Chapter-3

STOCHASTIC MODELING OF INS-DERIVED DOPPLER

3.1 INTRODUCTION

This chapter explains about stochastic modeling of INS-derived Doppler for GPS applications. Doppler frequency on GPS signal caused due to the user motion can be easily predicted accurately through co-locating an INS. One of the major difficulties in receiver tracking is that, during dynamic scenarios tracking loops lose lock because of the rapid changes in Doppler. Still, receiver can sustain tracking even in the dynamic condition by integrating the predicted Doppler from INS to tracking loops. Tracking loop bandwidth can be decreased to track only the oscillator dynamics as this is referred as the critical parameter in loop optimization [41]. Thus, in these types of integration systems, accuracy of the predicted Doppler estimate becomes imperative since it directly affects the carrier tracking loop bandwidth. As discussed in section 1.3, raw inertial sensor outputs are not error-free yet these are contaminated with two types of error sources, those are deterministic and stochastic. Generally, navigation parameters, attitude, velocity and position are modeled as the deterministic errors while residual biases from sensors are modeled as stochastic errors. Strapdown INS deterministic models are well-defined and acquired through linearising the mechanisation equations and consider only first-order terms. Here, higher order terms are
normally ignored. The commonly used model for estimating the stochastic errors are 1st and 2nd order Gauss-Markov (GM) and random walk. Yet the most popular and widely used is 1st order Gauss-Markov model [55]. As the deterministic models were well-defined, Kalaman filter identified the inertial errors accurately and eliminates then from raw measurements. However, stochastic errors can be estimated only by approximately since these are based on probability theory.

In ultra-tight integration, Doppler signal is derived from the inertial estimates and is further integrated with tacking loops for eliminating the dynamics from GPS signal. Once the total Doppler on GPS signal is removed, then carrier tracking loop bandwidth can be easily decreased to an order of 1 to 5 Hz based on the oscillator accuracy. Still, if the estimation of Doppler is not accurate then automatically results in the raise in loop bandwidth.

Regrettably the stochastic errors cannot be modeled very accurately and thus INS-derived Doppler still includes the residual biases. So, to improve its accuracy this section examines the usage of Autoregressive (AR) models based on discrete time-series techniques in order to model the stochastic errors of inertial sensors.

For further improving the accuracy of INS-derived Doppler estimates, a 2nd order Autoregressive (AR) model has been proposed
model inertial sensor random errors. As the 1st order Gauss-Markov was used popularly, it was mostly used as benchmark for comparing the efficiency of AR model. Further, optimal determination of model order and coefficients were discussed. Simulation experiments have been carried out for testing the efficiency of AR model. Result of these experiments proves than an improvement of about 45 to 50% can be achieved through 2nd order AR model than 1st order Gauss-Markov model.

3.2 DOPPLER ANALYSIS

Doppler frequency is generally defined as the frequency shift that is caused in received signal because of its relative velocity among transmitter and receiver. Consider a stationary source that transmits the electromagnetic signal at frequency $f_{tx}$. Then from this we can know that, this particular signal received through stationary receiver is at the same frequency as $f_{tx}$. Currently, let us consider a situation where transmitter is moving along the direction towards the receiver at a velocity of $v_{rel}$. The frequency that is received now is not the same as transmitted but the shift is perceived in received frequency. The mathematical model for this received frequency ($f_{dopp}^{\text{dyn}}$) is defined as:

$$f_{dopp}^{\text{dyn}} = f_{tx} \left( 1 - \frac{v_{\text{real}}}{c} \right)$$  \hspace{1cm} (3.1)

where,

$f_{tx}$ – Transmitted frequency
\( v_{rel} \) – Relative velocity between transmitter and receiver

\( \vec{a} \) – Line of sight unit vector between transmitter and receiver

\( c \) – Velocity of light

Eqn. (3.1) also holds good whenever the receiver moves towards its transmitter or else if both of the transmitter and receiver move with respect to each other. Even in GPS, same principle is applicable. Here, in this case the GPS satellite is a transmitter, where it transmits at a frequency \( f_{tx} \) of 1575.42 MHz and a received frequency \( f_{dopp} \) includes a Doppler component because of the motion of satellites and receiver. This Doppler signal in carrier frequency is used for computing the receiver velocity.

### 3.2.1 GPS Doppler Model

Velocity of GPS receiver is computed by measuring Doppler shift caused because of relative motion among the satellite and receiver. Initially, the Doppler signal doesn’t known by receiver, so that receiver must identify this frequency to extract navigation data.

The composite received signal includes both pseudo-random noise code and carrier frequency that are modulated through Doppler signal. As per Kaplan (1996) [7], the maximum Doppler frequency which can be expected by a stationary receiver on carrier frequency is
about ±6500 Hz. It must be noted that Doppler on code signal is less by a factor of 1540 (1575.42 MHz / 1.023 MHz) because of its lower frequency nature. After RF down conversion and digitization, receiver performs an operation known as acquisition through Frequency Locked Loop (FLL) and Delay Locked Loop (DLL) to coarsely find out the carrier frequency and code phase respectively.

To demodulate the navigation data these coarse find out the need to be refined to finer estimates. The fine tuning of carrier frequency is carried out through Costas Phase Locked Loop (CPLL) and code phase by narrowband DLL. The accurate Doppler approximates that are offered by CPLL are then used for velocity calculations.

Doppler frequency on GPS carrier signal can be modeled as:

\[ f_{\text{dopp}}^{\text{carr}} = f_{\text{dopp}}^{\text{dyn}} + f_{\text{dopp}}^{\text{clk}} \]  

\[ f_{\text{dopp}}^{\text{dyn}} = f_{\text{sat clk}} + f_{\text{rx clk}} + f_{\text{errors}} \]

where,

- \( f_{\text{dopp}}^{\text{dyn}} \) – Doppler frequency due to relative velocity among GPS satellite and receiver.
- \( f_{\text{sat clk}} \) – Doppler frequency due to satellite clock errors.
- \( f_{\text{rx clk}} \) – Doppler frequency due to receive clock errors.
- \( f_{\text{errors}} \) – Doppler frequency due to other combined errors.
Doppler frequency on GPS carrier signal is modeled as:

\[ f_{\text{dopp}}^{\text{code}} = f_{\text{dopp}}^{\text{carr}} / 1540 \]  ... (3.4)

### 3.2.2 INS Doppler Model

INS is a dead-reckoning device which provides velocity, position and attitude information when initialised. In an integrated GPS/INS framework, likely there is a chance to extract a Doppler signal from colocated INS measurements whenever it is combined with the GPS satellite ephemeris. Yet, the Doppler estimated through INS includes errors because of deterministic and stochastic errors of INS. Deterministic errors can be precisely estimated through Kalman filter and later removed from inertial measurements. Due to its random nature, only stochastic errors can only be estimated. Further the accuracy can be improved through better models.

The mathematical mode of Doppler estimated by INS is given by expression as:

\[ f_{\text{dopp}}^{\text{INS}} = f_{\text{dopp}}^{\text{dyn}} + f_{\text{dopp}}^{\text{INSbias}} + f_{\text{dopp}}^{\text{INSstoc}} \]  ...(3.5)

where,

- \( f_{\text{dopp}}^{\text{INS}} \) – Doppler frequency due to INS residual stochastic error
- \( f_{\text{dopp}}^{\text{dyn}} \) – Doppler due to relative velocity among the INS and GPS satellite
- \( f_{\text{dopp}}^{\text{INSbias}} \) – Doppler due to deterministic errors of INS
- \( f_{\text{dopp}}^{\text{INSstoc}} \) – Doppler due to stochastic errors of INS
From the above equation it can be noted that, first expression in Eqn. (3.5) is same as that of the received Doppler in Eqn. (3.1). It is occurred as both INS and GPS receiver are collocated and computes same receiver dynamics. There occurs an error which caused due to the finite distance among the GPS receiver antenna and INS although it is small. This type of effect is referred as lever-arm effect, which is not significant on Doppler measurements and thus can be neglected from analysis.

### 3.2.3 Ultra-tight Integration Doppler Model

In ultra-tight integration, Doppler signal predicted through INS is integrated with tracking loops in order to efficiently receive the high dynamic GPS signals. As the user-to-satellite dynamics remain same for both GPS and INS Doppler models (Eqns. (3.3) and (3.5)) and this type of integration approach requires a very small Doppler to be tracked. The residual Doppler which requires to be tracked by an ultra-tight system is given as:

\[
 f_{\text{res dopp}} = f_{\text{clk dopp}} - f_{\text{INSbias dopp}} - f_{\text{INSstoc dopp}} \quad \text{...(3.6)}
\]

where,

- \( f_{\text{res dopp}} \) – Doppler frequency due to GPS residual stochastic error
- \( f_{\text{clk dopp}} \) – Residual Doppler to be tracked by ultra-tight system
- \( f_{\text{INSbias dopp}} \) – Doppler due to deterministic errors of INS
- \( f_{\text{INSstoc dopp}} \) – Doppler due to stochastic errors of INS
Since the integration filter identifies and eliminates the systematic errors $f_{INS\text{bias}}^{\text{dopp}}$, the effective dynamics requiring tracking is given as:

$$f_{\text{res}}^{\text{dopp}} = f_{\text{clk}}^{\text{dopp}} - f_{INS\text{bias}}^{\text{dopp}}$$

...(3.7)

Now the receiver tracking loop requires tracking $f_{\text{clk}}^{\text{dopp}}$ of receiver oscillator and residual stochastic errors from INS-derived Doppler $f_{INS\text{bias}}^{\text{dopp}}$. As these Doppler apparatus are very minute when compared with the Doppler due to relative dynamics, tracking loop bandwidth can be easily reduced to nearly 1 to 3Hz. This bandwidth can be further reduced for the high quality oscillators like Oven Controlled Crystal Oscillators (OCXO).

### 3.3 STOCHASTIC MODELING OF INS DOPPLER

According to Tiao & Box (1981) [56], dynamical characteristics of a complex system are inferred from stochastic time series model fitted to observations. The deterministic errors of inertial sensors $f_{INS\text{bias}}^{\text{dopp}}$ are computed through integration Kalman filter and later removed from its measurements. Still, when these are dealing with the stochastic errors $f_{INS\text{stoc}}^{\text{dopp}}$, it is difficult to approximate them as accurately as deterministic errors since these are governed by probability laws. Doppler that is derived from inertial measurements includes stochastic errors. Two models are used in order to model these stochastic errors viz., 1st order Gauss-Markov and 2nd order Autoregressive model.
3.3.1 Gauss-Markov Model

This Gauss-Markov (GM) processes or models are commonly used in various applications since it describe several random processes with very good approximation [25]. All these models are related to a family of stochastic process by passing the white noise through filter. For modeling inertial sensor residual biases, the 1\textsuperscript{st} order GM model with the large correlation time is used. GM process is completely defined by its autocorrelation function as with any other random process is given as:

\[ R_x(t) = \sigma^2 e^{-\beta t} \]  \hspace{1cm} ... (3.8)

where,

\[ \sigma^2 \] – Variance of the sensors measurements

\[ \beta \] – Inverse process correlation time

The inertial sensor residual bias error is represented through 1\textsuperscript{st} order GM model.

Using the first-order differential equation:

\[ \dot{b}(t) = -\beta b(t) + \sqrt{2\beta_1 \sigma^2 \omega(t)} \]  \hspace{1cm} ... (3.9)

The discrete-form is used in inertial sensor error mode is represented as:

\[ b_{k+1} = (1-\beta_1 \Delta t) + \sqrt{2\beta_1 \sigma^2 \Delta t \omega_k} \]  \hspace{1cm} ... (3.10)

where,

\[ \Delta t \] – Sampling interval

\[ \omega_k \] – driving white noise
As discussed before, stochastic errors models are only the approximation and thus better models can be used if the performance improvement is achieved. As per Nassar (2003) [55], one of such technique is Autoregressive (AR) modeling that can be effectively applied to inertial sensor stochastic errors.

### 3.3.2 Autoregressive Model

This model is an infinite impulse response (IIR) filter or an all pole filter which use of stochastic properties of past behavior of variable in order to identify its behavior in the near future. The ‘N’th order AR model can be modeled in a time-series form [56, 57]:

\[
    u[n] = \sum_{k=1}^{N} \phi_k u[n - k] + \nu[n]
\]  

... (3.11)

where,

- \( u[n] \) – Signal to be modeled
- \( \phi_k \) – \( k \)th AR coefficient
- \( \nu[n] \) – Aero-mean Gaussian white noise

This equation states that the present sample \( \nu[n] \) is identified through \( N \) previous samples. Basically, the series \( \nu[n] \) is believed to be zero-mean, and any deviation from this is compensated by adding constant term \( k \) in front of summation. The transfer equation of Eqn. (3.11) is represented as:

\[
    H(z) = \frac{u[n]}{\nu[n]} = \frac{1}{1 - \phi_1 z^{-1} - \phi_2 z^{-2} - \ldots - \phi_k z^{-k}}
\]  

... (3.12)

and its representation is shown in below figure.
In Eqn. (3.11) with \( v[n] \) represents the inertial sensor stochastic errors \((\delta_{\text{INSstoc}})\), mathematical model for this error is given as:

\[
\delta_{\text{INSstoc}}[n] = \sum_{k=1}^{N} \hat{\phi}_k \delta_{\text{INSstoc}}[n-k] + \hat{\nu}[n] \quad \ldots(3.13)
\]

where,

\[
\hat{\phi}_k \quad \text{– estimate of } k^{\text{th}} \text{ AR coefficient}
\]

\[
\hat{\nu} \quad \text{– estimate of variables of white noise}
\]

\[
\delta_{\text{INSstoc}}[n] \quad \text{– inertial stochastic error estimate}
\]

Here, in the equation both \( \hat{\phi}_k \) and \( \hat{\nu} \) are estimates, so \( \delta_{\text{INSstoc}}[n] \) value cannot be identified precisely and thus a residual is formed that is defined as the difference among the measured and estimated which is given as:

\[
\delta_{\text{res}}[n] = \delta_{\text{INSstoc}}[n] - \hat{\delta}_{\text{INSstoc}}[n] = \hat{\nu}[n] \quad \ldots (3.14)
\]

From the equation, it can be seen that residual quantity is equal to the estimated innovations. The estimation of parameters \( \hat{\phi}_k \) and \( \hat{\nu} \) in Eqn. (3.13) is clearly discussed in next section.
3.4 ESTIMATION OF AR PARAMETERS

In the real time environment, there exists various number of models for computing the AR coefficients but albeit is preferred as it is the common method for deriving the equations where it includes a set of linear equations known as Yule-Walker (YW) equations which was written in matrix form and given by Box and Jenkins (1976) [58]:

\[
\begin{bmatrix}
I & r_1 & r_2 & r_3 & r_4 & \ldots & r_{N-1} \\
r_1 & I & r_2 & r_3 & r_4 & \ldots & r_{N-2} \\
r_2 & r_1 & I & r_2 & r_3 & \ldots & r_{N-3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
r_{N-1} & r_{N-2} & r_{N-3} & r_{N-4} & r_{N-5} & \ldots & I
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
\vdots \\
a_N
\end{bmatrix}
\begin{bmatrix}
r_1 \\
r_2 \\
r_3 \\
\vdots \\
r_N
\end{bmatrix}
\]

where,

\[ r_n \text{ - autocorrelation coefficient at delay } n \text{ and diagonal } r_0 = I. \]

In order to fit the inertial sensor stochastic errors into the AR process form, a good apriori choice of the model order is critical. As per Lardies (1996) [59], overestimation of model order causes either line splitting or erroneous peaks, in estimated spectrum and underestimation which leads to smoothed spectrum. Once the order of model is identified, AR coefficients are acquired through Yule-Walker equations.

3.4.1 Order Estimation

Presently there are numerous literatures that illustrate the criterion for choosing the order of model [60, 61, 62]. One of such
criteria is Schwarz’s Bayesian Criterion (SBC), where it is used for this study. As per Schwarz (1978) [63], SBC generally states that the order of AR model is chosen as the integer which reduces the criterion. The mathematical expression of SBC is given as follows:

$$SBC(k) = \frac{l_k}{m} - \left(1 - \frac{n_n}{N}\right) \log N$$

... (3.16)

where,

- $k$ – model order to be determined
- $l_k$ – log (det$\Delta_k$)
- $\Delta_k$ – residual cross-product matrix

Simulation package ARMFIT™ [64] is used to identify the order of filter. Estimated order is the appropriate model order, for which the value of SBC is minimum. In most of the cases, order doesn’t exceed 5, and thus upper limit is taken as 5 with lower limit being 1 and these results are presented in table 3.1 for the order ranging from 1 to 5.

**Table 3.1: SBC values for order 1 to 5**

<table>
<thead>
<tr>
<th>Order (k)</th>
<th>SBC(k)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 200$</td>
</tr>
<tr>
<td>1</td>
<td>0.1272</td>
</tr>
<tr>
<td>2</td>
<td>0.1006</td>
</tr>
<tr>
<td>3</td>
<td>0.1478</td>
</tr>
<tr>
<td>4</td>
<td>0.1747</td>
</tr>
<tr>
<td>5</td>
<td>0.2016</td>
</tr>
</tbody>
</table>

From the above table, it shows that the minimum criterion is achieved at an order of 2 and as a result the inertial sensor errors are modeled with AR (2). A test must be conducted for determining
whether the estimated order is suitable or not, this is done by verifying the whiteness of model output. If the estimated coefficients (described in next section) and if the order of model is correct then the autocorrelation sequence would represents the output of process as being white.

![Graph showing autocorrelation function with confidence intervals](image)

**Fig. 3.2: Original and 2\textsuperscript{nd} order AR model time series for inertial random errors**

AR modeled series from the above figure shows that the noise is indeed white (within 95% confidence interval denoted by two horizontal lines).

### 3.4.2 Coefficient Estimation

Once the order of model is estimated then the next step is to compute the coefficients of the model to complete AR model design. Expanding the Eqn. (3.11) for 2\textsuperscript{nd} order model with N=2 give the following expression:
\[ u[n] = \hat{\phi}_1 u[n-1] + \hat{\phi}_2 u[n-2] + \hat{\nu} \] ...(3.17)

where,

\[ \hat{\phi}_1 \text{ and } \hat{\phi}_2 - \text{ autoregression coefficients} \]

\[ \hat{\nu} - \text{ variance of white noise to be estimated} \]

\[ u - \text{ its equal to } \delta^{\text{INSstoc}} \]

From Eqn. (3.17), one can state that if AR coefficients are known then \( u[n] \) can easily calculated from the previous two samples \( u[n-1] \) and \( u[n-2] \). Eqn. (3.17) is represented as the 2-pole AR filter as shown in Fig. 3.3.

\[
\begin{align*}
Z^{-1} \quad & u[n] \\
Z^{-1} \quad & u[n-1] \\
\Sigma \quad & \phi_2 \\
\phi_1 u[n-1] \quad & - \\
\phi_2 u[n-2] \quad & + \\
\Sigma \quad & \\
\hat{\nu} \quad & + \\
\end{align*}
\]

**Fig.3.3: 2-Pole AR Filter**

Yule-Walker based autocorrelation method is used for determining the AR coefficients. Some of the steps are followed for estimating the coefficients, which are discussed below:

**Step-I:** Estimate the values of \( R_x(0) \), \( R_x(1) \) and \( R_x(2) \) for AR(2) model through time-averaging, which is represented as:
\[
\hat{R}_x(0) = \frac{1}{N} \sum_{t=1}^{N} (u[i]u[i+1])
\]  
... (3.18)

where,

\[N \quad \text{– 200 is number of samples}
\]

\[\hat{R}_x(i) \quad \text{– autocorrelation values}
\]

The parameter \(N\) is the trade-off among the accuracy and processing delay.

**Step-2:** From \(R_x(0)\), \(R_x(1)\) and \(R_x(2)\), estimate the values of \(\hat{\phi}_1\), \(\hat{\phi}_2\) and \(\hat{\nu}\) through the following equations:

1\(^{st}\) AR coefficient estimate:

\[
\hat{\phi}_1 = \frac{R_x(0)R_x(1) - R_x(1)R_x(2)}{R_x^2(0) - R_x^2(1)}
\]  
... (3.19)

2\(^{nd}\) AR coefficient estimate:

\[
\hat{\phi}_2 = \frac{R_x(0)R_x(2) - R_x(1)R_x(2)}{R_x^2(0) - R_x^2(1)}
\]  
... (3.20)

Estimate of variance of white noise

\[
\hat{\nu} = R_x(0) - \phi_1R_x(1) - \phi_2R_x(2)
\]  
... (3.21)

The values of autocorrelations for about 5 runs are represented in Table 3.2. From the mean values, the AR(2) parameters are computed as follows:

**Table 3.2: Autocorrelation estimates**

<table>
<thead>
<tr>
<th>Iteration No.</th>
<th>(R_x(0))</th>
<th>(R_x(1))</th>
<th>(R_x(2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4876</td>
<td>0.9768</td>
<td>0.9651</td>
</tr>
<tr>
<td>2</td>
<td>0.9154</td>
<td>0.8654</td>
<td>0.8612</td>
</tr>
<tr>
<td>3</td>
<td>0.9842</td>
<td>0.6172</td>
<td>0.6128</td>
</tr>
<tr>
<td>4</td>
<td>1.1860</td>
<td>0.7952</td>
<td>0.7864</td>
</tr>
<tr>
<td>5</td>
<td>1.2712</td>
<td>0.8149</td>
<td>0.8078</td>
</tr>
</tbody>
</table>
The mean values of estimated autocorrelation are given as:

\[ R_x(0) = 1.16884, \]
\[ R_x(1) = 0.8139 \]
\[ R_x(2) = 0.80666 \]

The more the iterations, accuracy of the results improves. Substitutions of these values into Eqns. (3.19) and (3.21) and solving results as:

1st AR coefficient estimate:
\[ \hat{\phi}_1 = 0.4188 \]

2nd AR coefficient estimate:
\[ \hat{\phi}_2 = 0.3984 \]

Estimate of variance of white noise
\[ \hat{\nu} = 0.5064 \]

Thus, the Eqn. (3.17) is represented as:
\[ \hat{\nu} = R_x(0) - \hat{\phi}_1 R_x(1) - \hat{\phi}_2 R_x(2) \] ... (3.21)
\[ u[n] = 0.4188u[n-1] + 0.3984u[n-2] + 0.5064 \] ... (3.22)

Hence, the Eqn. (3.22) defines the AR(2) model for inertial sensor’s stochastic errors.

3.5 SIMULATIONS

The simulations were conducted to test the INS-derived Doppler accuracy, with inertial sensor errors modeled as 1st order Gauss-Markov (GM) in first run and with 2nd order Autoregressive model
during its second run. Using GPSOFT™, a reference trajectory with the known dynamics was produced where it is shown in Fig. 3.4.

### 3.5.1 Trajectory 1 Tests

![IMU Trajectory](image)

**Fig. 3.4: Simulated trajectory 1 for testing the accuracy of INS-derived Doppler**

The trajectory includes segments, velocity, acceleration, 90° turns, pitch-up and acceleration is derived from inertial measurements. A white noise standard deviation of 0.008 ft/m was applied for accelerometer and white noise standard deviation of 0.125 deg/root-hour was employed for gyroscope measurements. Doppler values are calculated by merging this trajectory with satellite ephemeris. Absolute Doppler values are not utilized for aiding, and the only difference in Doppler among successive epochs is used to assist the tracking loops with Doppler value at $t_0$, which is considered as reference. For example, the Doppler values which are used for aiding are represented as:
\[
\begin{align*}
  d_1 &= d_{t1} - d_{t0} \\
  d_2 &= d_{t2} - d_{t1} \\
  &\vdots \\
  d_n &= d_{tn} - d_{t(n-1)}
\end{align*}
\] ...

**Fig. 3.5: Comparison of AR and GM estimated Doppler’s with original Doppler for trajectory 1**

The above figure represents the Doppler plot among the GM(1) and AR(2) modeled process. For contrast, an ideal Doppler is directly derived from trajectory is also shown. The proximity of AR(2) modeled Doppler to the ideal one confirms the hypothesis that AR(2) is better than GM(1) model for representing the inertial sensor’s stochastic errors. In Fig. 3.6, magnitude of both Doppler errors are represented.
The plot shows that GM modeled Doppler error is greater than the AR modeled error. The mean square is computed by calculating the mean of squared difference among the ideal Doppler and AR/GM estimated Doppler’s.

Mean square error of AR model

\[
\text{MSE}_{\text{AR}} = \frac{1}{N} \sum_{i=1}^{N} (\text{dopp}_{\text{AR}}[i] - \text{dopp}_{\text{ideal}}[i])^2 = 3.7434 \quad \ldots \quad (3.24)
\]

Mean square error of GM model

\[
\text{MSE}_{\text{GM}} = \frac{1}{N} \sum_{i=1}^{N} (\text{dopp}_{\text{GM}}[i] - \text{dopp}_{\text{ideal}}[i])^2 = 8.2185 \quad \ldots \quad (3.25)
\]

From the above analysis, N is selected as 200. From Eqns. (3.24) and (3.25), it can be inferred that mean square errors of AR-based modeling are nearly 45% less than GM-based modeling.
3.5.2 Trajectory 2 Tests

An another experiment was simulated by considering the constant velocity of 100m/s for 50 sec, the trajectory was plotted in Fig. 3.7.

![IMU Trajectory](image1)

**Fig. 3.7: Simulated trajectory 2 for testing the accuracy of INS-derived Doppler**

The stochastic errors are once more modeled with both AR and GM models and results are plotted in Fig. 3.8.

![Comparison of AR and GM estimated Dopplers with original Doppler for trajectory 2](image2)
Doppler errors magnitude is calculated and plotted in Fig. 3.9. From Fig. (3.8) and (3.9), it can be inferred that Doppler derived from AR modeling is more efficient than GM model.

![Graph showing the difference of Doppler errors between 2nd order AR and 1st order GM models](image)

**Fig. 3.9: Difference of Doppler errors between 2\textsuperscript{nd} order AR and 1\textsuperscript{st} order GM models**

Mean square error from AR model

\[ \text{MSE}_{\text{AR}} = 0.2977 \]

Mean square error from GM model

\[ \text{MSE}_{\text{GM}} = 0.5893 \]

In this trajectory, the mean square error of AR model is nearly 50% better than GM model. Thus, from the above analysis on trajectories, both the trajectories 1 and 2 shows that AR modeling of inertial sensor stochastic errors symbolizes a promising approach and also very effective for ultra-tight integration in terms of loop bandwidth reduction.
3.6 CONCLUSIONS

As the INS-derived Doppler also defines carrier tracking loop bandwidth in ultra-tight integrated system, and it’s imperative that this Doppler must be estimated accurately. There are two types of inertial sensor errors, deterministic and stochastic errors. The deterministic errors are well defined, and they are estimated through Kalman filter and eliminated from raw sensor measurements. The stochastic errors can be estimated approximately only because of their random nature. Although the 1st order Gauss-Markov (GM) model is trendy in modeling the stochastic errors, the discrete time series algorithms can implemented for improving the estimates. This section explores the 2nd order Autoregressive (AR) model to estimate the stochastic errors. The results of this model are compared with 1st order GM model. The estimation of AR model order and coefficients were also discussed. Simulation experiments were carried out in order to test the efficiency of AR model and those results are discussed. Thus, it can be concluded from the experiments that AR model offers 50% more efficiency when compare with GM model.