CHAPTER 5

ANALYSIS OF NEM SWITCHES

5.1 INTRODUCTION

Wireless Communication has led to an explosive growth of emerging civilian and military applications of Radio Frequency (RF), microwave and millimeter wave circuits and systems. Future personal and ground communication systems as well as satellite communication systems necessitates the use of highly integrated RF front ends, featuring small size, low weight, high performance and low cost. The key enabling technologies identified by researchers are RF Micro Electro Mechanical systems (RF MEMS) and RF Nano Electro mechanical systems (RF NEMS). NEMS technology for RF and microwave wireless applications is still in the infant stage of research. It is going to yield a new generation of very high performance RF NEMS passives to replace the existing off-chip to provide tremendous performance advantages over solid state components, especially for wireless applications.

RF switches are the key component in nearly all radios and wireless products as they determine the performance of the circuits and systems. Various solid-state switches, such as PIN diodes and FETs used in today’s RF application have limitations as slow switch speeds, low isolation in the OFF state, and high insertion loss in the ON state, high power to operate the switch. RF MEMS switches have wide spread applications in the millimeter wave range due to their lower losses and higher isolation than switches based
on semiconductor devices. However the switching time is in the range of 2-20 µs which is very slow, so they are not able to work for high speed switching applications. NEMS are integrated with microwave planar waveguides, such as coplanar waveguide, for developing NEMS based microwave switches (Dragoman et al 2007).

The basic building block of NEMS switches are Carbon Nano Tubes (CNTs). CNTs have very interesting properties because of their chemical composition (based on pure carbon), chemical bonding and mechanical structure. CNT can be used in NEMS switches because of its ultra-low mass, high directional stiffness and the ability to operate using electrostatic actuation. The first Nano tube-based NEMS switches were demonstrated for applications involving nano tweezers (Kim and Lieber 1999, Akita et al 2001), memory devices (Rueckes et al 2000), supersensitive sensors (Collins et al 2000, Adu et al 2001) and tunable oscillators. Nano relays are another promising application of nanotubes that offer high-performance switching, with high-speed operation at low actuation voltage and power.

The calculation of pull down voltage for carbon nanotube based nano electro mechanical switches has been carried out by (Dequesnes et al 2002). Pull in characteristics of several nanotube electromechanical switches such as double wall carbon nanotubes suspended over a graphene ground plane has also been reported (Dequesnes et al 2002). Static and dynamic pull in analysis of fixed-fixed and cantilever based carbon nanotube switches suspended on a ground plane using Molecular dynamics and Continuum models have been reported (Dequesnes et al 2004). Electromechanical switching devices have been fabricated employing vertically grown multi walled CNT. The devices show various switching characteristics depending on the length and the number of MWCNTs used (Jang 2005).
Electromechanical characteristics of CNT based NEM switches for high frequency applications was proposed (Kaul 2006). Radio frequency performance of the NEM switches using double clamped beam was studied (Dragoman et al 2007). Nano electromechanical and electromagnetic performances were obtained thus by opening the way for evolving agile reconfigurable microwave circuits.

5.2 ISSUES

Computational analysis of electrostatically actuated NEMS requires a repeated self-consistent analysis of the coupled electrostatic and mechanical energy domains (Tang et al 2005). The mechanical analysis is typically carried out in an un-deformed structure. However, the electrostatic analysis is performed on the deformed structures. Therefore, the geometry of the structures needs to be updated before an electrostatic analysis is performed during each iteration. The need to update the geometry of the structures could introduce several problems 1. Flat surfaces of the structures in the initial configuration could become curved due to deformation. This requires the development of complex integration schemes on curved panels (Wang et al 2000) to perform electrostatic analysis 2. When the structure undergoes a very large deformation, remeshing the surface as well as the interior of the deformed structure may become necessary ahead of electrostatic analysis.3. The interpolation functions, need to be recomputed whenever the geometry changes. Each of these issues significantly increases the computational effort making the self-consistent analysis of electrostatic NEMS is an extremely complex and challenging task.

A Lagrangian approach for electrostatic analysis of deformable conductors has been proposed and discussed. (Li and Aluru 2002, Shrivastava, Mukherjee 2005). For self-consistent mechanical and
electrostatic analysis, the deformation can be computed by performing a mechanical analysis using classical theories (Li and Aluru 2003) or classical theories with material properties extracted from atomistic simulation (Tang 2005), or by using a multiscale approach (Tang 2006). To overcome the difficulties in mesh based methods, an efficient approach to deformable conductors is done using meshless methods. (Belytschko et al 1996, Aluru and Li 2001, Jin et al 2004, Jin et al 2005, Li and Aluru 2002, Li and Aluru 2003a,b, Mukherjee and Mukherjee 1997, Telukunta and Mukherjee 2004). Static and dynamic pull in analysis of fixed-fixed and cantilever based carbon nanotube switches suspended on a ground plane using finite cloud method was reported (Dequesnes 2004).

This chapter proposes the formulation of mesh less method using reproducing kernel particle method to analyze the RF NEMS Switches realized on a CPW platform. The correctness of the method is verified by the proposed equivalent circuit model.

5.3 PROBLEM STATEMENT

The static analysis of a RF NEMS Switch is carried out on a CPW platform reduces to that of solving the Linear beam equation subjected to electrostatic forces with appropriate boundary conditions. The geometry of the double clamped CNT switch chosen for analysis is shown in Figure 5.1. The fixed ends of the switch have zero displacement variations. Upon the application of the electrostatic potential, the beam gets deformed and at a certain voltage defined as the pull in voltage the beam becomes unstable and collapses onto the bottom electrode. The problem is to analyze the double clamped RF NEMS switch to obtain the static pull-in voltages.
5.4 MATHEMATICAL FORMULATION

5.4.1 Governing Equation

The governing linear beam equation of a CNT subjected to electrostatic forces is given by (Dequesnes 2004)

\[
-\frac{\partial}{\partial x} \left[ \frac{E I}{\partial x^2} \right] = F 
\]

Figure 5.1  Double clamped CNT separated by a gap \( g_0 \) above the CPW line chosen for analysis

\[
\frac{\rho}{E I} \frac{\partial^2 w}{\partial t^2} + \frac{c}{E I} \frac{\partial w}{\partial t} + \frac{\partial^4 w}{\partial x^4} = \frac{\varepsilon_0 W L V^2}{2 r^2 E I} \left( 1 + \frac{2r}{\pi w} \right) 
\]  \quad (5.1)

where \( w \) is the transverse deflection of the tube, \( \rho \) is the material density, \( E \) is the Young’s modulus, \( I \) is the moment of inertia, \( \varepsilon_0 \) is the permittivity, \( c \) is the coefficient of damping term, \( L \) is the beam length, \( r \) is the gap between nanotube and CPW transmission line, and \( V \) is the applied voltage. Equation (5.1) can be modified as

\[
\frac{\rho}{E I} \frac{\partial^2 w}{\partial t^2} + \frac{c}{E I} \frac{\partial w}{\partial t} + \frac{\partial^4 w}{\partial x^4} - P(w) = 0 
\]  \quad (5.2)

where \( P(w) = \frac{\varepsilon_0 W L V^2}{2 E I r^2} \left( 1 + \frac{2r}{\pi w} \right) \)
5.4.2 Boundary Conditions

Boundary conditions for the double clamped CNT switch are given as

(i) \( w = 0, \frac{dw}{dx} = 0, \frac{d^2w}{dx^2} = 0 \) represents a fixed end and its slope.

(ii) \( \frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0 \) represents no connection (no restraint) and no load.

(iii) \( -\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w}{\partial x^2} \right) = F \) represents the application of a point load \( F \).

(5.3)

The boundary conditions on the gradient of the displacement (i.e. the slope) are treated through a Lagrange multiplier technique (Dequesnes 2004).

5.4.3 RKPM Formulation

The governing equation (5.1) has higher order derivative which involves some difficulty in imposing the boundary condition. So the strong form has to be converted to weak form using Lagrange multiplier technique.

Multiply the governing equation by an arbitrary function \( v \) such that it satisfies the boundary condition and integrate the governing equation over the domain \( \Omega \)

\[
\int_\Omega v \frac{\partial^2 w}{\partial t^2} d\Omega + \int_\Omega \frac{c}{EI} \frac{\partial w}{\partial t} d\Omega + \int_\Omega \frac{\partial^4 w}{\partial x^4} d\Omega - \int_\Gamma vP(w) d\Gamma + \int_\Gamma \delta \lambda (w_{,x} - w_{,x}) n d\Gamma = 0
\]

(5.4)

where \( \Omega \) is the domain, \( \Gamma \) is the boundary of the domain, \( \lambda \) is the Lagrange multiplier, \( \delta \lambda \) is the variation of the Lagrange multiplier and \( n \) is the unit outward normal. Integrating equation (5.4) by parts and noting that

\[
\dot{\lambda} = -w_{,xx}, \quad \delta \lambda = -v_{,xx}
\]
The weak formulation of equation (5.3) is summarized as,

\[
\int_{\Omega} \frac{\partial}{\partial t} \frac{\rho}{2} \dot{w}^2 d\Omega + \int_{\Omega} v \frac{c}{E I} \ddot{w} d\Omega + \int_{\Gamma} v_{,xx} w_{,xx} d\Omega - \int_{\Gamma} v_{,x} w_{,x} nd\Gamma
\]

\[- \int_{\Gamma} v_{,xx} w_{,x} nd\Gamma = \int_{\Omega} vP(w) d\Omega - \int_{\Gamma} v_{,xx} w_{,x} nd\Gamma \quad (5.5)\]

To obtain a matrix form from the equation (5.5), the displacement \( u \) and the function \( v \) are approximated by using the RKPM shape function, i.e.

\[
w = \sum_{B=1}^{NP} N_B w_B \quad (NP=1 \text{ to } 101)
\]

\[
v = \sum_{A=1}^{NP} N_A v_A \quad (NP=1 \text{ to } 101) \quad (5.6)
\]

Substituting (5.6) in (5.5)

\[
\int_{\Omega} \sum_{A=1}^{NP} N_A v_A \left( \frac{\partial}{\partial t} \frac{\rho}{2} \dot{w}^2 \right) d\Omega + \int_{\Omega} \sum_{A=1}^{NP} N_A v_A \left( \frac{c}{E I} \ddot{w} \right) d\Omega \cdot \int_{\Omega} \sum_{B=1}^{NP} N_B w_B d\Omega
\]

\[
+ \int_{\Omega} \sum_{A=1}^{NP} N_{A,xx} v_{A,xx} \sum_{B=1}^{NP} N_{B,xx} w_{B,xx} d\Omega
\]

\[- \int_{\Gamma} \sum_{A=1}^{NP} N_{A,x} v_{A,x} \sum_{B=1}^{NP} N_{B,xx} w_{B,xx} nd\Gamma - \int_{\Gamma} \sum_{A=1}^{NP} N_{A,xx} v_{A,xx} \sum_{B=1}^{NP} N_{B,x} w_{B,x} nd\Gamma
\]

\[
= \int_{\Omega} \sum_{A=1}^{NP} N_A v_A P(w) d\Omega - \int_{\Gamma} \sum_{A=1}^{NP} N_{A,xx} v_{A,xx} w_{x} nd\Gamma \quad (5.7)
\]

where \( N_A, N_B \) are the RKPM shape functions, \( w_A \) and \( v_A \) are the unknowns associated with particle \( A \). For any particle \( A \), a nonlinear residual equation can be written as

\[
R_A(w) = R_A^{dyn}(w) + R_A^{stat}(w) \quad (5.8)
\]

\( R_A^{stat}(w) \) can be written from equation (5.7) as
\[ R_A^{stat}(w) = \int_{\Omega} \left( N_{A,xx} \sum_{B=1}^{NP} N_{B,xx} w_B^N \right) d\Omega - \int_{\Gamma} \left( N_{A,xx} \sum_{B=1}^{NP} N_{B,xx} w_B^N \right) d\Gamma - \int_{\Omega} \left( N_{A,xx} \sum_{B=4}^{NP} N_{B,xx} w_B^N \right) d\Omega \]
\[ - \int_{\Omega} N_A P(w) d\Omega + \int_{\Gamma} N_{A,xx} w_B^N nd\Gamma \]  
\[ (5.9) \]

### 5.4.4 Static Analysis

For static analysis, the dynamic residual term in equation (5.8) is not considered and the residual \( R_A(w) \) is simply the static residual. Equation (5.9) (without the dynamic residual term) can then be solved by employing a Newton's method. The displacement increment within each Newton iteration can be computed by solving the following equation

\[ \frac{\partial R_A^{stat}}{\partial \Delta w_B} \Delta w_B = -R_A^{stat}(w) \]  
\[ (5.10) \]

where \( \frac{\partial R_A^{stat}}{\partial \Delta w_B} = J_{AB}(w) \)

Equation (5.10) can be modified as

\[ J_{AB}(w) \Delta w_B = -R_A^{stat}(w) \]  
\[ (5.11) \]

where \( J_{AB}(w) \in R^{(NP\times NP)} \) is the Jacobian matrix, \( \Delta w_B \in R^{(NP\times 1)} \) is the displacement increment vector, and \( R_A^{stat}(w) \in R^{(NP\times 1)} \) is the static residual vector. The entries of Jacobian matrix is given as

\[ J_{AB}(w) = \int_{\Omega} N_{A,xx} N_{B,xx} d\Omega - \int_{\Gamma} N_{A,xx} N_{B,xx} nd\Gamma - \int_{\Omega} N_{A,xx} N_{B,xx} nd\Gamma - \int_{\Omega} N_A \frac{\partial P(w)}{\partial u} N_B d\Omega \]

i.e.

\[ J_{AB}(w) = \int_{\Omega} N_{A,xx} N_{B,xx} d\Omega - \int_{\Gamma} N_{A,xx} N_{B,xx} nd\Gamma - \int_{\Gamma} N_{A,xx} N_{B,xx} nd\Gamma \]

\[ -\int_{\Omega} N_A \left[ \frac{E_0 wL^2}{EI} \left[ mw(g_0 - w_j)^{-1} - (g_0 - w_j)^{-2} + 2 \right] \right] N_B d\Omega \]  
\[ (5.12) \]
In matrix form equation (5.11) can be written as
\[
\begin{bmatrix}
J_{AB}(w)
\end{bmatrix}_{NP \times NP} \begin{bmatrix}
\Delta w
\end{bmatrix}_{NP \times 1} = \begin{bmatrix}
R_{s}^{\text{stat}}(w)
\end{bmatrix}_{NP \times 1}
\] (5.13)

Solving the system of equations (5.13), results in displacements at each point, which in turn can be used to calculate the down state capacitance in the ON state of the switch.

5.5 EQUIVALENT CIRCUIT APPROACH FOR DOUBLE CLAMPED CNT SWITCH

To show the validity of the RKPM analysis of NEMS switch, an equivalent circuit model is proposed and simulated to obtain the RF performance. The static down state capacitance found out using RKPM analysis is used in the proposed equivalent circuit model. The model available in (Muldavin and Rebeiz 2000) is used as a basis for obtaining the equivalent circuit model for RF NEMS switch realized on CPW platform.

\[Z_C \quad Z_C\]
\[\begin{array}{c}
\frac{1}{C_{d}, C_u} \\
L \\
R
\end{array}\]

Figure 5.2 Equivalent circuit model of RF NEMS Switch

The effect of capacitance between the double clamped CNT and the centre conductor of CPW in both down state and UP states of the switch are given in Figure 5.2 as \(C_{d}, C_u\). Due to the distribution of current in NEMS
switch, the effect of inductance \(L\) and series resistance \(R\) in the down state are included in the proposed equivalent circuit model.

The component values in the equivalent circuit model as in Figure 5.2 can be easily calculated using the standard formulas available in (Muldavin and Rebeiz 2000).

5.6 RESULTS AND DISCUSSIONS
5.6.1 Electrostatic Performance

The validity of the proposed analysis procedure is done using a double clamped CNT on a CPW platform having silicon substrate at a frequency of 25 GHz with the dimensions of the NEMS switch are taken from Dragoman et al 2007 to validate the proposed method. The length of the double clamped CNT is 3µm long, diameter of the CNT is 20nm, width and gap of the CPW line is 1µm. The initial gap \(g_0\) between the CNT and the CPW line is 20nm. The displacement and the slope are assumed constrained at both ends of the CNT. The switch is analyzed using RKPM by employing 101 sprinkled particles. The software code for the analysis procedure has been written in Matlab and the solution to the governing equation (5.1) along with the boundary condition (5.3) is obtained in the form of displacements. Once the displacements are known the downstate capacitance can be calculated using the standard formulas used chapter 3.

The deflections of the double clamped CNT with respect to the length of the beam as a function of series of applied voltages are presented in Figure 5.3 using RKPM analysis. For the double clamped CNT under consideration the pull-in voltage is computed to be 0.75volts. The pull in voltage reported in (Dragoman et al 2007) is 0.8 volts and the error is less than 5%. The values of the pull-in voltage obtained using the Intellisuite MEMSCAD is 2.6 volts as shown in Figure 5.4. Once the pull in has occurred
the contact or down state capacitance found using analysis is 28.4pF. The capacitance can be used in the equivalent circuit to obtain the RF performance.

![Graph](image)

Figure 5.3 Deflection of double clamped CNT for a series of applied voltages obtained using RKPM analysis. The pull in voltage is 0.75 volts.

![Graph](image)

Figure 5.4 Deflection of doubled clamped CNT for a series of applied voltages obtained using Intellisuite MEMS CAD. The pull in voltage is 2.6 volts.
5.6.2  Radio Frequency Performance

In order to determine the losses, RF performances of the switch in both the UP and DOWN states are to be found out. It is obtained using the transmission line model as introduced in section 5.4. Equivalent circuit model available in (Muldavin and Rebeiz 2000) for RF MEMS Switch is used to obtain the RF performance of the double clamped NEMS Switch. The values of the components in the equivalent circuit are calculated using the formulas available in (Muldavin and Rebeiz 2000). The model is simulated using ADS. The validity of the proposed approach is ensured by simulation of double clamped CNT using the HFSS layout simulator. The variation of scattering parameters with respect to frequency for double clamped CNT switch in the down state using equivalent circuit approach (ADS) and layout simulator (HFSS) is presented in Figure 5.5.

![Simulated magnitude of S11 and S21 of double clamped CNT in the Down state using equivalent circuit approach (ADS) and layout simulator (HFSS)](image)

**Figure 5.5**  Simulated magnitude of $S_{11}$ and $S_{21}$ of double clamped CNT in the Down state using equivalent circuit approach (ADS) and layout simulator (HFSS)
The variation of scattering parameters with respect to frequency for double clamped CNT switch in the Up state using equivalent circuit approach and layout approach is shown in Figure 5.6. The simulation results in a return loss of -15.5 dB in the UP condition as in figure 5.6 and agree well with the layout approach done using HFSS. The percentage of error is about 2.5%.

Figure 5.6  Simulated magnitude of $S_{11}$ and $S_{21}$ of double clamped CNT in the Up state using equivalent circuit approach (ADS) and layout simulator (HFSS)

The static capacitance obtained using Intellisuite and the proposed RKPM analysis is used in the equivalent circuit to obtain the radio frequency performance. Figure 5.7 presents the variation of scattering parameters with respect to frequency as a function of the static capacitance found using the proposed method (RKPM) and Intellisuite MEMCAD. The results obtained using RKPM analysis method agrees well with the Intellisuite MEMSCAD.
Figure 5.7  Performance comparison of double clamped CNT switch in Down state with the static capacitance obtained using Intellisuite and RKPM

5.6.2.1  Effect of variations of diameter

The diameter variations of CNT and the corresponding scattering parameter variations with respect to frequency are shown in Figure 5.8. The down state isolation for CNT switch varies with increasing diameters (length is kept constant at $L_m=3\mu m$). For a diameter change from 10nm to 30nm, the inductance value gets reduced and the capacitance value increases by a factor of 10.
5.6.2.2 Effect of variations of inductance and capacitance

Similar to RF MEMS Switch, the effect of inductance and resistance is negligible in the up-state position for NEMS switches. As the physical parameters of the switch such as radius of the tube (5nm - 15nm), gap between the CNT and the CPW line (0-20nm) varies, corresponding change in the capacitance and inductance also occurs. Figure 5.9 shows the variations of down state isolation of the CNT switch with respect to frequency as a function of the capacitance and inductance variations. The capacitance solely controls the response from 1 to 10 GHz (upto \( f_0/2 \)). Once the capacitance is determined, the inductance value controls the resonant frequency location. The inductance has a strong effect on the slope of \( S_{21} \) after \( f_0/2 \) and this can be used to fit an accurate model of the switch inductance.
Figure 5.9  Down state S parameters for a CNT switch with diameter of 20nm showing the capacitance and inductance variations
5.6.2.3 Effect of series resistance of the CNT

Figure 5.10 shows the effect of series resistance of the CNT on the scattering parameters as a function of frequency for a double clamped switch with \( C_d = 20.85 \) pF, \( L=3.02\)pH and \( R_s=0.38\)ohms. The response for \( R_s=0.05 \), 0.25, 0.5 ohms are included for comparison. It is seen that as the series resistance gets smaller, the resonance in \( S_{21} \) gets sharper and deeper (-54,-40,-34 dB, respectively). Also the series resistance has virtually no effect at \( f< 2f_r/5 \), thus it is important to measure the S parameters of the switch around the resonant frequency.

Figure 5.10  Down-state S parameters for a 20nm CNT switch showing the effect of series resistance

Table 5.1 gives a comparison of the simulated scattering parameters obtained using the equivalent circuit approach (ADS) with the capacitance found from RKPM analysis method and layout approach done using High
frequency simulation software (HFSS). The down state isolation of the double clamped CNT switch is almost the same.

Table 5.1 Comparison of the scattering parameters obtained using equivalent circuit approach and layout simulator

<table>
<thead>
<tr>
<th></th>
<th>ON State (dB)</th>
<th>OFF State (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S_{11}</td>
<td>S_{21}</td>
</tr>
<tr>
<td>Layout approach (HFSS)</td>
<td>-0.2</td>
<td>-36</td>
</tr>
<tr>
<td>Equivalent circuit approach (ADS)</td>
<td>-0.131</td>
<td>-36.47</td>
</tr>
</tbody>
</table>

5.7 CONCLUSION

In this chapter a double clamped RF NEMS switch has been proposed and analyzed using reproducing kernel particle method to obtain the pull-in characteristics. The meshless approach is shown to be accurate by comparing the pull-in voltage and peak deflections available in the literature. Electrostatic analysis of the switch is done to obtain the down state capacitance. To calculate the losses an equivalent circuit approach using CLR model is proposed. The down state capacitance obtained using RKPM method is used in the proposed RLC equivalent circuit. The equivalent RLC model is simulated using ADS to obtain the RF performance of the switch in both UP and DOWN states. The simulation results in an isolation of 36.47 dB in the DOWN state and a minimum insertion loss of 0.01 dB. The influence of each component of the equivalent circuit model is also studied and it is found that the inductance and resistance of the switch in the down state has large effect on the isolation.