CHAPTER 4

Evolution of Ion Acoustic Rarefactive Pulse and its Nonlinear Effects

4.1 Introduction

This chapter describes evolution of a rarefactive ion acoustic wave and its nonlinear effects on the plasma particles. Ion acoustic waves are dispersive in nature. Weakly nonlinear ($\delta n/n < < 1$) and dispersive ($k\lambda_D < < 1$) ion-acoustic waves are described by the KdV equation. A compressive perturbation is known to evolve into a soliton due to the balance between the nonlinear steepening and dispersion effects. The KdV equation admits soliton type of solutions. These solitons have been studied experimentally by many workers under different experimental conditions. Ikezi and Taylor, (1970), Hershkowitz et al., (1972), Watanabe, (1975), reported the studies of ion acoustic solitons in uniform quiescent plasmas. John and Saxena, (1976), Dahiya et al. (1978) studied these solitons in a plasma with density gradients and John et al. (1977) studied the formation of ion acoustic solitons in plasma with rf fields.

The balance between nonlinearity and dispersion cannot be achieved for rarefactive perturbations and they do not evolve into solitons. The KdV equation predicts, however, that an initial depletion in density will evolve into a wave train. This analytical deduction has been confirmed by the numerical and experimental results of Okutsu and Nakamura (1979). They launched compressive and rarefactive pulses and their combinations in the form of sinusoidal waves and observed the wave train associated with the rarefactive perturbation, no solitons were observed. Since the rarefactive perturbations do not evolve into solitons very few work on the rarefactive perturbations have been reported (John et al., 1976, Appert and Vaclavik, 1977, Saxena et al., 1981).

In the experiment by Saxena et al.(1981), a rarefactive pulse of large (10%) amplitude evolved into the predicted wave train. Apart from this, the leading pulse was observed to steepen and fission as it propagated away from the launcher. This behavior could not be explained on the basis of the KdV equation. The rarefactive wave is associated with a negative potential and large amplitude rarefactive wave may significantly alter particle distri-
butions by reflecting electrons of certain energy range and trapping ions. In order to understand the propagation characteristics of highly nonlinear rarefactive ion-acoustic waves, it therefore seems important to invoke the Vlasov picture. The modification of particle distributions may give rise to BGK (Bernstein et al., 1957) type structures.

Some of the known BGK structures are the electron and ion phase space holes. Numerical as well as theoretical investigations of the two stream instabilities (Roberts et al. 1967, Kako et al. 1971) predicted that these instabilities evolve into a train of vortex like structures in electron phase space. Experiments in a highly magnetized plasma (Saeki et al. 1970) have confirmed these predictions. The electron phase space vortices appear as stable a BGK equilibria (Bernstein et al. 1957).

In these theoretical investigations the ions were treated as an immobile charge neutralizing background. The development of corresponding configurations in ion phase space was numerically studied by Sakanaka (1972). Numerical integration of Ion-Vlasov equation was carried out assuming electrons to be Boltzmann distributed, as exemplified by the ion two stream region behind electrostatic ion shocks.

A unified theoretical treatment of both the ion and electron phase space vortices was presented by Bujarbarua and Schamel (1981). Hans Pécsei et al. (1984) have reported the first experimental observation of a stable phase space vortex in the ion two stream region behind electrostatic ion-acoustic shocks. These observations have been supported by numerical simulations based on the stationary solution of ion Vlasov equation using a water-bag distribution model (Pécsei et al. 1984). The observation of ion hole was also reported by Tsikis et al. (1985) in a double plasma machine.

The present chapter discusses the launching and evolution of large amplitude rarefactive ion acoustic wave in a linear unmagnetised device. These perturbations are associated with large negative potential structures and hence the ions approaching this potential well will just slip into the well. This would create a depletion in the ion density or a hole in the ion phase space distribution. Observations on the ion velocity distribution associated with these large amplitude waves are also presented. Particle trapping and modification of distribution function in the presence of rarefactive pulse is observed. There are also indications of phase space vortex formation both in the main pulse and the wave train like structures of the trailing end of the wave.
4.2 Experimental Setup

The experiments were carried out in a linear, unmagnetised, multifilament plasma device, described in chapter 2. Argon plasma with average axial density \(5 \times 10^{9}/\text{m}^3\), electron temperature in the range of 2.0–3.0 eV, and ion temperature \(\sim 0.2\) eV.

The waves were launched by applying a square pulse of repetition rate 1 kHz to the launcher as described in chapter 2. The rise time of the applied pulse was \((\sim 1.0 \mu\text{s})\) which was faster than inverse of ion plasma frequency but slower than electron plasma frequency. The launcher bias being superimposed on the floating potential of the launcher to ensure minimal plasma disturbance by the launcher between the pulses. The repetition rate of 1 kHz was chosen so that the effect of successive pulses do not interfere with each other.

4.3 Results and Discussions

4.3.1 Unperturbed Ion Velocity Distribution Measurement

The unperturbed velocity distribution of plasma ions was made using RPA described in chapter 2. The first grid of the analyzer was left floating so that it acquired the floating potential to reflect part of the electron population incident on it. The second grid was biased to a negative potential to reflect the high energy electrons from plasma. The third grid was used as the energy selector for the ions and hence a variable bias was applied to it.

Fig. (10) in chapter 2, shows the block diagram of the electronic circuits involved. The collector current was fed to a resistor and the signal was amplified and fed to a box car integrator to improve on signal to noise ratio. The time varying bias for selector grid was derived from a function generator. The I–V signal was digitized and stored on a micro computer for further analysis.

Fig. (25) shows a typical I–V curve and its first derivative with respect to the voltage on the selector grid. In order to get these curves a five point running averaging had been performed on the dI/dV to get rid of the noise due to differentiation. This smoothing broadens the velocity distribution estimates. Hence the ion temperatures were calculated from a logarithmic fit to the analyzer data. This was done by plotting \(\log(I_C)\) against the selector potential and then fitting a straight line to the resulting curve, shown in fig.(26). The inverse of slope of this curve was taken as a measure of the ion temperature. Table 1 lists the measured ion temperatures.
Figure 25: The I-V characteristics of RPA and the ion energy distribution. The curve shown has been taken at an axial position of 42.0 cm and radial position of 0.0 cm.

Figure 26: Straight line fit to the Log(I)-V curve shown in the figure (25).
at different axial and radial locations. The axial location is measured from one end of the device. The radial locations had been measured with respect to the center of the cylindrical chamber.

Table 1.

Ion temperature measured at various radial and axial locations with both the plasma sources on

<table>
<thead>
<tr>
<th>radial location</th>
<th>Axial location</th>
<th>Ti from ln(I) (eV)</th>
<th>error %T&lt;sub&gt;i&lt;/sub&gt;</th>
<th>Ti from di/dv (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.0 cm</td>
<td>42 cm</td>
<td>.4</td>
<td>10</td>
<td>0.8</td>
</tr>
<tr>
<td>9.0 cm</td>
<td>42 cm</td>
<td>.25</td>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td>0.0 cm</td>
<td>42 cm</td>
<td>.2</td>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td>-9.0 cm</td>
<td>42 cm</td>
<td>.2</td>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td>-18.0 cm</td>
<td>42 cm</td>
<td>.3</td>
<td>10 to 14</td>
<td>0.7</td>
</tr>
<tr>
<td>-22.0 cm</td>
<td>42 cm</td>
<td>.4</td>
<td>10 to 14</td>
<td>1.0</td>
</tr>
<tr>
<td>18.0 cm</td>
<td>52 cm</td>
<td>.36</td>
<td>10</td>
<td>0.8</td>
</tr>
<tr>
<td>8.0 cm</td>
<td>52 cm</td>
<td>.31</td>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td>12.0 cm</td>
<td>52 cm</td>
<td>.28</td>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td>0.0 cm</td>
<td>52 cm</td>
<td>.26</td>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td>-9.0 cm</td>
<td>52 cm</td>
<td>.23</td>
<td>10 to 14</td>
<td>0.6</td>
</tr>
<tr>
<td>-22.0 cm</td>
<td>52 cm</td>
<td>.35</td>
<td>10 to 14</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 2.

Ion temperature measured at various radial and axial locations with only second plasma source off

<table>
<thead>
<tr>
<th>radial location</th>
<th>Axial location</th>
<th>Ti from ln(I) (eV)</th>
<th>error %T&lt;sub&gt;i&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.0 cm</td>
<td>42 cm</td>
<td>0.38</td>
<td>20</td>
</tr>
<tr>
<td>0.0 cm</td>
<td>42 cm</td>
<td>0.48</td>
<td>61</td>
</tr>
<tr>
<td>-9.0 cm</td>
<td>42 cm</td>
<td>0.50</td>
<td>76</td>
</tr>
</tbody>
</table>

It can be seen from the above tables that the distributions near the wall of the vessel are wider. In another set of measurements the one of the two
plasma source was switched off. The measurements were carried out far away from the source. It was found that again the distributions were wider. These results are shown in table 2. The fact is that near the wall and far away from the source the plasma density is small and the signals have poor signal to noise ratio. Hence the uncertainties involved in the calculations are more. The ion temperature has been calculated from both a logarithmic fit to the data and from the full width at half maxima of the distribution function. It can be concluded from these results that, the determination of temperature from distribution function gives an overestimate in lower density range, because of the broadening of the distributions due to the noise.

4.3.2 Rarefactive Ion-Acoustic Wave

The fig.(27) shows the density perturbation of amplitude 1.2% as a function of time at different axial distance from the launcher. As can be seen from this figure this perturbation started out as a symmetric pulse and broadened as it moved away from the launcher. The velocity of this pulse was found to be $3.0 \times 10^5$ cm/sec. This pulse suffered damping and dispersion on further propagation in the system. In the fig. (27), presence of an oscillatory tail in the trailing edge was observed. These oscillations can be seen clearly in traces 8 to 9, as the total time duration is more in these records. These oscillations have a frequency ~ 30 kHz and were found to be present in all higher amplitude perturbations as well (see fig. (28)). These oscillations appeared as a wave train consisted of alternate peaks of density compression and depletion with a prominent compressive peak as the first peak. This wave train type of response will be described in a separate section. These oscillations will be described in detail in a separate section.

The pulses of initial amplitudes ~ 6.0% are shown in fig.(28). This also started out as symmetric pulses and broadened as it moved away from the launcher. This pulse also damped on further propagation. It was observed that the leading edge of this pulse was sharper than the trailing edge.

Perturbations of amplitude 7.0% also started out as symmetric pulses, showing all the essential features as described in the above paragraph. In addition this and all higher amplitude pulses showed some additional features. It was found that the pulse started becoming asymmetric at a distance of 25-30 $\lambda_D$ from the launcher. Beyond this the initial pulse splitted into two pulses separated by a local maxima (fig.29). The splitted pulses moved at different speeds, one moving at a speed little lower than the acoustic speed.
Figure 27: Example of rarefactive pulse propagation in the plasma, for initial perturbations $\delta_n \sim 1.2\%$. The successive traces are time records taken at different axial positions separated from each other by $14.0 \lambda_D$. The vertical scaling for traces 1 to 6 is $1.2\%$/div. marked. The trace 7 has a vertical scale of $0.52\%$/div., trace 8 is at $0.26\%$/div. and traces 9 to 15 are at $0.13\%$/div.
Figure 28: Example of rarefactive pulse propagation in the plasma, for initial perturbations $\delta n/n \sim 6.0\%$. The successive traces are time records taken at different axial positions separated by $14.9\lambda_D$. Note that the quantity plotted in these figures is electron current and the signal in this particular set is inverted. So a density depletion in this curve is actually an increase in the density. Vertical scaling for traces 1 to 3: 1.3%/div., traces 4 to 8: 0.66%/div., traces 9 & 10: .33%/div., 11 to 13: .35%/div. and traces 14 & 15: 0.18%/div.
Similar results had been reported earlier by Saxena et al. (1981). However the differences between the two case will be discussed later in the following sections. It was also seen that the trailing edge becomes sharper than the leading edge. As the pulse moves away from the launcher both the minimas show a broadening and damping. Further propagation lead to breaking into more pulses. Similar behavior was also shown by pulses of higher amplitudes. Fig.(30) shows initial pulse of amplitude 9% and fig.(31) shows the pulses of the initial amplitude \( \sim 10.5\% \).

The fissioning in the experiment reported by Saxena et al. (1981) started appearing only after the wave had traveled about 65-70 \( \lambda_D \) in the system. Whereas in the present experiment, the first split occurred very close to the launcher around 25-30 \( \lambda_D \) from it. A very remarkable difference was found in the relative widths of these two pulses after the fissioning. The first peak which was moving at a speed faster than the ion acoustic speed suffered a higher dispersion in the experiment of Saxena et al. (1981). The second peak remained narrower throughout its propagation. It appeared as if the first peak was the one getting splitted into more pulses as it generated more fissioning for higher amplitudes. In the present experiment it was found that the second peak was broader and the first one narrower. It started as smaller amplitude but suffered less damping compared to the second one. In the subsequent fissioning the second peak splitted into two and more. Similar behavior was shown by amplitudes of the order of 12%.

The fig.(32) show the position versus time of arrival of different peaks. These figures show that the different peaks travel at different speeds. It was found that one of the peaks acquire a speed little higher than the other one. In the case of initial amplitude \( \theta_n \sim 9.0\% \), it is found that one trough travels with a speed \( \sim 0.9C_s \) and the other with \( \sim 1.2C_s \). In all other higher initial amplitudes the troughs move with different speeds.

The fig.(33) shows the plot of amplitudes of perturbation received at different axial positions away from the launcher, for the main pulse. The rate of damping found is much faster compared to the Landau damping. The Landau damping length \( X_\alpha \) of ion acoustic waves is given by

\[
X_\alpha^{-1} = \frac{e}{\sqrt{2}} \left( \frac{T_e}{T_i} \right)^{3/2} \exp \left( \frac{-T_e}{2T_i} \right)
\]

The \( X_\alpha \) for our system comes out to be \( \sim 175.0 \) cms, whereas the damping length observed is much shorter typically within 8.0-10.0 cms.
Figure 29: Example of rarefactive pulse propagation in the plasma, for initial perturbations $\Delta n \sim 7.0\%$. The successive traces are time records taken at different axial positions separated by $14.0\lambda_D$. Vertical scale for traces 1 & 2: 1.5\% /div., traces 3 & 4: 1.62\% /div., 5 to 7: 1.7\% /div., 8 & 9: .84\% /div., trace 10: .34\% /div. and traces 11 to 13: .17\% /div.
contd. Figure 29.
Figure 30: Example of rarefactive pulse propagation in the plasma, for initial perturbations $\frac{\delta u}{u} \approx 9.0\%$. The successive traces are time records taken at different axial positions separated by $14.0\lambda_p$. The vertical scale for traces 1 to 6: $1.6\%$/div., 7 & 8: $0.64\%$/div. and traces 9 to 11: $0.34\%$/div.
contd. Figure 30.
Figure 31: Example of rarefactive pulse propagation in the plasma, for initial perturbations $\delta n \sim 10.5\%$. The successive traces are time records taken at different axial positions separated by $14.0\lambda_D$. The separation between the traces 3 and 4 was $7.0\lambda_D$. Vertical scale for traces 1 to 3: $3.26\%$/div., 4 to 6: $1.6\%$/div., 7 to 9: $1.7\%$/div., and trace 10: $.44\%$/div.
contd. Figure 31.
4.3.3 Numerical Solution of KdV Equation

Since the weakly dispersive ion acoustic waves are governed by KdV equation, a numerical investigation of KdV equation, was carried out, to describe the above mentioned features observed in the experiment. The KdV equation in a frame moving with the ion acoustic speed is given as

\[ \frac{\partial \phi}{\partial \tau} + \phi \frac{\partial \phi}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \phi}{\partial \xi^3} = 0 \]  

(1)

where \( \phi \) is the normalized potential or density perturbation and \( \xi = (x - ut), \quad \tau = t \)

are the stretched coordinates.

In the numerical technique, used to solve this equation for a given initial condition (Saxena & Sen 1981), the time integration by using partially corrected Adams-Bashforth scheme (Gazdag 1976). The space operator is split into linear and nonlinear part (Tappert 1974). The space integration is performed in the Fourier space, but the nonlinear terms are computed in the real space representation. The constants of motions are used to check the accuracy of the integration.

Writing equation (1) as

\[ \frac{\partial \phi}{\partial \tau} + \beta \phi \frac{\partial \phi}{\partial \xi} + \delta \phi \frac{\partial^3 \phi}{\partial \xi^3} = 0 \]  

(2)

\( \beta \) and \( \delta \) are constant coefficients and in our case for ion acoustic waves in plasmas \( \beta = 0 \) and \( \delta^2 = 1/2 \). The equation (2) is invariant under the transformation

\[ \xi \rightarrow \lambda \xi; \quad \tau \rightarrow \lambda^3 \xi \quad \text{and} \quad \phi \rightarrow \lambda^{-2} \phi \]

This invariance can be used to normalize the scale length \( \lambda \) to unity in the direction of \( \xi \). The initial value problem is essentially looking for the time evolution of a given initial condition of the form

\[ \phi(\xi, 0) = \phi_0 f(\xi/L) \]  

(3)

where \( \phi_0 \) is the initial amplitude, \( L \) is the width and \( f(\xi/L) \) is the functional form of the initial pulse. It can be a square pulse, half sine, secant hyperbolic or exponential to suit the experimental condition. A new set of variables can be defined (Berezin and Karpman 1966):
Figure 32: Time of flight measurements for different peaks in the main pulse. (1) is the case of initial $\delta n/n \sim 1.2\%$ (2) is for 5 % (3) 7 % (4) 9 % (5) 10.5 %. The fissioning was observed for amplitudes 7.0% and higher. Different curves in each set correspond to the different peaks after fissioning, moving at different speeds.
Figure 33: The relative amplitude of signal received at the probes at different axial positions away from the launcher. This figure shows the data for the main pulse with different starting amplitudes. The first point on each curve represents the initial amplitude in terms of $\% \delta n/n$.  The x-axis is distance, measured from first probe position, which is 2.0 cms away from the launcher.
\[ \dot{\xi} = \xi/L, \quad \dot{\tau} = \phi_0 \tau/L \text{ and } \dot{\phi} = \phi/\phi_0 \]

Under this new set of variables the KdV equation can be written as

\[ \frac{\partial \phi}{\partial \tau} + \beta \frac{\partial \phi}{\partial \xi} + \frac{\delta^2}{\phi_0 L^2} \frac{\partial^3 \phi}{\partial \xi^3} = 0 \]  

(4)

Under these transformations, all initial conditions are transformed into the same initial conditions i.e. unit amplitude and unit width initial condition. All initial conditions with same value of \( \phi_0 L^2 \) evolve identically in the scaled coordinate system but differently in the laboratory coordinate system. This scaling applies for all times and not just in the asymptotic region. Initial conditions can be described by functional forms of unit width and the behavior in laboratory coordinate system can be recovered by multiplying the space, time and amplitude in the scaled coordinates by the scaling parameters given below:

Scaling parameter for amplitude = \( \phi_0 \)
Scaling parameter for time = \( L/\phi_0 \)
Scaling parameter for space = \( L \)

The Fourier transform of equation (2) gives

\[ \frac{\partial \phi}{\partial \tau} - ik^3 \delta^3 \phi_k + ik < \phi^2 \beta/2 >_k = 0 \]  

(5)

where \( \phi_k \) is the \( k^{th} \) Fourier component of \( \phi(\xi, \tau) \) and \( < \phi^2 \beta/2 >_k \) is the \( k^{th} \) component of the nonlinear term. These terms can be expressed as

\[ \phi_k = \int_{-\infty}^{\infty} \phi(\xi, \tau) \exp(-ik\xi) d\xi \]  

(6)

\[ < \phi^2 \beta/2 >_k = \frac{\beta}{2} \int_{-\infty}^{\infty} \phi^2(\xi, \tau) \exp(-ik\xi) d\xi \]  

(7)

Eqn. (5) consists of two parts
a linear part
\[ \frac{\partial \phi}{\partial \tau} - ik^3 \delta^2 \phi_k \]
and a nonlinear part
\[ ik < \phi^2 \beta/2 >_k \]
The non-linear part can be treated as a source term. Thus the linear part can be written as

\[ \phi_k(\tau) = \phi_k(\tau) \exp(i\Omega \tau) \]  

where \( \Omega = k^3 \beta^2 \). Taylor series expansion gives

\[ \phi_k(\tau + \Delta \tau) = \left[ \phi_k(\tau) + \frac{\partial \tilde{\phi}_k}{\partial \tau} |_{\tau} \Delta \tau + \frac{1}{2} \frac{\partial^2 \tilde{\phi}_k}{\partial \tau^2} |_{\tau} \Delta \tau^2 + \ldots \right] \exp(i\Omega (\tau + \Delta \tau)) \]  

(9)

The derivatives can be evaluated from the equation for \( \tilde{\phi}_k \), which can be obtained by substituting eqn. (8) in eqn. (5)

\[ \frac{\partial \tilde{\phi}_k}{\partial \tau} = -ikS_k(\tau) \exp(-i\Omega \tau) \]  

(10)

where \( S_k(\tau) = \langle \phi^2 \beta / 2 \rangle_k \). If \( \Delta \tau \) is chosen in such a way that \( S_k(\tau) \) does not change significantly over this interval then eqn. (10) can be integrated to yield

\[ \tilde{\phi}_k(\tau + \Delta \tau) = \tilde{\phi}_k(\tau) + \frac{k}{\Omega} S_k(\tau) [\exp(-i\Omega \Delta \tau)-1] \exp(-i\Omega \tau) \]  

(11)

Thus the second term of eqn. (9) can be written as

\[ \frac{\partial \tilde{\phi}_k}{\partial \tau} |_{\tau} \exp(i\Omega \tau) \Delta \tau = [\phi_k(\tau + \Delta \tau) - \tilde{\phi}_k(\tau)] \exp(i\Omega \Delta \tau) \]

\[ = \frac{k}{\Omega} \left[ 1 - \exp(i\Omega \Delta \tau) \right] S_k(\tau) \exp(-i\Omega \Delta \tau) \]  

(12)

In similar fashion the higher derivatives can be calculated. Since \( S_k(\tau) \) is a convolution term and hence constructing the nonlinear term in real space and then taking the Fourier transform is convenient. To prevent aliasing errors only the lowest \( n/2 \) Fourier modes are retained over a grid of \( n \) points in the space coordinate. The solution of eqn. (5) is obtained in terms of the Taylor series of eqn. (9) and truncating the series at appropriate term.

It has been shown (Gazdag 1973) that this method is unstable if the in the process of truncation derivatives higher than second \( (\partial^2 \tilde{\phi}_k / \partial \tau^2) \) are neglected, however it is stable if 3\( rd \) and 4\( th \) derivatives are retained. The computations of these terms can be involved and hence following Gazdag (1976) a two step time differencing scheme has been used, which is stable with terms only up to first time derivatives (Saxena & Sen 1981). The scheme
consists of two steps a predictor step and a corrector step. The equation (12) is written as

$$\frac{\partial \phi_k}{\partial \tau} |_{\tau} \exp(i\Omega \tau) \Delta \tau = G_k(\tau) = \frac{k}{\Omega} \left[ 1 - \exp(i\Omega \Delta \tau) \right] S_k(\tau) \exp(-i\Omega \Delta \tau)$$ \hspace{1cm} (13)

Let us denote the results from predictor step by the superscript p. The predictor step according partially corrected second order Adams–Bashforth Scheme is given by

$$\phi_k^p(\tau + \tau \Delta \tau) = \phi_k(\tau) \exp(i\Omega \Delta \tau) + \frac{3G_k^p(\tau) - G_k^p(\tau - \Delta \tau)}{2}$$ \hspace{1cm} (14)

and the corrector step is given by

$$\phi_k(\tau + \tau \Delta \tau) = \phi_k(\tau) \exp(i\Omega \Delta \tau) + \frac{3G_k^c(\tau) + G_k^c(\tau + \Delta \tau)}{2}$$ \hspace{1cm} (15)

In the first time step the predictor is calculated according to the equation

$$\phi_k^p(\tau + \tau \Delta \tau) = \phi_k(\tau) \exp(i\Omega \Delta \tau) + G_k(\tau)$$ \hspace{1cm} (16)

The inverse Fourier transform of $\phi(\tau + \tau \Delta \tau, \xi)$ gives the solution of KdV equation at the time step ($\tau + \Delta \tau$).

$$\phi(\tau + \tau \Delta \tau, \xi) = \frac{1}{\sqrt{2\pi}} \text{Re} \int_{-\infty}^{\infty} \tilde{\phi}_k(\tau + \Delta \tau) \exp[i(\Omega \tau + k\xi)] dk$$ \hspace{1cm} (17)

The accuracy of the integration is checked by monitoring the conservation of following invariant quantities

$$I_1 = \int_{-\infty}^{\infty} \phi(\xi, \tau) d\xi$$ \hspace{1cm} (18)

$$I_1 = \int_{-\infty}^{\infty} \phi^2(\xi, \tau) d\xi$$ \hspace{1cm} (19)

$$I_1 = \int_{-\infty}^{\infty} \left[ \frac{\partial^2 \phi(\xi, \tau)}{3} - \left( \frac{\partial \phi}{\partial \xi} \right)^2 \right] d\xi$$ \hspace{1cm} (20)

These constants are conserved to better than 1 part in $10^6$ and are sensitive to the size of the time step used. The time step chosen should satisfy

$$\pi^2 \delta^2 k_{\text{max}} \Delta \tau \leq 1.$$

The KdV equation was solved for rarefactive pulses, keeping the amplitude ($\delta n/n$) constant at 10%, and varying the widths. The shape of the
initial pulse was taken as \( \text{sech}^2(\xi/\xi_0) \). The amplitude was negative for rarefactive pulses and can be made positive if one wishes to solve the KdV equation for compressive pulses.

These pulses propagate to left in the ion acoustic frame, implying a speed less than the ion acoustic speed in the laboratory frame. The leading edge gets stretched, and the trailing edge develops into a wavetrain. The number of peaks in this wave train increases as it propagates further. The number of peaks generated also increase with decrease in the width of initial pulse. The numerical solutions showed the development of oscillatory behavior for initial perturbations with \( \leq 15\lambda_D \). These are the Airy function type of oscillations with high frequency. Also the width of these oscillations were found to be changing with distance. The experimental perturbation was close to \( 3\lambda_D \).

The validity of KdV equation demands \( k\lambda_D \ll 1 \). Hence the experimentally observed values may not come under the validity range of KdV equations. However the features observed do not matches with the numerical results.

The oscillations in the trailing edge also behave differently as the variations in the time period of these oscillations was not detected in the numerical calculations. The fig.(34) shows the numerical results for amplitude= 10% and width = 5, 10. The fissioning of the main pulse also could not be reproduced. In order to understand the behavior fully a solution of atleast full fluid equations seems inevitable. However since the potential structures involved in the experiments are negative and large in amplitude the particle trapping effect could be possible. Thus only a kinetic prescription can only be expected to be closer to the reality.

4.3.4 The oscillations in the trailing edge

The first peaks in the figures (29) to (33) are the main rarefactive ion acoustic wave, which started as a symmetric pulse for smaller perturbations and dispersed as the pulse moved in the system. The main pulse maintains its negative potential structure and its trailing edge resembles to a shock like structure. For very low amplitude \( \sim 1.2\% \) it was found that along with the symmetric rarefactive pulse some high frequency oscillations is seen towards the upstream region. These high frequency oscillations damp out very fast and only the main peak survives. A low frequency oscillation with \( f \sim 30\,kHZ \) was seen to be present towards the trailing edge. The frequency of these oscillations were found to be independent of the initial amplitude. Once formed they travelled in the system to almost the end of the device, without any appreciable change in the width. The amplitude decayed as the
Figure 34: The evolution of rarefactive pulse according to the calculations based on KdV equation. The amplitude of the pulse is 10% and widths 5.0λ₀ and 10.0λ₀ (on next page). The wave steepens on the trailing edge and generates ion acoustic wave train. The Airy function type of response appears, with large amplitude and more peaks, very early in time, for pulse of shorter width, whereas for wider pulses the same builds up at much later time.
contd. Figure 34.
pulses moved away from the exciter. This decay was similar in nature to the decay of the main pulse, as shown in the fig.(35).

These oscillations do not agree to the usual Airy function type of response as reported by Ikezi et al. (1973). The Airy function response comes at a frequency $\sim \omega_{pi}$ (300 kHz for the parameters of present experiment). It was demonstrated by Ikezi et al.(1973) that the period of oscillation of such a function should increase with propagation distance as $\propto x^{1.3}$. Whereas in the present experiment it was observed that the time period of these structures remain same over a large distance of propagation through the plasma. These oscillations move with a speed little smaller than the speed of the main pulse. Fig.(36) shows the time of flight of measurements of these oscillations. The speed of the first trough is found to be $2.7 \times 10^6$ cm/sec.

These structures were found to be present, all through the machine, in the plasma even at larger distance from the launcher. These structures have shoulders of equal amplitude. Since in the present experiment the launched frequency is $\sim \omega_{pe}^{-1}$, it should have evolved into a shock, if these oscillations had damped out, leaving the density above zero level. The initial discontinuity in density during the impulse near the launcher leads to the formation of a very sharp peaked electric field. This electric field would accelerate the ions from this region locally and create a region of density depletion. This is similar to the numerical calculations of Sakanaka (1972), for the two stream instability. Here the electric field generated by the initial discontinuity in the density evolved into a shock followed by a hole in the ion phase space. These holes appeared as regions of density depletions in front of the shock. These holes were found to be moving at a constant speed. Stable holes in ion phase space with width $\sim 100 \lambda_D$, was observed.

The formation of these shocks is facilitated by the continuous feeding of the beam in the system. It is well known that if there is a current in the system in the presence of a solitary wave structure with negative potentials, then the electrons are reflected back by this structure and thus the current will be impeded. The reflection of these electrons leaves behind an ion rich region, which creates a positive potential region behind the negative potential structure. This leads to the formation of double layer (Hasegawa and Sato, 1982).

A rarefactive ion acoustic instability was found to evolve in a double layer in the presence of an electron current in the system (Sekar et al., 1984). In this experiment also a similar oscillations were observed at positions close to the separation grid. These oscillations were damped after about 40 $\lambda_D$ and a double layer was formed. The double layers are BGK solutions in the
Figure 35: Amplitude of the oscillations in the trailing edge as a function of space. First point in each curve correspond to the amplitude at position 2.0 cm away from the launcher (marked as 0.0 in the figure). Four traces correspond to the four peaks generated by the launched $\delta u/n \sim 7\%$. 

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Figure 36: Time of flight measurements of the oscillations in the trailing edge. Only two cases for initial amplitudes $\delta n/n \sim 6\%$ (top) and initial amplitudes $\delta n/n \sim 10.5\%$ (bottom).
presence of a current in the plasma.

In the experiment reported in the present thesis there was no current sustained in the system. Initial ions are drawn by the launcher and electrons are reflected. The current drawn initially is thus not sustained for sufficient time. Thus formation of the double layer does not take place in this experiment, because the density compression created initially in the trailing edge, does not have enough ions to maintain its level and the system probably evolves into another stable BGK structure, a hole in the ion phase space. The phase space holes are understood to be the initial stages of a double layer formation (Hasegawa and Sato, 1982) and are BGK modes of such systems. All the features mentioned above indicates that the density depletions observed in the trailing edge are associated with vortex like structures in ion phase space. The features are also in agreement with the measurements of Pécsei et al. (1984).

4.3.5 Perturbed Ion Velocity Distribution Measurements

In order to investigate this phenomenon further the ion energy distribution was measured at various positions in the pulse. This was done using a boxcar integrator and RPA. In the measurements the collector current $I(\phi)$ is sampled and averaged using the boxcar integrator and the bias on the ion energy selector is swept slowly. The ion velocity distribution for the unperturbed plasma was first measured. Then in the presence of the wave the measurements were performed. The boxcar integrator was operated in a fixed delay mode. The delay corresponding to the different positions in the waveform. The boxcar output in this way was the I–V curve of the analyzer, and measured the velocity distribution at the various positions in the wave pulse. The boxcar output was digitized and stored on a microcomputer. The digital signal was differentiated to give the velocity distribution. The fig.(37) shows the differentiated signals. In the above figures the collector current versus the bias potential on the ion selector grid of the analyzer, has been plotted, rather than the energy. This was done because in the measurements the current collected by the collector of of the analyzer of area $A$ is given by

$$I(\phi) = \int_{-\infty}^{\infty} A \text{en} f(x, v, t) dv$$

where $\frac{1}{2}mv^2 = e(\phi - \phi_{pl}(x, t))$, $\phi_{pl}$ is an uncertain quantity varying in
Figure 37: First derivative of the RPA characteristics curve taken at positions marked as 1, 2, 3... in the top curve, with respect to the applied bias on the selector grid.
the pulse $\epsilon(k\phi = T(\nabla n/n)$ hence to avoid any such uncertainty the x-axis of above figures, has been given as $\phi$ and not the energy.

A comparison of above curves with the unperturbed velocity distribution shown in fig. (25) shows that ion distribution in the acoustic pulse was modified in a manner similar to the one predicted by Sakanaka (1972). The slope of the distribution changed close to an ion energy of 0.8 eV.

However, as distinct from Sakanaka's results, the measured distribution in this case never reached zero. As discussed in the previous section since there is no current in the plasma there can not be enough reflected electrons to form the ion rich region behind the negative potential structures. Thus the initial depletion of ion density remains depleted. The trapping does not look like a complete trapping. There could be particles leaking from the potential well of the wave. Thus the observed fissioning could be due to trapping and detrapping in the velocity space. Thus the main pulse was associated with a vortex in the ion hole with unequal shoulders.

4.4 Conclusions

The rarefactive ion acoustic waves investigated in the linear unmagnetised plasma have shown several interesting characteristics of excitation and propagation. The smaller amplitude pulses behave in accordance with the predictions of KdV equation. The waves with amplitude $\sim 1.2\%$ and more show several features which do not agree with the KdV equation.

The rarefactive perturbations excited by a fast step ($\omega \sim \omega_{pi}$, are found to evolve into holes in ion phase space. Both asymmetric and symmetric holes, have been observed in the presence of these large amplitude rarefactive ion acoustic waves. The asymmetric holes were obtained in the main ion acoustic pulse region, where they have shoulders of unequal amplitude. On the other hand, a train of fully developed symmetric holes were observed in the region behind ion acoustic pulse in the two stream region.

Numerical solutions of KdV equation, were obtained for the rarefactive perturbations. These solutions reveal the excitation of Airy function type of oscillations, in the trailing edge of the main pulse. These oscillations behave differently as compared to the one observed experimentally. It is found that narrow pulses excite more number of peaks with larger amplitude. But these oscillations occur at ion plasma frequency. The frequency observed experimentally is order of magnitude below ($\omega_{pi} \sim 300$ kHz, for our parameters). The validity of KdV equation for such narrow pulses is questionable.
The measurement of ion velocity distribution in these pulses reveal a strong modification of the distribution function near an energy corresponding to 0.8 eV.

In our measurements the hole velocities are found to be same as the ion acoustic speed. Hole widths observed are of ~ 200λ_D and they are found to move till the end of the device, showing a stable configuration. Relative density associated with the hole varies upto 18%.

The experimental observations match quite well with numerical calculations of Sakanaka (1972), for two beam instability. In agreement with numerical results of Sakanaka (1972), the usual short wavelength Airy function type of oscillations, were not seen. The width of the vortex is too large to confuse with Airy type oscillations. Sakanaka has given one example of a hole with width 100 λ_D, we have observed a hole with width ~ 200λ_D.