alpha} (E + \frac{\mathbf{v}_\alpha \times \mathbf{B}}{c}) - \left( \frac{v_{th,\alpha}^2}{n_\alpha} \right) \nabla n_\alpha, \quad (7.1) \]

\[ \frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla) \mathbf{v}_e = \frac{e}{m_e} (E + \frac{\mathbf{v}_e \times \mathbf{B}}{c}) - \left( \frac{v_{th,e}^2}{n_e} \right) \nabla n_e, \quad (7.2) \]

\[ \frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0, \quad (7.3) \]

\[ \frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0, \quad (7.4) \]

\[ \nabla \cdot \mathbf{E} = 4\pi e (n_\alpha - n_e), \quad (7.5) \]
\[ \mathbf{V} \times \mathbf{E} = \left( \frac{1}{c} \right) \mathbf{B} \quad \text{and} \quad \mathbf{V} \times \mathbf{B} = \left( \frac{4 \pi}{c} \right) \mathbf{J} - \left( \frac{1}{c} \right) \mathbf{E}, \quad (7.6) \]

where the suffixes \( \alpha \) and \( \epsilon \) denote the two-ion species and electrons respectively. \( \mathbf{E}, \mathbf{v} \) and \( \mathbf{B} \) are the total electric field, velocity of the particle and total magnetic field respectively. \( \mathbf{J} \) is the current density while \( n_{\epsilon}(\alpha) \) is the total number density of the electrons (ions). \( v_{\text{th}, \alpha}(\epsilon) \) corresponds to the thermal velocity of the particles. \( e, m \) and \( c \) are the charge, mass of the particles and the speed of light respectively. Equations (7.1) to (7.6) are the momentum transfer equations for ions and electrons, continuity equations for ions and electrons, Poisson equation and Maxwell's equations respectively.

7.3 Linear Dispersion Relation of an Alfven wave

Splitting the variable quantities as

\[ \mathbf{E} = \mathbf{E}_0 (\omega, \mathbf{k}_0) + \mathbf{E}_{\alpha} (\omega, \mathbf{k}) + \mathbf{E}_1 (\omega_1, \mathbf{k}_1) \]
\[ \mathbf{B} = \mathbf{B}_s + \mathbf{B}_0 (\omega, \mathbf{k}_0) + \mathbf{B}_1 (\omega_1, \mathbf{k}_1) \]
\[ \mathbf{v} = \mathbf{v}_0 (\omega, \mathbf{k}_0) + \mathbf{v}_{\alpha} (\omega, \mathbf{k}) + \mathbf{v}_1 (\omega_1, \mathbf{k}_1) \]
\[ n = n_0 + n_{\alpha} (\omega, \mathbf{k}) \quad (7.7) \]
where $E_0$, $v_0$, $B_0$ and $n_0$ are the electric field of the pump, zeroth order velocity of the particle, uniform static magnetic field and zeroth order number density of the particles respectively. $E_{ia}$, $B_0$, $v_{ia}$ and $n_{ia}$ are the electric field of the ion-acoustic wave, magnetic field of the pump, velocity of the particle due to the excited wave and number density of the particle due to the excited wave respectively. $E_1$, $B_1$ and $v_1$ are the electric field, magnetic field of the scattered Alfven wave and velocity of the particles due to the scattered Alfven wave respectively. $\omega_0$, $\omega$ and $\omega_1$ are the frequencies of the pump, the excited ion-acoustic wave and the scattered Alfven wave respectively. $k_0$, $k$ and $k_1$ are the wave numbers of the pump, the excited ion-acoustic wave and the scattered Alfven wave respectively.

Using equations (7.1), (7.2) and (7.7), one gets the zeroth order velocities of the electrons $v_{e0}$ and the ions $v_{i0}$ as

$$v_{e0x} = \frac{\omega_{ce} e E_{0y}}{m_e (\omega_0^2 - \omega_{ce}^2)} - \frac{i\omega_0 e E_{0x}}{m_e (\omega_0^2 - \omega_{ce}^2)},$$

(7.8a)
\[ v_{eoy} = -\frac{i\omega_0 e E_{oy}}{m_e (\omega_0^2 - \omega_{ce}^2)} + \frac{\omega_{ce} e E_{ox}}{m_e (\omega_0^2 - \omega_{ce}^2)} , \quad (7.8b) \]

\[ v_{eoz} = -\frac{i e E_{oz}}{m_e \omega_0} , \quad (7.8c) \]

\[ v_{aox} = \frac{i\omega_0 e E_{ox}}{m_\alpha (\omega_0^2 - \omega_{ca}^2)} - \frac{\omega_{ca} e E_{oy}}{m_\alpha (\omega_0^2 - \omega_{ca}^2)} , \quad (7.8d) \]

\[ v_{aoy} = \frac{i\omega_0 e E_{oy}}{m_\alpha (\omega_0^2 - \omega_{ca}^2)} + \frac{\omega_{ca} e E_{ox}}{m_\alpha (\omega_0^2 - \omega_{ca}^2)} \quad (7.8e) \]

\[ v_{aoz} = -\frac{i e E_{oz}}{m_\alpha \omega_0} . \quad (7.8f) \]

where \( \omega_{ce(\alpha)} \) is the electron (ion)-cyclotron frequency.

Now substituting these values in the current density expression \( J = nev \) and using the Maxwell's equations, one gets
\[
(k_o^2 c^2 - \omega_o^2) \left( \frac{w_{pe}^2 \omega_o^2}{(\omega_o^2 - \omega_{ce}^2)} + \frac{\omega_{pa}^2 \omega_o^2}{(\omega_o^2 - \omega_{ca}^2)} \right) - \left( \frac{w_{ce} \omega_o^2 \omega_{pa}}{(\omega_o^2 - \omega_{ca}^2)} + \frac{w_{pe} \omega_{ca} \omega_o^2}{(\omega_o^2 - \omega_{ce}^2)} \right) + \left( \frac{w_{pe} \omega_{ca} \omega_o^2}{(\omega_o^2 - \omega_{ce}^2)} + \frac{w_{pa} \omega_{ca} \omega_o^2}{(\omega_o^2 - \omega_{ce}^2)} \right) = 0
\]

\[
0 \ , \ 0 \ , \ \left( \frac{-\omega_o^2}{c^2} + \frac{(w_{pe}^2 + \omega_{pa}^2)}{c^2} \right)
\]

\[
\begin{align*}
E_{ox} \\
E_{oy} = 0 \\
E_{oz}
\end{align*}
\]

where \( \omega_{pe}(\alpha) \) is the plasma frequency \( = \sqrt{\frac{4\pi e^2 n_e(\alpha) \alpha}{m_e(\alpha)}} \).

For linearly polarised transversed wave, \( E_{oy} = 0 \), equation (7.9) can be reduced to
\[ \left[ k_o^2 c^2 - \omega_o^2 + \frac{\omega_{pe}^2 \omega_o^2}{\omega_o^2 - \omega_{ce}^2} + \frac{\omega_{p\alpha}^2 \omega_o^2}{\omega_o^2 - \omega_{c\alpha}^2} \right] E_{ox} = 0 \]  

(7.10a)

and

\[ \left[ \frac{1}{\omega_o^2} \frac{\omega_{ce}^2 \omega_{pe}^2}{\omega_o^2 - \omega_{ce}^2} + \frac{1}{\omega_o^2} \frac{\omega_{c\alpha}^2 \omega_{p\alpha}^2}{\omega_o^2 - \omega_{c\alpha}^2} \right] E_{ox} = 0 \]  

(7.10b)

Under the condition \( \omega_o > \omega_{ce}, \omega_{c\alpha} \) and neglecting the ion dynamics, equation (7.9) reduces to the well-known dispersion relation of the ordinary electromagnetic wave

\[ \omega_o^2 = k_o^2 c^2 + \omega_{pe}^2 + \omega_{p\alpha}^2 . \]

Moreover, using the approximation \( \omega_o < \omega_{ce}, \omega_{c\alpha} \) the equation (7.9) reduces to the usual dispersion relation of the slow Alfven wave given by

\[ \omega_o^2 = k_o^2 v_A^2 \]

where Alfven speed \( v_A = \frac{B_o}{\sqrt{(4 \pi n_o m_o)}} \).

7.4 Nonlinear Dispersion Relation of an Alfven wave

With the help of equation (7.7), now equations (7.1) and (7.2) can be rewritten as
\[
\frac{\delta v_{\alpha(ia)}}{\delta t} = \frac{1}{m_{\alpha}} (eE_{ia} + F_{p\alpha}) - \left( \frac{v_{th,\alpha}}{n_{a0}} \right) v_{n\alpha(ia)} \\
+ \frac{e}{m_{\alpha}c} (v_{\alpha(ia)} \times B_{s}) 
\] 
(7.11a)

and

\[
\frac{\delta v_{e(ia)}}{\delta t} = -\frac{1}{m_{e}} (eE_{ia} + F_{pe}) - \left( \frac{v_{th,e}}{n_{e0}} \right) v_{n e(ia)} \\
- \frac{e}{m_{e}c} (v_{e(ia)} \times B_{s}) 
\] 
(7.11b)

where \( \vec{F}_{p\alpha} \) and \( \vec{F}_{pe} \) are the ponderomotive force on the ions and electrons respectively and are expressed as

\[
\vec{F}_{p\alpha} = \left[ \frac{e}{c} (\vec{v}_{\alpha(ia)} \times \vec{B}_{0}) - m_{\alpha} (\vec{v}_{\alpha(ia)} \cdot \nabla) \vec{v}_{\alpha(ia)} \right] \text{ and} 
\]

\[
\vec{F}_{pe} = \left[ \frac{e}{c} (\vec{v}_{e(ia)} \times \vec{B}_{c}) + m_{e} (\vec{v}_{e(ia)} \cdot \nabla) \vec{v}_{e(ia)} \right].
\]

Following the procedure of separation of variables into homogeneous and varying parts and after linearization, we get the low frequency number density
for the ions [from equations (7.1la) and (7.3)] as

\[
n_{\alpha}(ia) = \frac{-k^2 \beta_{\alpha}}{4 \pi e} \left( \varphi + \varphi_{p\alpha} \right)
\]  

(7.12a)

where \( \beta_{\alpha} = \frac{\omega_{p\alpha}^2}{k^2 v_{th,\alpha}^2 - \omega^2} \).

(7.12b)

Here \( \varphi \) and \( \varphi_{p\alpha} \) are the potential and the ponderomotive potential for ions given by

\[
\dot{E}_{ia} = \nabla \varphi \quad \text{and} \quad \dot{F}_{p\alpha} = -\nabla \varphi_{p\alpha}.
\]

Similarly from equations (7.11b) and (7.4) the low-frequency number density for the electrons is

\[
n_{e}(ia) = \frac{k^2 \beta_e}{4 \pi e} \left( \varphi + \varphi_{pe} \right)
\]  

(7.12c)

where

\[
\beta_e = \frac{\omega_{pe}}{k^2 v_{th,e}^2 - \omega^2}.
\]  

(7.12d)

Here \( \varphi_{pe} \) is the ponderomotive potential for electrons given by \( \dot{F}_{pe} = -\nabla \varphi_{pe} \).

With the help of equations (7.5), (7.11) and (7.12),
one can get $n_{\alpha}(ia)$ of ions and $n_{e}(ia)$ of electrons as

\begin{equation}
\begin{aligned}
n_{\alpha}(ia) &= \frac{k^2(\beta_1+\beta_2)}{4\pi e \varepsilon} \left[ (\phi_{p1}+\phi_{p2}) + \beta_e (\phi_{pe}-\phi_{p1}-\phi_{p2}) \right] \\
\end{aligned}
\tag{7.13a}
\end{equation}

and

\begin{equation}
\begin{aligned}
n_{e}(ia) &= \frac{-k^2\beta_2}{4\pi e \varepsilon} \left[ \phi_{pe}+\beta_1 (\phi_{p1}-\phi_{pe}) + \beta_2 (\phi_{p2}-\phi_{pe}) \right]. \\
\end{aligned}
\tag{7.13b}
\end{equation}

where

\begin{equation}
\varepsilon = (1+\beta_{\alpha}+\beta_e). \\
\tag{7.13c}
\end{equation}

$\beta_1$ and $\beta_2$ correspond to two different ion species. $\phi_{p1}$ and $\phi_{p2}$ are the ponderomotive potential for first and second ions respectively.

The high-frequency nonlinear current density $\vec{J}_1$ at $(\omega, k_1)$ is given by

\begin{equation}
\begin{aligned}
\vec{J}_1(\omega, k_1) &= -n_{e0}e\vec{v}_e + n_{\alpha0}e\vec{v}_{\alpha} - \frac{1}{2} n_{e}(ia)e\vec{v}_e^{*} + \frac{1}{2} n_{\alpha}(ia)e\vec{v}_{\alpha}^{*} \\
\end{aligned}
\tag{7.14}
\end{equation}

where the asterix represents the complex conjugate of the quantity, $\vec{v}_{\alpha}$ and $\vec{v}_e$ are the velocity of the ions and
the electrons respectively due to the scattered wave.

Using equations (7.8a), (7.8b) and (7.14), we get the x-component of \( J_1 \) as

\[
J_{1x} = (A_1 + A_2) E_{1x} B_1 v_{eox}^* + B_2 v_{aox}^* \tag{7.15}
\]

where

\[
A_1 = \frac{-ie^2 n_{e} \omega_1}{m_e (\omega_{ce} - \omega_1)} \tag{7.16a}
\]

\[
B_1 = \frac{k^2 \beta_{e}}{8 \pi \varepsilon} [\phi_{pe} + \beta_1 (\phi_{pl} - \phi_{pe}) + \beta_2 (\phi_{p2} - \phi_{pe})] \tag{7.16b}
\]

\[
A_2 = \frac{-ie^2 n_{\alpha} \omega_1}{m_\alpha (\omega_{ca} - \omega_1)} \tag{7.16c}
\]

and

\[
B_2 = \frac{k^2 (\beta_1 + \beta_2)}{8 \pi \varepsilon} [\phi_{pl} + \phi_{p2} + \beta_a (\phi_{pe} - \phi_{pl} - \phi_{p2})] \tag{7.16d}
\]

From equations (7.5), (7.13), (7.15) we obtain

\[
\varepsilon \phi = - (\beta_e \phi_{pe} + \beta_a \phi_{pa}) \tag{7.17a}
\]
\[ D_1 \cdot \vec{E}_1 = L \left( \vec{v}_\alpha \beta + \vec{v}_e \vec{v}_e^* \right) \]  \hspace{1cm} (7.17b)

where

\[ L = -i\omega_1 k^2 \phi / 2c^2 , \]  \hspace{1cm} (7.18a)

\[ D_1 = \left[ k_1^2 c^2 - \omega_1^2 \epsilon_1 - \frac{\omega_p e \omega_1}{(\omega_{ce}^2 - \omega_1^2)} - \frac{\omega^2_{p1} \omega_1^2}{(\omega_{c1}^2 - \omega_1^2)} \right] I , \]  \hspace{1cm} (7.18b)

\[ \epsilon_1 = \left( 1 - \frac{\omega_p^2}{\omega_1^2} \right) , \]  \hspace{1cm} (7.18c)

and \( I \) is the unit dyadic. Equation (7.18) can be reduced as \( \omega_1^2 \left( 1 + 4\pi n_0 m / B_0^2 \right) - k_1^2 c^2 = 0 \) which is the dispersion relation of the scattered Alfven wave.

On eliminating \( \vec{E}_1 \) and \( \phi \) from equation (7.17), the nonlinear dispersion relation of an Alfven wave is obtained as

\[ \xi = \mu / |D_1| \]  \hspace{1cm} (7.19)
where

\[ \mu = -\frac{e^2 k^2 E_0^2}{8\omega^2} \left( C_1 + C_2 \right) \]  

(7.20a)

\[ C_1 = \frac{\omega^2}{m_e k^2 v_{th,e}^2} \left( \frac{1}{m_e} + \frac{\omega_{pl}^2}{\omega^2} \left( \frac{1}{m_1} - \frac{1}{m_e} \right) \right) \]  

(7.20b)

and

\[ C_2 = \frac{\omega_{pl}^2}{m_1 \omega_0^2} \left( \frac{1}{m_1} + \frac{\omega_{pe}^2}{k^2 v_{th,e}^2} \left( \frac{1}{m_e} - \frac{1}{m_1} \right) \right) \]  

(7.20c)

\[ \frac{\omega_{p2}^2}{m_2 \omega_0^2} \left( \frac{1}{m_2} - \frac{\omega_{pe}^2}{k^2 v_{th,e}^2} \left( \frac{1}{m_e} - \frac{1}{m_2} \right) \right) \]  

Here \( m_1 \) and \( m_2 \) are the mass of first and second ions respectively. \( \omega_{pl} \) and \( \omega_{p2} \) are the plasma frequency of first and second ions respectively.

When the ponderomotive potential is neglected, the non-linear dispersion relation of an Alfvén wave reduces to that of an ion-acoustic wave as
\[ \omega^2 = -\frac{\omega_p^2}{\omega} \frac{1}{1 + \frac{\omega^2}{\omega_p^2}} \frac{1}{k^2 v_{th,e}^2} \]  

(7.21)

which gives the same result as obtained by Cramer and Donnelly 1981 when quasi-neutrality \( n_e = n_\alpha \) is considered.

Assuming the conditions \( \omega < \omega_o \) and \( v_{th,e} > \frac{\omega}{k} > v_{th,\alpha} \) in equations (7.12b) and (7.12d), one gets

\[ \beta_\alpha = \frac{-\omega_p^2}{\omega^2} \]  

(7.22a)

and

\[ \beta_e = \frac{\omega_p^2}{k^2 v_{th,e}^2} \]  

(7.22b)

7.5 Growth Rate of Ion-Acoustic Wave

In the absence of linear damping, the growth rate of the low-frequency ion-acoustic wave can be determined following Sharma and Tripathi 1979 as
For a given pump intensity, the growth rate of the low-frequency ion-acoustic wave is given by [from equations (7.19) and (7.23)]

\[
\gamma = \left[ R(M - N)(P + Q) \frac{E_0^2}{\omega} \right]^{1/2}
\]  

(7.24)

where

\[
R = \frac{e^2 \omega^2}{32 \omega_0 \omega \rho a},
\]  

(7.25a)

\[
N = \frac{\omega^2}{m_e k^2 v_{th,e}^2},
\]  

(7.25b)

\[
N = \frac{\omega^2}{m_\alpha \omega^2},
\]  

(7.25c)

\[
P = \frac{\omega^2}{m_e (\omega_0^2 - \omega_{ce}^2) k^2 v_{th,e}^2},
\]  

(7.25d)

and

\[
Q = \frac{\omega^2}{m_\alpha (\omega_0^2 - \omega_{ca}^2)}.
\]  

(7.25e)
In the case of a "strong" pump for which $M \gg N$ and $P \gg Q$, the equation (7.24) reduces to

$$\gamma = \sqrt{\frac{RMP E_o^2}{2}}$$  
(7.26)

For a "weak" pump ($N \ll M$ and $P \ll Q$), $\gamma$ becomes

$$\gamma = \sqrt{\frac{RNQ E_o^2}{2}}$$  
(7.27)

From the values of $\gamma$, it may be noted that the growth rate of the low-frequency perturbation increases more sensitively with the pump power in the "weak" pump regime than that in the "strong" pump regime. From equation (7.24), the decay-type instability occurs when the frequency of the perturbing wave is smaller than that of the large-amplitude wave, that is, $\omega < \omega_0$.

The growth rate of the ion-acoustic wave in the presence of two types of ions is obtained from equation (7.24) as

$$\gamma_1^2 = \frac{e^2 k^2 E_o^2}{32 \omega_0^3 (\omega_{p1}^2 + \omega_{p2}^2)} \left[ A_1 + A_2 \right]$$  
(7.28)

where
and in the absence of second ions, the equation (7.24) becomes as

\[ \gamma_2^2 = \frac{\gamma^2}{\omega_0^2 \omega_3^2} \left[ \frac{B_1 + B_2}{\omega_{pl}} \right] \]  \hspace{1cm} (7.30)

where

\[ B_1 = \frac{4 \pi e^2 n_{eo}}{m_e^2 k^2 v_{th,e}^2 (\omega_0^2 - \omega_{ce}^2)} \left\{ \frac{1}{m_e} + \frac{\omega_{pl}^2}{\omega^2} \left( \frac{1}{m_1} - \frac{1}{m_e} \right) \right\} \]  \hspace{1cm} (7.31a)

and
Comparison of equations (7.28) and (7.30) yields

\[ \left( \frac{\gamma_1}{\gamma_2} \right)^2 = 1 + \left( \frac{\omega_B^2}{\omega_p l} \right) \left[ \frac{\omega_0}{\omega_p l} \left( \frac{m_2}{m_1} \right)^{\frac{1}{2}} \left( \frac{m_2}{m_e} \right)^{\frac{1}{2}} \left( \frac{\omega_0^2 - \omega_{ce}^2}{\omega_0^2 - \omega_{ce}^2} \right) \right] \]

\[ \frac{1}{1 + \left( \frac{n_{2o}}{n_{1o}} \right) \left( \frac{m_1}{m_2} \right)} \]

(7.32)

where \( n_{1o} \) and \( n_{2o} \) are the number density of first and second ions respectively.

It is seen from equation (7.24) that in the collisionless magnetoactive plasma, the growth rate of the excited mode well above the threshold is proportional to the pump amplitude, that is, in the absence of \( E_0 \), the growth rate disappears. One can thus infer that the amplitude of the pump field must be finite in order to get stimulated Brillouin scattering (SBS) which is also physically the basis of the study of SBS phenomenon. Further, the effect of the magnetic field is to increase the growth rate of the excited wave but the finite electron temperature tends to reduce the growth rate of the excited wave.
7.6 Results and Discussions

Hung 1974 studied the parametric interaction of two Alfven waves and an acoustic wave in a single-ion species plasma using the normal-mode approach. He has considered a large-amplitude Alfven wave acting as a pump which excites another Alfven wave and an acoustic wave. It is noticed that the maximum growth rate of the excited waves increases much more sensitively with the pump power in the weak pump regime than in the strong pump regime. In the present chapter, we have analytically investigated the same problem in a multi-ion species plasma considering the Alfven wave as a pump wave. The present formulation leads to an expression for the growth rate similar to that of Hung 1974. Besides this, one more interesting result is that due to the introduction of heavier ions species in the plasma, the growth rate of the excited ion-acoustic wave gradually decreases. Thus, it can be controlled by increasing or decreasing the concentration of heavier ions in the plasma, that is, the heavier ions can suppress the instability of the excited wave.
For \( n_{10} \gg 10^5 \text{ m}^{-3} \), we get

\[
\left( \frac{\gamma_1}{\gamma_2} \right)^2 \approx \left[ 1 - \left( \frac{n_{20} m_1}{n_{10} m_2} \right) \right] \left[ \frac{m_e \omega_{ce}^2 - m_2 \omega_0^2}{m_e \omega_{ce}^2 + m_1 \omega_0^2} \right]
\]

and when \( n_{10} \ll 10^5 \text{ m}^{-3} \), we get

\[
\left( \frac{\gamma_1}{\gamma_2} \right)^2 \approx \left[ 1 - \left( \frac{n_{20} m_1}{n_{10} m_2} \right) \right] \left[ \frac{m_e \omega_{ce}^2 - m_2 \omega_0^2}{m_e \omega_{ce}^2 + m_1 \omega_0^2} \right] \left[ 1 + \left( \frac{n_{20}}{n_{10}} \right) \left( \frac{m_1}{m_2} \right) \right]^{-1}.
\]

Calculations have been made for the case of Cesium-Chloride plasma at \( 10^5 \text{ k} \) when \( m_e = 0.91 \times 10^{-30} \text{ Kg.} \), \( m_\alpha = 0.66 \times 10^{-26} \text{ Kg.} \), \( k = 25 \times 10^{-4} \text{ m} \), \( \omega_0 = \omega_1 \sim 10^6 \text{ sec}^{-1} \), \( n_{10} \) is equal to \( 10^7 \text{ m}^{-3} \) and \( 10^3 \text{ m}^{-3} \). The results of the calculations have been plotted in Fig. 7.1 which illustrates the nature of the variation of \( (\gamma_1/\gamma_2) \) as a function of \( (n_{20}/n_{10}) \). It is evident from the figure 7.1 that the growth rate of the excited ion-acoustic wave decreases with the increasing number density of heavier ions species in a plasma. The best results can be achieved by using moderately low values of the pump amplitude and by choosing large carrier concentration in the medium.
Fig. 7.1: Variation of $(\gamma_1/\gamma_2)$ with $(n_{20}/n_{10})$ in a Cesium-Chloride plasma at $10^5$ K.
References


