2.1 INTRODUCTION

The stand-alone Inertial Navigation System (INS) cannot uphold navigation for lengthy periods because of its growth of navigation errors along with time. Generally, there are two ways for reducing these errors. In the first way, it makes use of high quality navigation grade inertial sensors (which are matched with the navigation accuracy requirements) in addition to the associated navigation filter implementation. The second way is to just augment INS with non-inertial navigation sensors/technologies, like Tacan (Tactical Air navigation), Omega, Global Positioning System (GPS) and Jtids (Joint Tactical Information Distributed System). Even though the first way controls the error growth with time, yet the size and cost of sensors limits the usage of high quality INS for most of the applications. Whereas the second approach is more feasible because of its smaller size, low cost and provides a satisfactory system performance. GPS is preferred as the best sensor for the INS augmentation due to its global coverage, bounded accuracy and has an ability to operate in all weather conditions.

Some of the benefits of the GPS/INS integration related to the individual standalone systems include: (i) Higher navigation data
rates, (ii) Smoother trajectory, (iii) Good short-term and long-term accuracies, (iv) Better availability and (v) Greater integrity [19, 25, 29].

Mostly the integrated systems are widely used in some of the specialized applications like mobile mapping, vehicle guidance & control, aerial photogrammetry/imaging & gravimetry, etc. [30, 31, 32, 33, 34].

In order to offer the more robust navigation output, both INS and GPS technologies are integrated which exhibit the complementary characteristics, they are exploited under system integration. Few of its complementary features are discussed below:

- The measurement errors of GPS tend to be of high frequency whereas in the case of INS the errors predominantly fall in low frequency region.
- GPS is considered as a long-term stable system having abounded navigation accuracy while INS can just operate for a short period with acceptable navigation accuracy.
- The atmospheric, multipath and orbital errors of GPS are in medium-to-low frequency band whereas INS Schuler-type errors have a narrow bandwidth at low frequency band and also the errors in vertical position and velocity of INS rises along with time.

Basically there are three types of architectures for integration, which are loosely-coupled, tightly-coupled and ultra-tightly coupled.
The GPS measurements in all of these configurations calibrate the inertial sensor errors and then correct the raw inertial measurements. Based on the type of GPS measurements used for calibration, this system is categorized into one of the types that are mentioned above. For instance, if the selected type is loosely-coupled configuration then the position and velocity data from GPS receiver are utilized in integration filter while if the chosen type is tightly-coupled configuration then pseudo range and delta pseudo measurements are used in integration filter. Finally, in the case of ultra-tightly coupled configuration, I (in-phase) and Q (Quadrature) measurements from correlator are utilized in integration filter. In general, the error state vector of integration Kalman filter includes INS position, attitude, velocity, scale factor parameters, biases and drifts. On the other hand, even additional error states can be added to improve the performance of system.

Hence, from the above context one can easily understood the integration of GPS receiver with the INS in ultra-tight mode. Reader can easily acquire knowledge on the ultra-tight tracking loops as it clearly explains the overview on various integration architectures and then these tracking loops are provided. Further of this section explains the dynamic and thermals stress analysis of both the carrier and code tracking loops and also explains how they are optimized concurrently in ultra-tight configuration. INS is mostly configured in strapdown mode for low cost applications so a brief analysis is done
on strapdown mechanization algorithm. The Kalman filter algorithm is discussed with its prediction and correction approach as it is used for the integration of GPS and INS. The simulation experiments are conducted for investigating the effects of aiding the inertial signals with code and carrier tracking loops are also presented in this chapter.

2.2 INERTIAL NAVIGATION SYSTEM (INS) MECHANISATION

In general, mechanization is defined as the process of converting raw inertial sensor measurements to navigation parameters. Accelerometers are used for computing the particular force (acceleration plus gravity) in the body-fixed axis frame. Through the double integration process, both the velocity and position are derived from a particular force. The platform consisting of integrated devices travels with respect to certain navigation frame thus the accelerometer measurements are resolved with respect to this reference frame. From the gyroscopes, the attitude measurements provide the essential transformations for converting the accelerometer measurements in a body-fixed axis to navigation frame. Basically, the mechanization process can be categorized into three types, viz., Local-level, Space-stable and Strapdown [26].

Any one of the above mechanization can be utilized based on the computational complexities, interfaces with the other subsystems and navigation frame requirements. Although, there are various
mechanizations mostly strapdown mechanisation is preferred because of its lower complexity.

2.2.1 Strapdown Mechanisation

The inertial sensors are fixed to the platforms in this mechanization that is INS frame coincides with the vehicle or body-frame. From the body-frame, the accelerometer measurements are acquired and converted to the values in navigation frame through the mathematical rotations which are define through the attitude measurements of gyroscope. According to Lapucha (1990) [35], once the navigation frame is defined then at the same time the specific force and attitude measurements are further converted to position, velocity and attitude parameters. This process is carried out through the integration of differential equations for navigation parameters [28, 29, 35].

Fig.2.1: Strapdown mechanization system for navigation frame
The time differential navigation equations for the strapdown INS are written as follows:

Position vector estimation in n-frame

\[
\dot{r}^n = -\omega_{en}^n r^n + v^n \quad \ldots (2.1)
\]

Velocity vector estimation in n-frame

\[
v^n = f^n - (2\omega_{ie}^n + \omega_{en}^n) v_e^n + g^n \quad \ldots (2.2)
\]

Estimation of DM to transform the specific force vector from the b-frame to n-frame

\[
\dot{C}_b^n = C_b^n \omega_{ib}^b - \omega_{in} C_b^n \quad \ldots (2.3)
\]

with rotation rate vector in e-frame

\[
\omega_{ie}^n = \begin{bmatrix}
\Omega \cos L \\
0 \\
-\Omega \sin L
\end{bmatrix} \quad \ldots (2.4)
\]

with rotation rate vector in n-frame

\[
\omega_{ie}^n = \begin{bmatrix}
\dot{\lambda} \cos L \\
-\dot{L} \\
-\dot{\lambda} \sin L
\end{bmatrix} \quad \ldots (2.5)
\]

Wavelength estimate

\[
\dot{\lambda} = \frac{v_E}{(R_o + h) \cos L} \quad \ldots (2.6)
\]

Longitude estimate

\[
\dot{L} = \frac{v_N}{(R_o + h)} \quad \ldots (2.7)
\]

where,

\[r^n \quad - \quad \text{Position vector in the n-frame}\]

\[v^n \quad - \quad \text{Velocity vector in the n-frame}\]

\[g^n \quad - \quad \text{Gravity vector in the n-frame}\]
\( f^n \) – Specific force in the n-frame

\( C^b_n \) – Direction cosine matrix (DCM) to transform the specific force vector from the b-frame to the n-frame

\( \omega_{ie}^n \) – Rotation rate vector in the e-frame

\( \omega_{en}^n \) – Rotation rate vector in the n-frame

\( \omega_{ib}^n \) – Rotation rate vector in the b-frame

\( \omega_{in}^n \) – Rotation rate vector in the n-frame

\( v_E \) – Velocity vector in the e-frame

\( v_N \) – Velocity vector in the n-frame

\( R_o \) – Radius of the earth

\( L \) – Latitude

\( h \) – Height above the ellipsoid

Once the specific force vector \( f^n \) is acquired then it is a relatively straightforward process to numerically integrate the equations (2.1) and (2.2) for identifying the position and velocity. Even though, the rotation matrix can be computed directly through the integration of equation (2.3) but a numerical quaternion updating approach is used because of its reduced computational burden. Few of the excellent references for the quaternion update are Lee et al. (1988), Da (1997) and Savage (2001) [28, 31, 36]. Thus, from the above context it can be easily stated that Strapdown Mechanisation is mostly used by various manufacturers due to its various features and the benefits provided by it. From this process one can easily calculate the velocity and position.
2.3 INTEGRATION ARCHITECTURES

Generally there are three architectures, which are loosely-coupled, tightly-coupled and ultra tightly-coupled. Based on the architecture type, various GPS measurements are considered as the input to complementary Kalman filter and the other input is taken from the strapdown mechanization algorithm that converts the raw inertial sensor measurement to position, velocity and attitude parameters. This filter approximately calculates the biases, drifts and scale factor errors of inertial sensors and sends this back to the inertial unit for correction.

Fig. 2.2: Architecture of loosely, tightly and ultra-tightly coupled
In ultra-tight integration, the Doppler signal is also produced by the filter and fed back to tracking loops where as in the case of loosely-coupled system, it treats both the INS and GPS components as the two independent navigation systems. From both these systems, position (P) and velocity (V) are estimated and combined in the external Kalman filter in order to finally estimate the inertial sensor errors. However, as the GPS is referred as the standalone navigation system, tracking on the minimum of four satellites where these are maintained for an effective integrated system. For a 3-dimensional navigation, whenever the number of visible satellites are dropped below four for a then the updates of measurements doesn’t take place and lead to dependence on the INS alone operating in a free-inertial mode. So, this is considered as one of the most limitations of the system. This limitation can be overcome through the tightly-coupled configuration where the GPS is treated as the sensor rather than considering as the navigation system and from the receiver both pseudo ranges (ρ) and delta pseudoranges (Δρ) are integrated with the INS measurements [29, 31]. This approach offers a better flexibility in GPS data processing and INS predicts the navigation parameters at each epoch. For further reference, Phillips et al. (1996), Jekeli (2001), Scherzinger (2000) [37, 38, 39] gave an overview on the various levels of integration.

### 2.3.1 Comparison of GPS/INS Integration Architecture

As shown in Fig.2.2, the main difference among the three architectures is GPS measurements that are utilized in the integration
of Kalman filter during the measurement update process. At the same time as the process model that propagates the inertial states remains same for all the three architectures, the matrix H of measurement model which relates the states and measurements varies. In loosely-coupled integration, since both the state variables and measurements are position and velocity the relationship matrix H is unit matrix. In the case of tightly-coupled integration, GPS measurements which are used are pseudo ranges (ρ) and delta pseudo ranges (˙ρ). Since in this case, as the inertial states is velocity and position the matrix H is defined as the first order partial derivatives ρ of and ˙ρ with respect to position and velocity respectively, i.e., $\frac{\partial \rho}{\partial x}$ and $\frac{\partial \dot{\rho}}{\partial x}$.
However, relationship matrix (H) is more complex in the case of ultra-tightly coupled system as there exist a complicated relationship among the correlator measurements I and Q and inertial states P and V. It must be noted that they are related through the phase and frequency errors of tracking loops that are occurred due to missing of composite signal with locally generated phase and frequency components. The detailed analysis of this relationship and structure of matrix H is explained in later Chapter 4. The relationship between the states and the measurements for all the three architectures are shown in Fig.2.3.

2.3.2 Ultra-Tight GPS/INS Integration System

The measurements I and Q from the receiver correlator are used in the ultra-tight GPS/INS integration scheme. Correlator is just the baseband processor of GPS receiver that digitally processes the composite GPS signal for demodulating the navigation data. During this demodulation process, the correlator produces the two signals I and Q by multiplying the incoming digitised IF (Intermediate Frequency) signal with phase quadrature signals (0° and 90°) from Numerically Controlled Oscillator (NCO). And then these signals consequently mixed with the code and estimates the early (E), prompt (P), and late (L) for acquiring $I_E$, $P_E$, $I_L$ and $Q_E$, $P_E$, $Q_L$ respectively. Kaplan (1996) and Tsui (2000) [7, 40] stated that code/carrier acquisition and tracking loops makes use of signals for precise phase alignment by using the code and carrier discriminator algorithms. For
calibrating the inertial sensor error $I_P$ and $Q_P$ signals are used as measurements in integration Kalman filter after accurate tracking. The INS corrected data and GPS satellite ephemeris information are combined to derive a Doppler value which is fed back in to the carrier tracking loop.

![Diagram](image)

**Fig. 2-4: GPS INS Integrated system in ultra-tight configuration**

A Doppler value is derived by merging the corrected INS data with the GPS satellite ephemeris data. Then this value is fed back to carrier tracking loop. This Doppler value (that is measure of the receiver dynamics) can be successfully integrated with the receiver tracking loops for mitigating the Doppler on the GPS signal before entering into tracking loops (here, satellite dynamics are ignored as these can be estimated from navigation message data). As per Weiss &
Kee (1995) [41], once the majority of Doppler Effect is eliminated from GPS signal, the tracking bandwidth particularly the carrier tracking loop can be significantly reduced since the receiver need to track only the residual motion implied by oscillator errors. Thus, it can be stated that the integration of GPS and INS enables the system to recognize the Doppler value and send back to the carrier tracking loop. Once, this Doppler effect is eliminated, carrier tracking loop is reduced.

2.4 PERFORMANCE OF ULTRA-TIGHT TRACKING LOOPS

In the GPS receiver, the tracking loops must track both the pseudorandom noise code and carrier frequency for extracting the 50Hz navigation message information that is in turn used for position computation. Under normal signal strength and moderate dynamic conditions, the tracking loops that include Costas Phase Locked Loop (CPLL) and Delay Locked Loop (DLL) perform well in these conditions. However, the performance of tracking loops degrades considerably whenever the signal strength is distorted or weak because of obstruction and also when the dynamics exceed certain threshold. In the stressed scenarios, tracking bandwidth and dynamic stress are considered as the two conflicting parameters where these need to be optimized in receiver operating [42]. Conventional receiver is optimised either to handle high dynamics or to receive the weak signals but not both. Simultaneous optimization can be carried out by adopting the augmentation techniques. In ultra-tight configuration, the Doppler signal is derived from INS integrated with the tracking loops. This
method not only eliminates the platform dynamics from GPS signal but also decreases the tracking loop bandwidth and thereby improving its overall system performance.

### 2.4.1 Carrier Tracking Loop

As the Doppler Effect occurs in the received GPS signal, the two step process is preferred for the baseband processor for tracking this signal. The steps are coarse acquisition and fine tracking.

Acquisition loop guess the accurate Doppler frequency either through frequency domain or time domain search techniques [40]. Once the estimation of coarse frequency is acquired to an accuracy of nearly 20 to 30 Hz, further it is sent to tracking loop for refinement. Navigation data occurrence on carrier reduces the usage of simple Phase Locked Loop (PLL). As a result, CPLL [43] consists of dual PLL architectures that is protected to the data modulation and is typically used. In this particular architecture, composite signal is multiplied with the in-phase and quadrature signals from NCO for producing I and Q signals (doubt). After combining with the code signals [7, 40] both I_P and Q_P components are integrated over the pre-detection interval, and merged in a phase discriminator algorithm for generating the error signal. This signal is not affected by data modulation as the information is present on both of the quadrature components. Errors signal is low pass filtered and fine-tunes the NCO frequency in such a way that the output of multiplier becomes zero.
Conventional receiver make use of carrier tracking loop bandwidth of nearly 10 to 20 Hz having a loop order of 2 in order to track the moderate dynamic signals. The dynamics such as acceleration and jerk are reflected as Doppler rate and change of Doppler rate, respectively on the GPS signal. Whenever there exist excessive dynamics same as in the case of vehicle undergoing rapid maneuvers, both the loop order and conventional tracking loop bandwidth many not be adequate for tracking the high Doppler rates. For acceleration, the second order loop is more sensitive that is the steady-state tracking error doesn’t become zero for continuously accelerating signal. Thus, both the tracking loop order and loop bandwidth have to be raised [42, 44, 45, 46]. Naturally, the bandwidth must be raised beyond 20 Hz and a loop order 3. The third order loop design presents a stability challenges and raises in loop bandwidth results in more thermal noise in measurements. Another approach for addressing the high dynamics in order to augment the tracking loops with external information. It not only restricts the higher Doppler rates but also facilitates the significant reduction in tracking the bandwidth and also leads to various benefits like increased signal-to-noise ratio (SNR), reduction in integration period for weak signals and increased immunity to jamming and RF interference.

In ultra-tight configuration, the augmented approach is used where by INS-derived Doppler data are integrated with carrier tracking loop in Fig.2.5.
For approximating the inertial sensor errors, both I and Q signals from tracking loops are fed to integration Kalman filter. Further, in order to remove the platform dynamics from GPS signal the Doppler is estimated from INS and then integrated with the carrier NCO. GPS signal that is stripped of the major Doppler Effect is referred to be almost dynamic-free. In GPS signal, residual dynamics is mainly because of local oscillator error. As Doppler is comparatively small because of oscillator error, so for improving the accuracy of raw measurements the carrier bandwidth is reduced significantly.
Fig. 2.6: Conventional vs. ultra-tight tracking loop bandwidth

The above figure further illustrates the components that contribute to ultra-tight and conventional receivers tracking loop bandwidth. In conventional receivers, it is the receiver dynamics which contributes greatest part of bandwidth where as in the case of ultra-tightly integrated receivers its only residual biases from integration filter and Doppler from local oscillator where it affects the bandwidth. Thus, it can be stated that conventional receiver make use of carrier tracking loop bandwidth for tracking the moderate dynamic signals.

2.4.1.1 Bandwidth and Dynamic Analysis

In PLL design mainly two conflicting parameters are considered which are dynamic stress error and thermal error [7]. At the same time as the thermal error is directly proportional with the loop bandwidth so the dynamic stress errors has the inverse effect. In
order achieve the higher accuracies; the bandwidth must be kept low however this limits the high dynamic performance. The bandwidth need to rise for achieving the high dynamic operation where it in turn raises the thermal noise in measurements. Thus in the conventional receiver, bandwidth is optimized either for dynamic performance or thermal performance.

The mathematical expression for thermal noise or thermal stress is given by:

\[
\sigma_{\text{thermal}} = \frac{360}{2\pi} \sqrt{\frac{B_n}{c/n_o}} \left( 1 + \frac{1}{2TC/n_o} \right)
\]  

... (2.8)

where,

- \( B_n \) – Loop bandwidth in Hz
- \( c/n_o \) – Carrier-to-noise ratio
- \( T \) – Pre-detection integration time (typically 1 msec)

From the equation (2.8), it can be observed that the thermal noise added to loop is directly proportional to the loop bandwidth, while both signal power and pre-detection interval are inversely proportional to the thermal noise that is higher integration intervals and signal power diminish the thermal effect. The mathematical expression of dynamic stress error is represented as:

\[
\sigma_{\text{dynamic}} = 0.2809 \frac{\ddot{a}}{B_n^2} (\text{deg})
\]  

...(2.9)

where,

- \( \ddot{a} \) – Line-of-sight acceleration (m/s²)
The above equation states the dynamic stress error is inversely proportional to the loop bandwidth. Simply it can be said that, higher the bandwidths, lower the dynamic errors and vice-versa. The 3-sigma error includes both the thermal and dynamic stress errors component which is expressed as:

\[ 3\sigma_{PLL} = 3\sigma_{\text{thermal}} + \sigma_{\text{dynamic}} < 45^\circ \quad \ldots (2.10) \]

Above equation (2.10) is simply the sum of (2.8) and (2.9) where it must not exceed a phase of 45°, which is the threshold for lock detection. As well, on the closer inspection on (2.8) and (2.9) it can be seen that it is the paradoxical situation which needs a trade-off for either dynamics or thermal performance.

Table 2.1: PLL tracking errors for various signal strengths and dynamics

<table>
<thead>
<tr>
<th>Band</th>
<th>0.1g @ 30dB-Hz</th>
<th>0.5g @ 30dB-Hz</th>
<th>1g @ 30dB-Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thermal noise</td>
<td>Dynamic Error</td>
<td>Total Error</td>
</tr>
<tr>
<td>3</td>
<td>3.84</td>
<td>57.86</td>
<td>61.7</td>
</tr>
<tr>
<td>6</td>
<td>5.43</td>
<td>14.46</td>
<td>19.89</td>
</tr>
<tr>
<td>9</td>
<td>6.65</td>
<td>6.42</td>
<td>13.07</td>
</tr>
<tr>
<td>12</td>
<td>7.68</td>
<td>3.61</td>
<td>11.29</td>
</tr>
<tr>
<td>15</td>
<td>8.59</td>
<td>2.31</td>
<td>10.9</td>
</tr>
<tr>
<td>18</td>
<td>9.41</td>
<td>1.60</td>
<td>11.1</td>
</tr>
</tbody>
</table>

It is evident from the Table 2.1 that as the dynamics increases the loop bandwidth also increases to limit the errors. For example, if the acceleration is 0.1 g then the carrier tracking loop bandwidth is maintained at 6 Hz while at an acceleration of 1 g the bandwidth must be set to be greater than 15 Hz. The bold letters in the table represents the situations whenever the errors exceed threshold of 45°.
degrees, and the tracking loop cannot maintain the lock. Also it can be seen from the table that the dynamics have more effect on bandwidth rather than on thermal noise.

![Graph](image)

**Fig. 2.7: Thermal and dynamic stress errors on carrier phase**

The Fig.2.7 represents the relationship among the phase error and thermal and dynamic errors by assuming 0.5 g dynamics and signal strength of 27.5 dB-Hz. In ultra-tight integration, both thermal and dynamic stress parameters are optimised concurrently through integration of INS-derived Doppler into tracking loops. The simulation experiments shows the performance of ultra-tight carrier loop compared with the conventional stand-alone loop.
Fig.2.8: Simulated trajectory for evaluating code and carrier tracking loops

A reference trajectory (represented in Fig. 2.8) is produced through Matlab simulation package GPS\textsuperscript{TM} [47]. This trajectory symbolize a flight path and includes the following segments such as pitch up, acceleration, 90 deg turns, straight, level and roll. Measurements of inertial sensor are resulted from this trajectory and errors added are 10.0 mg acceleration bias and 1.0 deg/sec gyro bias. Additionally, the GPS signals are simulated by just merging the satellite ephemeris data.

**Performance**: Fig.2.9 represents the performance of carrier tracking loops in both the ultra-tight and conventional modes. A Matlab software GPS receiver from DataFusion\textsuperscript{TM} [95] is utilized for tracking the GPS signals and the information is gathered for
5 seconds. In first pass, receiver was configured in conventional mode that is the carrier tracking bandwidth is set to 13 Hz and no external aiding is provided.

From Fig. 2.9, it can be observed that Doppler values range from 1105 to 1210 Hz, and the difference is 105 Hz. Because of high rate of Doppler change, the carrier phase loop lost lock and tracking was switched to Frequency Locked Loop (FLL) mode. While in second pass, receiver was configured in ultra-tight mode that is carrier tracking bandwidth is set to 3 Hz and INS-derived Doppler is integrated with carrier NCO. Fig. 2.9 represents that Doppler values vary from 1150 to 1145 Hz that is only a difference of 5 Hz. Hence, the integration of external Doppler data diminishes the Doppler Effect in GPS signals which enters the tracking loop.
Residual error which must be tracked is mainly because of oscillator error and any residual errors in INS-derived Doppler estimate. From the figure 2.9, it can be observed that the carrier NCO stays almost constant thereby facilitating a significant bandwidth reduction.

2.4.2 Code Tracking Loop

Modulation of C/A code on navigation data results in a wide spectrum through the signal transmitter. Whenever the signal is received at antenna then it is mixed with noise. Thus, the process of dispreading should take place at the correlator for extracting the navigation information from C/A code and noise. Through Delay Locked Loop (DLL), tracking of C/A code is performed where it includes code generator, Discriminator algorithms and code NCO and discriminator algorithms as shown in the Fig.2.10.

Fig.2.10: Tracking loop in Ultra-tight code
The digitized IF signal multiplied with the receiver produced replica of code. The code generator produces the replica signals in three various phases which are Early (E), Prompt (P) and Late (L) that are typically separated by about $\frac{1}{2}$ chip spacing. Here, in this case the early phase is $\frac{1}{2}$ chip advanced with respect to prompt phase while the late phase is $\frac{1}{2}$ chip retarded with respect to prompt phase. All these three signals are multiplied with the each component of I and Q signals and then low pass filtered or integrated. In the code discriminator algorithms (A-1), the filtered I and Q outputs are combined where the correction signal is produced to align the code generator with incoming code phase. Despreading process eliminates the C/A code from signal and extracts the 50 Hz navigation message data.

![Fig.2.11: Code Autocorrelation for Early, Late and Prompt](image-url)
Whenever the prompt phase is precisely aligned for incoming code phase in ideal scenario, the prompt arm correlation output results in triangle with sharp peak to zero offset and linearly ramp down to zero at 1 chip offset in Fig. 2.11.

2.4.2.1 Bandwidth and Dynamic Analysis

The performance of code loop gets affected with both the dynamic stress and thermal noise but not to the same extent they affect carrier tracking loop. This is mainly due to the lower code loop bandwidths. The code chipping frequency is 1.023 MHz in contrast with the carrier frequency of 1575.42 MHz as a result the Doppler effect on code frequency is 1540 (1575.42 MHz / 1.023 MHz) times lesser than that of carrier. In order to track the loop, the 3-sigma errors of merged dynamic and thermal errors must not be greater than the correlator spacing (d) in chips which are given by Kaplan (1996) [7]:

\[ 3\sigma_{\text{DLL}} = 3\sigma_{\text{IDL}} + R_e \leq d \] (chips) \hfill \ldots(2.11)

The thermal error DLL is given as:

\[
\sigma_{\text{IDL}} = \left[ \frac{B_L d}{2C} \left( 1 + \frac{2}{T(2-d)c} \right) \right]^{\frac{1}{2}} \hfill \ldots(2.12)
\]

where,

- \( B_L \) – loop bandwidth
d – chip spacing

\( T \) – pre-detection integration period

\( \frac{C}{n_o} \) – (SNR) BL (Ratio-Hz)

\[
\frac{C}{n_o} = 10 \log_{10} \frac{C}{n_o} \text{ (dB-Hz)}
\] ... (2.13)

As per Kaplan (1996) [7], dynamic stress error is given as:

\[
R_e = \frac{dR^n}{dt^n} n_o^n \text{ } \frac{dR^n}{\omega_o^n} \] ... (2.14)

where, \( \frac{dR^n}{dt^n} \) is the line-of-sight dynamics which is expressed in chips/sec [7].
The Fig.2.12 represents both the thermal error and dynamic stress error for receiver with dynamics of 1g (from Eqns. 2.12 and 2.14). Here, the total error (sum of both thermal and dynamic errors) is plotted. It must be noted that the dynamics effect on code loop is very less when compare with the effect of thermal errors. For the threshold of (1/6)\textsuperscript{th} of chip, it can be noticed that code loop loses locks at about 23dB-Hz.

For testing the performance of code tracking loop, simulation experiments have been performed through test trajectory which is shown in Fig. 2.8. Similar to the carrier tracking loop performance explained in section 2.4.1.1, then experiments were carried out for both the ultra-tight and conventional modes through software receiver. In conventional mode, the code loop tracking bandwidth is set for 2 Hz having the code spacing of 0.5 chips and carrier tracking bandwidth of 13 Hz.

![Fig.2.13: Code tracking loop performance (conventional mode)](image)

*code bandwidth=2Hz, code spacing=0.5 chips*
From the Fig.2.13 it can be easily stated that the code power reduces below the threshold and results in loss of lock.

![Graph showing code power over time](image)

**Fig.2.14: Code tracking loop performance (ultra-tight mode) code bandwidth=1Hz, code spacing=0.25 chips.**

In the same way, the experiment is carried out for tracking loops that are configured in ultra-tight mode that is the carrier is amplified with the INS derived Doppler and aids the code tracking loops. Here, code bandwidth is set to 1hz having the code spacing of 0.25 chips and carrier bandwidth at 3 Hz. The code loops performance represented in Fig.2.14 shows that these code loops do not lose lock even at the presence of dynamics. This can be recognized as an effective reduction of Doppler Effect in GPS signal in ultra-tight tracking mode. Thus, from this context it can be understood that the existence of lock in ultra-tight mode when compare to conventional mode. In ultra-tight mode, code loops doesn’t lose lock even in the presence of dynamics but where as in the case of conventional mode the code power reduce and results in loss of lock.
2.5 GPS/INS INTEGRATION KALMAN FILTER ANALYSIS

According to Kalman, 1960 [48], the Kalman filter is the optimal algorithm that approximates the states of system with uncertain dynamics and noisy measurements. For navigation applications based on INS and GPS the states normally refers to the velocity, position, attitude, acceleration however it may also comprises correlated error sources like drifts and biases from both INS and GPS sensors. The uncertain dynamics corresponds to unpredictable platform motion and any unpredictable noises associated with sensor measurements. The noisy measurements generally refer to sensors measurements that are mixed with noise.

One of the main valuable aspects of Kalman filter is its ability to integrate the noisy measurements from multiple sensors and even offers an optimal output. Complementary filter approach makes use of two filters with dissimilar transfer function that is, the first filter is designed with the high pass transfer function \( G_1(s) \), while the other filter is designed with low pass transfer function \( G_2(s) \). The properties of two filters are represented [25, 49, 50]:

\[
\lim_{s \to 0} G_1(s) = 0 \quad \text{... (2.15)}
\]

\[
\lim_{s \to \infty} G_2(s) = 0 \quad \text{... (2.16)}
\]

\[
G_1(s) + G_2(s) = 1 \quad \text{... (2.17)}
\]

where,

\[ s \quad \text{– the Laplace frequency domain parameter} \]
G_1(s) – high pass transfer function
G_2(s) – low pass transfer function

In integrated GPS/INS system, INS measurements are fed to filter with high pass characteristics since the error spectrum of INS naturally consists of systematic low frequency error (n_1). While GPS measurements are fed to low pass filter (because of its high frequency error (n_2) spectrum).

\[ \hat{y} = y(s) + G_1(s)n_1(s) + G_2(s)n_2(s) \]  ... (2.18)

**Fig.2.15: Complementary filter for estimating the signal**

The complementary filter architecture that is used in integration of GPS and INS are shown in Fig.2.15. From INS and GPS, the redundant measurements ‘y’ contaminated with errors n_1 and n_2 are sent to the two components of complementary filter and further filter computes the \( \hat{y} \). INS measurements \( Z_{INS} \) are processed through high pass filter \( G_1(s) \) and merged with the GPS measurements \( Z_{GPS} \) that are again processed through low pass filter \( G_2(s) \) in order to calculate \( \hat{y} \). The mathematical expression for overall transfer function of filter is:
The equivalent of this setup which is utilized in GPS/INS integrated system is the difference and feed-forward complementary filter as represented in Fig.2.16. The filter processes difference among the INS and GPS measurements, $n_1 - n_2$ and approximate the INS errors $\hat{n}_1$. The GPS estimates are then deducted from INS measurements for acquiring the estimated $\hat{y}$.

**2.5.1 Process and Measurement Models**

For estimating the state parameter in Kalman filters, mathematical (or functional) and stochastic models are essential. The functional models in discrete form comprise the system dynamics model:

$$x_{k+1} = \phi_k x_k + G_k \omega_k \quad k = 0, 1, 2 \ldots \quad \ldots (2.19)$$

and the observation model as:

$$y_k = H_k x_k + v_k \quad k = 1, 2, 3, \ldots \ldots$$

where,
\( x_k \) – n-dimensional state vector at epoch \( k \)

\( \phi_k \) – n-by-n state transition matrix at epoch \( k \)

\( G_k \) – n-by-p system matrix at epoch \( k \)

\( \omega_k \) – p-dimensional process noise at epoch \( k \)

\( y_k \) – m-dimensional measurement vector at epoch \( k \)

\( H_k \) – m-by-n observation vector at epoch \( k \)

\( \nu_k \) – m-dimensional measurement noise at epoch \( k \)

Moreover, the random noise parameter \( \omega_k \) and \( \nu_k \) required to satisfy the following conditions as per Brown & Hwang (1997) [25]:

\[
E[\omega_k \omega_l^T] = \begin{cases} Q_k & k = 1 \\ 0 & \text{otherwise} \end{cases} \quad \ldots (2.21)
\]

\[
E[\omega_k \omega_l^T] = \begin{cases} R_k & k = 1 \\ 0 & \text{otherwise} \end{cases} \quad \ldots (2.22)
\]

\[
E[\omega_k \omega_l^T] = 0 \quad \text{for all } k, l \quad \ldots (2.23)
\]

If the system that needs to estimated fits the form that is represented by Eqns. (2.18) and (2.19) and assure the stochastic properties in Eqns. (2.21) to (2.23) then Kalman filter can be easily initialized with the system state vector and covariance matrix as given in the Eqns. (2.24) and (2.25). It is followed by the two step recursive computation, which are prediction update and measurement update as given in the Eqns. (2.26) to (2.30).
**Initialization:**

Initialisation of State

\[ \hat{x}_0 = E[x_0] \quad \ldots \text{(2.24)} \]

Initialisation of Covariance

\[ P_0 = E[ (x_0 - \hat{x}_0) (x_0 - x)_1^T ] \quad \ldots \text{(2.25)} \]

**Prediction Update:**

State Update

\[ \hat{x}_o (-) = \phi_{k,k-1} \hat{x}_{k-1} (+) \quad \ldots \text{(2.26)} \]

Covariance Update

\[ P_k (-1) = \phi_{k,k-1} P_{k-1}(+) \phi_{k,k-1}^T + Q_{k-1} \quad \ldots \text{(2.27)} \]

**Measurement Update:**

Kalman Gain

\[ K_k = P_k(-) H_k^T [H_k P_k(-) H_k^T + R_k]^{-1} \quad \ldots \text{(2.28)} \]

State Update

\[ \hat{x}_k (+) = \hat{x}_k (-) + k_k (y_k - H_k \hat{x}_k (-1)) \quad \ldots \text{(2.29)} \]

Covariance Update

\[ P_k (+) = P_k(-) - k_k H_k P_k (-) \quad \ldots \text{(2.30)} \]

As per Brown & Hwang (1997) [25], system plant noise covariance matrix \((Q_{k-1})\) is defined as:

\[ Q_{k-1} = E[\omega(k-1)\omega^T(k-1)] \]

\[ = E \left\{ \int_{(k-1),T}^{kT} \phi_{k,k-1}(\xi) G(\xi) \omega(\xi) d\xi \int_{(k-1),T}^{kT} \phi_{k,k-1}(\eta) G(\eta) \omega(\eta) d\eta \right\} \]

\[ = \int_{(k-1),T}^{kT} \int_{(k-1),T}^{kT} \phi_{k,k-1}(\xi) G(\xi) E[\omega(\xi)\omega^T(\eta)] G(\eta) \phi_{k,k-1}(\eta)^T d\xi d\eta \quad \ldots \text{(2.31)} \]
Here in this study, however $Q_{k-1}$ is approximated through first order approximation of transition matrix:

$$Q_{k-1} \approx \phi_{k,k-1} G Q G^T \phi_{k,k-1}^T \Delta t$$

For further study on Kalman filter algorithm, one can make use of these references [25, 51, 52, 53, 54]. Thus, from this context one can easily understand the Kalaman filter analysis through GPS/INS integration, where this filter is mainly used for estimating the system states with the uncertain dynamics and noisy measurements. Further, it even discusses its process and measurement models.

### 2.6 CONCLUSION

This section clearly illustrates the complementary features of both the technologies that are GPS and INS. It even emphasized how these are exploited in integrated system for achieving more robust navigation performance. A brief idea on all the three integration architectures i.e., loosely-coupled, tightly-coupled and ultra-tightly coupled is provided. Further, a theoretical analysis is carried out on tracking loops in ultra-tight configuration is given. Loop bandwidth optimization for both dynamics and thermal performance is being discussed with a range of dynamic and signal strengths.

In addition to the above, Simulation experiments are carried out with the known trajectory for calculating the performance of tracking loops in ultra-tight configuration. Results from receiver are configured
in the conventional manner that is used for their reference. Thus, the
results proves that the ultra-tightly configured receiver can receive the
high dynamics signal with carrier tracking bandwidth of only 3 Hz and
code bandwidth of 1 Hz while the conventional receiver fails to track
such a signal even with the carrier bandwidth of 13 Hz.