CHAPTER 4

Deteriorating Inventory Model for Demand Declining Market using the strategy of trade credit period

4.0 Introduction

In this chapter we discuss deteriorating inventory models for demand declining market using different components which are related to selling price. We divide this chapter in two different sections.

In section 4.1 we characterize the inventory model for deteriorating items in declining market when the supplier offers a permissible delay in payments to the retailer to settle the account against the purchases. In this model demand of a product is assumed to be decrease with time. It is assumed that the retailer generates revenue on unit selling price which is necessarily higher than the unit purchase cost. The algorithm
is exhibited for a retailer to determine the optimal procurement quantity which minimizes the total inventory cost per time unit. The model is supported by a numerical example. The sensitivity analysis is carried out to observe the changes in the optimal solution.

Section 4.2 is extension of the inventory model which we discussed above. Here also an attempt is made to develop optimal ordering and pricing policy for a retailer when the supplier offers a credit period to settle the account. The demand of a product is declining with time and retailer’s selling price subject to constant deterioration. In market, the retailer tries to sell the product at much higher price than the purchase cost. An algorithm is developed to determine the optimal selling price and the ordering quantity to maximize the retailer’s profit. The numerical examples are given to support the development of the mathematical model. The sensitivity analysis of critical parameters is carried out to observe the changes in the decision variables and objective function.

4.1: Optimal ordering policy for deteriorating items under the delay in payments in demand declining market
Additional assumptions and notations which are used to derive model 4.1 is as follows:

4.1.1: Assumptions and Notations

4.1.1.1 Assumption

- The inventory system under consideration deals with the single item.
- The planning horizon is infinite.
- The demand of the product is declining function of the time.
- Shortages are not allowed and lead-time is zero.
- The deteriorated units can neither be repaired nor replaced during the cycle time.
- The retailer can deposit generated sales revenue in an interest bearing account during the permissible credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day-to-day expenditure, and paying the interest charges on the unsold items in the stock.

4.1.1.2 Notations:
\( R(t) = a(1 - bt) \): the annual demand as a decreasing function of time where \( a > 0 \) is fixed demand and \( b \) (\( 0 < b < 1 \)) denotes the rate of change of demand

\( C \) the unit purchase cost

\( P \) the unit selling price with \( P > C \)

\( h \) the inventory holding cost per unit per year excluding interest charges

\( A \) the ordering cost per order

\( M \) the permissible credit period offered by the supplier to the retailer for settling the account

\( l_c \) the interest charged per monetary unit in stock per annum by the supplier

\( l_e \) the interest earned per monetary unit per year

Note: \( l_c > l_e \)

\( Q \) the order quantity (a decision variable)

\( I(t) \) the inventory level at any instant of time \( t, 0 \leq t \leq T \)

\( T \) the replenishment cycle time (a decision variable)

\( \theta \) constant deterioration. where \( 0 < \theta < 1 \)

\( K(T) \) the total inventory cost per time unit

**4.1.2 Mathematical formulation**
The inventory level; \( I(t) \) depletes to meet the demand and deterioration. The rate of change of inventory level is governed by the following differential equation:

\[
\frac{dI(t)}{dt} + \theta I(t) = -R(t), \quad 0 \leq t \leq T
\]  

(4.1.2.1)

with the initial condition \( I(0) = Q \) and the boundary condition \( I(T) = 0 \).

Using, \( I(T) = 0 \) the solution of differential equations (4.1.2.1) is

\[
I(t) = \left(\frac{a}{\theta} + \frac{b}{\theta^2}\right) \left(e^{\theta(T-t)} - 1\right) - \frac{bT}{\theta} \left(e^{\theta(T-t)}\right) + \frac{bT}{\theta}, \quad 0 \leq t \leq T
\]  

(4.1.2.2)

and the order quantity

\[
Q = I(0) = \left(\frac{a}{\theta} + \frac{b}{\theta^2}\right) \left(e^{\theta(T)} - 1\right) - \frac{bTe^{\theta(T)}}{\theta}
\]  

(4.1.2.3)

The total cost of inventory system per time unit consists of the following

Ordering cost;

\[
OC = A/T
\]  

(4.1.2.4)

Cost due to deterioration per time unit;

\[
DC = \frac{c}{T} \left[Q - \int_0^T R(t) \, dt\right] = \frac{c}{T} \left[\left(\frac{a}{\theta} + \frac{b}{\theta^2}\right) \left(e^{\theta(T)} - 1\right) - \frac{bTe^{\theta(T)}}{\theta} - aT + \frac{bt^2}{2}\right]
\]  

(4.1.2.5)

Inventory holding cost per unit per unit time

\[
IHC = \frac{h}{T} \int_0^T I(t) \, dt
\]  

(4.1.2.6)
4.1.3 Theoretical results

Regarding interest charges and earned, two cases may arise based on the length of cycle time $T$ and credit period $M$ which is exhibited in Fig.4.1.1 and Fig.4.1.2.

\[
= \frac{-h}{2\theta^3 T} \left[ -2a\theta e^{\theta(T)} - 2b e^{\theta(T)} + 2bT \theta e^{\theta(T)} + 2a\theta + 2a\theta^2 T + 2b - b\theta^2 T^2 \right]
\]
**Case 1: $M \leq T$.**

The retailer sells $R(M)M$ units by the end of the permissible tread credit $M$ and has $CR(M)M$ to pay the supplier. For the unsold items in the stock, the supplier charges an interest rate $I_c$ from time $M$ onwards. Hence, the interest charged, $IC_1$ per time unit is

$$IC_1 = \frac{C_{I_c}}{T} \int_M^T I(t) \, dt$$

$$= \frac{C_{I_c}}{2\theta^3 T} \left[ -2a\theta e^{\theta(T-M)} - 2a\theta^2 M - 2be^{\theta(T-M)} - 2bM\theta + 2bT\theta e^{\theta(T-M)} + bM^2\theta^2 + 2a\theta + 2a\theta^2 T + 2b - bT^2\theta^2 \right]$$

(4.1.3.1)
During \([0, M]\), the retailer sells the product and deposits the revenue into an interest earning account at the rate \(I_e\) per monetary unit per year. Therefore, the interest earned, \(IE_1\) per time unit is

\[
IE_1 = \frac{PL_e}{T} \int_0^M R(t) t \, dt = \frac{PL_e}{T} \left[ \frac{aM^2}{2} - \frac{bM^3}{3} \right] \tag{4.1.3.2}
\]

**Case 2: \(T \leq M\)**

Here, the retailer sells \(R(T)T\) a unit in all by the end of the cycle time and has \(C R(T) T\) to pay the supplier in full by the end of the credit period \(M\). Hence, interest charges

\[
IC_2 = 0 \tag{4.1.3.3}
\]

and the interest earned per time unit is

\[
IE_2 = \frac{PL_e}{T} \left[ \int_0^T R(t) t \, dt + R(T)T(M - T) \right] \\
= \frac{PL_e}{T} \left[ \frac{aT^2}{2} - \frac{abT^3}{3} + aTM - aT^2 - abT^2M + abT^3 \right] \tag{4.1.3.4}
\]

As a result, the total cost of an inventory system per time unit in both the cases is given by

\[
K_i(T) = OC + PC + IHC + IC_i - IE_i \quad \text{where, } i = 1, 2 \tag{4.1.3.5}
\]

Hence, the total cost; \(K(T)\) of an inventory system per time unit is
\[
K(T) = \begin{cases} 
K_1(T), & M \leq T \\
K_2(T), & M \geq T 
\end{cases}
\] (4.1.3.6)

For \( T = M \), we have

\[
K_1(M) = K_2(M)
\]

\[
= \frac{1}{M} \left[ c \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) (e^{\theta T} - 1) - \frac{abCM}{\theta} e^{\theta T} - aM + \frac{abCM^2}{2} - P_{le} \left( \frac{aM^2}{2} - \frac{abM^3}{3} \right) \right] +
\]

\[
+ A + \frac{ha}{2\theta^3} (2\theta + 2b - 2bM\theta)e^{\theta M} - 2\theta - 2M\theta^2 - 2b + bM^2\theta^2
\]

\] (4.1.3.7)

The optimum value of \( T = T_1 \) is the solution of

\[
\frac{\partial K_1(T)}{\partial T} = \frac{c}{T} \left( (a - abT)e^{\theta T} + abT - a \right) - \frac{c}{T^2} \left( \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) (e^{\theta T} - 1) - \frac{abTe^{\theta T}}{\theta} + \frac{abT^2}{2} - aT \right) +
\]

\[
\frac{A}{T^2} + \frac{ha}{2T^2\theta^3} [(2\theta^2 - 2bT\theta^2)e^{\theta T} - 2\theta^2 + 2b\theta^2T] + \frac{Cl_{le}}{2T^3\theta^3} [(2\theta^2 - 2bT\theta^2)e^{\theta(T-M)} - 2\theta^2 + 2b\theta^2T] - \frac{ha}{2T^2\theta^3} [(2\theta + 2b - 2bT)e^{\theta T} - 2\theta - 2\theta^2T - 2b + b\theta^2T^2] + \frac{P_{le}}{T^2} \left( \frac{aM^2}{2} - \frac{abM^3}{3} \right)
\]

which minimizes \( K_1(T) \) provided.

\[
K_1(T_1) = \frac{c}{T_1} \left( \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) (e^{\theta T_1} - 1) - \frac{abT_1e^{\theta T_1}}{\theta} + \frac{abT_1^2}{2} - aT_1 \right) - \frac{P_{le}}{T_1^2} \left( -\frac{abM^3}{3} + \frac{aM^2}{2} \right) + \frac{A}{T_1^2} +
\]

\[
+ \frac{Cl_{le}}{2T_1^3\theta^3} [(2\theta - 2b\theta T_1 + 2b)e^{\theta(T_1-M)} + 2M\theta^2 + 2bM\theta + b\theta^2T_1^2 - 2b - 2\theta - 2\theta^2T_1] +
\]

\[
+ \frac{ha}{2T_1^2\theta^3} [(2\theta + 2b - 2bT)e^{\theta T_1} - 2\theta - 2\theta^2T_1 - 2b + b\theta^2T_1^2] - \frac{Cl_{le}abM^2}{2T_1\theta
\]

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(4.1.3.9)

The optimum value of $T = T_2$ is the solution of

$$
\frac{\partial K_2(T)}{\partial T} = \frac{C}{T} \left( (a - abT) e^{\theta T} + abT - a \right) - \frac{C}{T^2} \left[ \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) (e^{\theta T} - 1) - \frac{abT^2 e^{\theta T}}{\theta} + \frac{ab T^2}{2} - a T \right] + \frac{A}{T^2} \\
+ \frac{ha}{2T^3 \theta^3} \left[ (2\theta^2 - 2bT\theta^2) e^{\theta T} - 2\theta^2 + 2b\theta^2 T \right] + \frac{P_I e}{T^2} \left( \frac{aT^2}{3} - \frac{ab T^3}{3} + a(1 - bT) (M - T) \right) \\
+ \frac{P_I e}{T} \left[ -ab T^2 + a T - abT (M - T) + a(1 - bT) (M - T) - a(1 - bT) T \right] \\
- \frac{ha}{2T^3 \theta^3} \left[ (2\theta + 2b - 2bT) e^{\theta T} - 2\theta - 2\theta^2 T - 2b + b\theta^2 T^2 \right] 
$$

which minimizes $K_2(T)$ provided.

(4.1.3.10)

$$
K_2(T_2) = \frac{C}{T_2} \left[ \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) (e^{\theta T_2} - 1) - \frac{abT_2 e^{\theta T_2}}{\theta} + \frac{ab T_2^2}{2} - a T_2 \right] + \frac{A}{T_2} \\
+ \frac{ha}{2T_2^3 \theta^3} \left[ (2\theta + 2b - 2bT_2 \theta) e^{\theta T_2} - 2\theta - 2\theta^2 T_2 - 2b + b\theta^2 T_2^2 \right] \\
- \frac{P_I e}{T_2} \left( \frac{ab T_2^3}{3} + \frac{aT_2^2}{2} + a(1 - bT_2) T_2 (M - T_2) \right) 
$$

(4.1.3.11)

4.1.4 Computational algorithm

To obtain optimal solution, decision maker is advised to observe the following steps.

**Step1:** Initialize all parametric values.

**Step2:** Compute $T_1$ from equation (4.1.8),

If $M < T_1$ then $K_1(T_1)$ from equation (4.1.3.9) gives minimum cost; else go to step3.

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Step3: Compute $T_2$ from equation (4.1.3.10),

If $M > T_2$ then $K_2(T_2)$ from equation (4.1.3.11) gives minimum cost for decision maker; else go to step 4.

Step4: $K_1(M) = K_2(M)$ from equation (4.1.3.7) is the minimum cost.

Step5: stop.

4.1.5 Numerical example and observation

Here we derive two examples relative to above discussed cases

Example1 (for case 1) Consider the parametric values

$$[a, b, A, C, P, h, I_c, I_v, M, \theta] = [1000, 0.2, 250, 20, 40, 1, 0.12, 0.09, 30/365, 0.10]$$

Using the algorithm exhibited in section 4, $T_1 = 0.3185$ years which is greater than $M = 0.082$ Years. Hence corresponding minimum cost is $K_1(T_1) = 1396.44$

(see Fig.4.1.5.1) and optimum procurement units are 313.
Example 2 (for case 2) Consider the parametric values:

\[ [a, b, A, C, P, h, I_e, M, \theta] = [600, 0.1, 50, 30, 35, 1, 0.15, \frac{60}{365}, 0.20] \]

Then \( T_2 = 0.122 \) years which is less than \( M = 0.1644 \) years. Hence using algorithm stated in section 4.1.4, the minimum cost is \( K_2(T_2) = 405.36 \) (see Fig.4.1.5.2) and optimum purchase quantity is 74 units.
Next, we carry out sensitivity analysis by varying parameters $b, \theta, M$ as -40%, -20%, 20%, 40% for case1. The corresponding changes in the cycle time, purchase quantities and total cost are exhibited in table 4.1.5.1.

**Table 4.1.5.1**: Sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>% change in $T$</th>
<th>% change in $Q$</th>
<th>% change in $K_1$</th>
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<td>-1.2</td>
<td>1.1</td>
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<td>-20</td>
<td>-1.2</td>
<td>-0.6</td>
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<tr>
<td></td>
<td>20</td>
<td>1.2</td>
<td>0.6</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>2.7</td>
<td>1.2</td>
<td>-1.2</td>
</tr>
<tr>
<td>$b$</td>
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<td>8.7</td>
<td>8.9</td>
<td>-8.9</td>
</tr>
<tr>
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<td>4.1</td>
<td>4.1</td>
<td>-4.3</td>
</tr>
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<tr>
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<td>-</td>
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<td>-</td>
</tr>
<tr>
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<td>0.9</td>
<td>0.6</td>
<td>5.9</td>
</tr>
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</tr>
<tr>
<td></td>
<td>40</td>
<td>-1.1</td>
<td>-1.2</td>
<td>-6.2</td>
</tr>
</tbody>
</table>

-stands for infeasible solution

It is observed that as rate of change of demand increases, cycle time increases while total cost of an inventory system decreases. Increases in deterioration rate forces retailer to buy more number of units frequently and hence increases total cost of an inventory system. Increases in delay period decreases retailer’s cycle time and total cost of inventory system. The reduction in total cost is obvious because retailer can
earn more interest during this permissible delay period for settlement of accounts against his dues.

Next section is carried out new results and observation when the demand rate is price sensitive time dependent.

4.2 Optimal pricing and ordering policy for deteriorating inventory under trade credit in demand declining market

4.2.1 Assumptions and Notations

Assumption and notation are as follows:

4.2.1.1 Assumption

- The inventory system under consideration stocks single item only.
- The planning horizon is infinite.
- The demand of the product is decreasing function of the time and the selling price.
- Shortages are allowed and lead-time is zero.
• The deteriorated units can neither be repaired nor replaced during the cycle time.

• The retailer can deposit generated sales revenue in an interest bearing account, during the allowable credit period. At the end of this period, the retailer settles the account for all units sold keeping the difference for day-to-day expenditure, and starts paying the interest charges on the unsold items in the stock.

4.2.1.2 Notations

\[ R(t,P) = a(1 - bt)P^{-\eta}; \] where \( a > 0 \) is fixed demand and \( b \) is the rate of change of demand, \( 0 \leq b < 1 \) and \( \eta > 0 \) is mark up parameter.

\[
\begin{align*}
C & \quad \text{the unit purchase cost} \\
P & \quad \text{the unit selling price with } (P > C) \\
h & \quad \text{the inventory holding cost per unit per year excluding interest charges} \\
A & \quad \text{the ordering cost per order} \\
M & \quad \text{the permissible credit period offered by the supplier to the retailer for settling the account}
\end{align*}
\]
\[ I_c \] the interest charged per monetary unit in stock per annum by the supplier

\[ I_e \] the interest earned per monetary unit per year

Note: \( I_c > I_e \)

\[ Q \] the order quantity (a decision variable)

\[ I(t) \] the inventory level at any instant of time \( t, 0 \leq t \leq T \)

\[ T \] the replenishment cycle time (a decision variable)

\[ \theta \] constant deterioration. where \( 0 < \theta < 1 \)

\[ Z(t) \] the total profit per unit time.

### 4.2.2 Mathematical formulation

The retailer’s inventory level depletes due to demand and deterioration of units in inventory. The rate of change of inventory level is governed by the differential equation

\[
\frac{dI(t)}{dt} + \theta I(t) = -R(t, P), \quad 0 \leq t \leq T
\]  

(4.2.2.1)

with the initial condition \( I(0) = Q \) and the boundary condition \( I(T) = 0 \).
Using, $I(T) = 0$ the solution of differential equations (4.1.2.1) is

$$I(t) = \left[ \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) (e^{\theta(T-t)} - 1) - \frac{abT}{\theta} (e^{\theta(T-t)}) + \frac{abT}{\theta^2} \right] P^{-\eta}, \quad 0 \leq t \leq T \quad (4.2.2.2)$$

And the order quantity

$$Q = I(0) = \left[ \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) (e^{\theta(T)} - 1) - \frac{abT e^{\theta(T)}}{\theta} \right] P^{-\eta} \quad (4.2.2.3)$$

The total profit per time unit comprises of the following:

Sales revenue; per time unit is $SR = \frac{PQ}{T} \quad (4.2.2.4)$

Purchase cost of procuring $Q$-units per time unit is $PC = \frac{CQ}{T} \quad (4.2.2.5)$

Inventory holding cost; $IHC$ per time unit excluding interest charges is:

$$IHC = \frac{-abP^{-\eta}}{2\theta^3 T} \left[ 2\theta e^{\theta(T)} + 2be^{\theta(T)} - 2bT \theta e^{\theta(T)} - 2\theta - 2\theta^2 T - 2b + b\theta^2 T^2 \right] \quad (4.2.2.6)$$

Ordering cost; $OC$ per order is $OC = \frac{A}{T} \quad (4.2.2.7)$

**4.2.3 Theoretical Results**

Regarding interest charges and earned, two cases may arise based on the length of $T$ and $M$; Viz $M \leq T$ and $M > T$.

**Case 1:** $M \leq T$
Using the assumption the retailer sells $R(M)M$ - units by the end of the permissible tread credit $M$ and has $CR(M)M$ to pay the supplier. For the unsold items in an inventory system, the supplier charges an interest rate $I_c$ during the period $[M,T]$.

Hence, the interest charged, $IC_1$ per time unit is

$$IC_1 = \frac{C_I e}{T} \int_M^T I(t) \, dt = \frac{C_I e}{2\theta^3 T} \left[ 2\theta e^{\theta(t-M)} + 2\theta^2 M + 2b e^{\theta(t-M)} + 2bM \theta 
- 2bT \theta e^{\theta(t-M)} - bM^2 \theta^2 - 2\theta - 2b + bT^2 \theta^2 \right]$$

(4.2.3.1)

During $[0, M]$ the retailer sells the product and deposits the revenue into an interest earning account at the rate $I_e$ per monetary unit per year. Therefore, the interest earned, $IE_1$ per time unit is

$$IE_1 = \frac{P I_e}{T} \int_0^M R(t, P) \, dt = \frac{a P^{-\eta} I_e}{T} \left[ \frac{M^2}{2} - \frac{bM^3}{3} \right]$$

(4.2.3.2)

**Case2: $T \leq M$**

Here, the retailer sells $R(T)T$ a unit in all by the end of the cycle time and has $CR(T)T$ to pay the supplier in full by the end of the credit period $M$. Hence, interest charges

$$IC_2 = 0$$

(4.2.3.3)
and the interest earned per time unit is

\[ IE_2 = \frac{P_{T_e}}{T} \left[ \int_0^T R(t, P) \, dt + R(T, P)T(M - T) \right] \]

\[ = \frac{aP - \eta + ie}{T} \left[ \frac{aT^2}{2} - \frac{bT^3}{3} + TM - T^2 - bT^2M + bT^3 \right] \quad (4.2.3.4) \]

As a result, in both the scenario, retailer's profit in inventory system per time unit is given by

\[ Z_i(T, P) = SR - OC - PC - IHC - IC_i + IE_i \quad \text{Where,} \quad i = 1, 2 \quad (4.2.3.5) \]

For, \( T = M, Z_1(M, P) = Z_2(M, P) \quad (4.2.3.6) \]

The necessary condition for \( Z_1(T, P) \) to be optimum,

\[ \frac{\partial Z_1(T, P)}{\partial P} = 0 = \]

\[ \frac{P - \eta}{T} \left[ \frac{CaT\eta}{2P\theta^3} \left( (2\theta - 2bT\theta + 2b)e^{\theta(T-M)} - 2\theta - 2\theta^2T - 2b + b\theta^2T^2 + 2\theta^2M + 2bM\theta - b\theta^2M^2 \right) \right] \]

\[ - \frac{CaT\eta}{2P\theta^3} \left( (2a - 2bT\theta + 2b)e^{\thetaT} - 2\theta - 2\theta^2T - 2b + b\theta^2T^2 \right) + \left( -\frac{abT}{\theta} + \frac{ab\theta}{\theta} - \frac{Ca\eta T}{P\theta} \right) e^{\thetaT} \quad (4.2.3.7) \]

\[ \frac{\partial Z_1(T, P)}{\partial T} = 0 = \left[ \frac{Pe^{\thetaT}}{T} (a - abT) - \frac{P(e^{\thetaT-1})}{T^2} \left( a + \frac{ab}{\theta} \right) - \frac{P_{abT}e^{\thetaT}}{\theta^2T^2} \left( a - abT \right) + \frac{ha}{\theta} - \right. \]

\[ \left. \frac{CaT \theta e^{\thetaT}}{T^2} + \frac{C(e^{\thetaT-1})}{T^2} \left( a + \frac{ab}{\theta} \right) - \frac{ha}{\theta^3T} \left( (2 - abT)\theta^2e^{\thetaT} - 2\theta^2 - 2b\theta^2T^2 \right) \right] - \frac{P_{T_e}}{T^2} \left( \frac{-abM^2}{3} + \frac{a\theta^2}{2} \right) + \]

\[ \frac{ha}{\theta^3T^2} \left( (2 - bT)\theta e^{\thetaT} - 2\theta + 2\theta^2T - 2b \right) + \frac{C_{leT}}{2\theta^2T} \left( 2(1 - bt)\theta^2e^{\theta(T-M)} - 2\theta^2 - 2b\theta^2T \right) + \]
\[
\frac{C_{1e}a}{2\theta^3 r^2} \left( 2(1 - bt) \theta e^{\theta(T - M)} - 2\theta - 2\theta^2 T - 2b + b\theta^2 T^2 + 2M \theta^2 + 2bM \theta - bM^2 \theta^2 + 2be^{\theta(T - M)} \right) \right] P^{-\eta} + \frac{A}{T^2}
\]

(4.2.3.8)

The obtained \((T, P)\) maximizes profit \(Z_1\) provided

\[
\frac{\partial^2 Z_1(T, P)}{\partial T^2} < 0, \quad \frac{\partial^2 Z_1(T, P)}{\partial P^2} < 0 \quad \text{and} \quad \frac{\partial^2 Z_1(T, P)}{\partial T \partial P} - \left( \frac{\partial^2 Z_1(T, P)}{\partial T \partial P} \right)^2 > 0
\]

(4.2.3.9)

Similarly, the necessary condition for \(Z_2(T, P)\) is

\[
\frac{\partial Z_2(T, P)}{\partial T} = 0 = \left[ \frac{pe^{\theta T}}{T} (a - abT) - \frac{p(e^{\theta T} - 1)}{T^2} \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) - \frac{pa \theta e^{\theta T}}{T^2} - \frac{c \theta e^{\theta T}}{T} (a - abT) + \frac{pe^{\theta T} - 1}{T^2} \left( \frac{a}{\theta} + \frac{ab}{\theta^2} \right) - \frac{h a}{\theta^3 T^2} \left( 2 - abT \right) \theta e^{\theta T} - 2\theta^2 + 2b\theta^2 T \right) + \frac{h a}{\theta^3 T^2} - \frac{c \theta e^{\theta T}}{T^2} + \frac{pl e}{T} \left( -abT^2 + a(1 - bT)T(M - T) \right) - \frac{pl e}{T^2} \left( -abT^3 + aT^2 + a(1 - bT)T(M - T) \right) \right] P^{-\eta} + \frac{A}{T^2}
\]

(4.2.3.10)

\[
\frac{\partial Z_2(T, P)}{\partial P} = 0 = \left[ \frac{\partial}{\partial T} \left( \frac{a^2 T - a b T^3}{2} + a(1 - bT)T(M - T) \right) - \frac{pl e}{T^2} \left( \frac{a^2 T - a b T^3}{2} + a(1 - bT)T(M - T) \right) + \frac{h a}{2p \theta^3} \left( 2a - 2bT \theta + 2b \right) e^{\theta T} - 2\theta - 2\theta^2 T - 2b + b\theta^2 T^2 \right]
\]

(4.2.3.11)
The complexities of the expression suggest that it is not easy to get good closed form for the necessary and sufficient conditions. One can solve equations by mathematical software. The obtained \((T, P)\) maximizes the profit if and only if

\[
\frac{\partial^2 Z_2(T, P)}{\partial P^2} < 0, \quad \frac{\partial^2 Z_2(T, P)}{\partial T^2} < 0 \quad \text{and} \quad \frac{\partial^2 Z_2(T, P)}{\partial P \partial T} - \left(\frac{\partial^2 Z_2(T, P)}{\partial P \partial T}\right)^2 > 0
\]

(4.2.3.12)

4.2.4 Computational Algorithm

The decision maker can use following steps to maximize his profit.

**Step1:** Take parametric values in proper units.

**Step2:** Calculate \(P\) and \(T\) using (4.2.3.7) and (4.2.3.8). If \(M < T\) then \(Z_1(T, P)\) is maximum else go to step 3.

**Step3:** Compute \(P\) and \(T\) from (4.2.3.10) and (4.2.3.11). If \(M > T\) then \(Z_2(T, P)\) is maximum profit else go to step 4.

**Step4:** Compute \(P\) from (4.2.3.6). Here, \(Z_1(M, P) = Z_2(M, P)\) is maximum profit.

4.2.5 Numerical example and observation

**Example1:** For
\[ a = 1000000, \ b = 0.2, \ \eta = 2.0, \ h = \$1.00 / \text{unit/year}, \ c = \$20.00 \text{ per unit}, \ A = \$250 / \text{order}, \ l_c = 0.12 / \$ / \text{year}, \ l_e = 0.09 / \$ / \text{year}, \ \theta = 0.10, \ M = 30/365 \text{ year}, \] the optimal solution is \( P = \$40.92 \text{ per unit} \) and cycle time \( T = 0.391 \text{ year} \). The maximum profit per time unit is \( \$11340.60 \) and optimum purchase quantity is 229 units per order. For \( P = \$40.92 \) and \( T = 0.391 \text{ years} \)

\[
\frac{\partial^2 Z_1}{\partial P^2} = -241.04, \quad \frac{\partial^2 Z_1}{\partial T^2} = -1.911 \times 10^{-5} \text{ and } \left( \frac{\partial^2 Z_1}{\partial P^2} \right) \left( \frac{\partial^2 Z_1}{\partial T^2} \right) - \left( \frac{\partial^2 Z_1}{\partial P \partial T} \right)^2 = 4.55 \times 10^7 > 0
\]
guarantees maximum profit. The 3D - plot (See fig. 4.2.5.1) drawn in the range \([35, 55]\) for \( P \) and \([0.1, 0.9]\) for \( T \) depicts that obtained profit \( Z_1(0.391, 40.92) = \$11340.60 \) is maximum.
Fig4.2.5.1 Concavity of profit $M \leq T$

**Example2:** Take $a = 150, 00,000$, $b = 0.1$, $\eta = 2.0$, $h = \$ 1.24 / unit / year$, $C = \$ 20.00 per unit$, $A = \$ 250 / order$, $l_e = 0.09/ \$ / year$, $\theta = 0.20$, $M = 60/ 365$ year, the optimal solution is $P = \$ 39.83 per unit and cycle time $T = 0.136$ year. The maximum profit per time unit is $\$ 189368.90 and optimum purchase quantity is 1301 units per order. For $P = \$ 39.83$ and $T = 0.136$ years

\[
\frac{\partial^2 z_2}{\partial p^2} = -14.30 < 0, \quad \frac{\partial^2 z_2}{\partial T^2} = -8169.31 \quad \text{and} \quad \left(\frac{\partial^2 z_2}{\partial p^2}\right)\left(\frac{\partial^2 z_2}{\partial T^2}\right) - \left(\frac{\partial^2 z_2}{\partial p \partial T}\right)^2 = 1.14 \times 10^5 > 0
\]
guarantees maximum profit. The 3D-plot (see fig. 4.2.5.2) drawn in the range [30,45] for $P$ and [0.1,0.5] for $T$ depicts that obtained profit $Z_2(0.136,39.83) = $189368.90 is maximum.

Next, we carry out sensitivity analysis by varying parameters $b, \theta, M$ as -40%, -20%, 20%, 40% the case $M < T$. The corresponding changes in the cycle time, purchase quantities and total cost are exhibited in table 4.2.5.1.
Table 4.2.5.1 Sensitivity analysis

<table>
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<th>parameter</th>
<th>% changes</th>
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<tr>
<td></td>
<td>$T$</td>
</tr>
<tr>
<td>$M$</td>
<td></td>
</tr>
<tr>
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</tr>
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</table>

It is observed that when delay period increases, retailer’s cycle time and selling price decreases while procurement quantity and profit increases. Thus, it is rightly said that offer of trade credit attracts more customers. The proposed model is very sensitive
to deterioration rate and demand decreasing rate. As demand decrease, cycle time and profit decrease because retailer has to put an order of smaller size.

4.3 Conclusion

The effect of delay period offered by the supplier to retailer is analyzed when the demand of the product is decreased with respect to time in the market. The units in inventory are assumed to deteriorate at a constant rate. It is observed that incentive of credit period is advantageous to the retailer for lowering the total cost or to increase their profit of an inventory system. But when demand is decreasing function of both time and retail price it is more sensitive with respect to deterioration rate and demand decreasing rate.