CHAPTER 3

Deteriorating Inventory Models using DCF approach for Demand Declining Market under the scenario of trade credit

3.0 Introduction

In this chapter, we discuss deteriorating inventory models for demand declining market using different components which are related to selling price. We divide this chapter in two sections.

In section 3.1, we discuss the article in which ordering policy is decided by a vendor of a deteriorating product where the decision is influenced by decreasing demand of the product, the time value of money, inflation and allowable credit period. Depending on the length of the payment period, the vendor can earn interest on the revenue generated by the sale of the product. The optimizing present value of the total cost is the objective function. The effects of deterioration rate, inflation and decreasing demand rate on decision variable and objective function are studied.
In section 3.2, a mathematical model is developed for an inventory system dealing with deteriorating items. We expand this model by allowing shortages to production rate. It examines how ordering policy of vendor is influenced by demand of the product which is facing declining market conditions and by economy that is passing through an inflationary pressure. It is further assumed that demand can be completely backlogged once the next replenishment of stock arrives. Proposed mathematical model aims at optimizing total inflated cost of vendor and analyzing combined effect on total inflated cost due to deterioration, declining market conditions, inflation, trade credit and backlogging of the units using discounted cash flow approach. The paper includes with interesting observations about ordering policy of the vendor.

3.1: A deteriorating inventory model for demand declining market under inflation and trade credit

There are additional assumptions and notations which are used to derive model is as follows
3.1.1: Assumptions and Notations

3.1.1.1 Assumption

- The inventory system deals with single item.
- The demand is decreasing function of time.
- Replenishment rate is infinite.
- Lead – time is zero and shortages are not allowed.
- The deteriorated units can neither be repaired nor replaced during the cycle time.

3.1.1.2: Notations

\[ A \] \quad \text{Ordering cost per order}

\[ C \] \quad \text{Unit cost of an item at time } t = 0, \$ \text{ / unit}

\[ C_0 \] \quad \text{present value of the inflated price of an item, } S \text{ / unit}

\[ CD \] \quad \text{total cost of deterioration per cycle}

\[ CD_f \] \quad \text{total cost of deterioration per cycle with inflation}

\[ IHC \] \quad \text{total inventory holding cost per cycle}
\( IHC_f \) total inventory holding cost per cycle with inflation

\( R(t) \) Demand rate where \( a > 0 \) is fixed demand and \( 0 \leq b < 1 \) denotes rate of change

number of units deteriorated during a cycle time \( T \)

\( D(T) \)

\( f \) inflation rate

\( i \) inventory carrying charge fraction per unit per time unit

\( I_e \) interest earned at time \( t = 0 \), $ / $ / unit time

\( IE \) total interest earned per cycle

\( IE_f \) total interest earned per cycle under inflation

\( I_c \) interest charged at time \( t = 0 \), $ / $ / unit time

\( IC \) total interest charged per cycle

\( IC_f \) total interest charged per cycle under inflation

\( r \) discount rate representing the time value of money

\( R \) $f - r$, Present value of inflation rate
\( \theta \)  
Deterioration rate 0 \( \leq \theta \leq 1 \), of the on-hand inventory

\( Q \)  
Optimal order quantity, units / order

\( T \)  
Length of the inventory cycle (decision variable)

### 3.1.2 Mathematical formulation

Let \( I(t) \) be the inventory level at any instant of time \( t \). The inventory level changes due to demand and deterioration of units. The rate of change of inventory level; \( I(t) \) at any instant of time \( t \) during the period \([0, T]\) is given by the differential equation

\[
\frac{d}{dt}I(t) + \theta I(t) = -R(t), \quad 0 \leq t \leq T
\]  

(3.1.2.1)

with the boundary condition \( I(0) = Q \) and \( I(T) = 0 \).

Using \( I(T) = 0 \), the solution of equation (3.1.2.1) is given by

\[
I(t) = \frac{a}{\theta}(e^{\theta(T-t)} - 1) + \frac{ab}{\theta^2}(e^{\theta(T-t)} - 1) - \frac{abT}{\theta}(e^{\theta(T-t)}) + \frac{abt}{\theta}
\]  

(3.1.2.2)

and boundary condition \( I(0) = Q \) gives the purchase quantity as

\[
Q = \frac{a}{\theta}(e^{\theta T} - 1) + \frac{ab}{\theta^2}(e^{\theta T} - 1) - \frac{abT}{\theta}(e^{\theta T})
\]  

(3.1.2.3)

The number of units deteriorated during a cycle is given by

\[
D(T) = Q - R(T) T
\]  

(3.1.2.4)
Under inflation rate $f$, the item cost $C$ will be $Ce^{ft}$ and for a discount rate $r$ the present value factor is $e^{-rt}$ for time $t$. Hence, the present value of the inflated amount of the product at time $t = 0$ is given by

$$C_0 = Ce^{(f-r)t} = Ce^{Rt}, \text{ where } R = f - r$$  \hspace{1cm} (3.1.2.5)

As order is placed initially i.e. at time $t = 0$, it will not affected by inflation and hence ordering cost is given by

$$OC = A$$ \hspace{1cm} (3.1.2.6)

The present value of $Q$– units procured in the beginning of the cycle is given by $CQ$ and the present value of the units sold is $\int_0^T C_0 R(T) \, dt$. Hence, the present value of cost of deterioration per cycle under inflation is given by

$$CD_f = CQ - \int_0^T C_0 R(T) \, dt$$ \hspace{1cm} (3.1.2.7)

The inventory holding cost under inflation per cycle is

$$IHC_f = i \int_0^T C_0 I(t) \, dt$$ \hspace{1cm} (3.1.2.8)
3.1.3 Theoretical results

The length of the offered credit period creates two cases which are as follows:

Case 1: Payment is made at or before the total depletion of inventory \((M \leq T)\), and

Case 2: Payment is made at after total depletion of inventory \((M > T)\)

**Case 1**: \(M \leq T\)

For this case the present value (at \(t = 0\)) of interest charged rate and interest earned at
time \(t\) are

\[
I_c(t) = (e^{lt} - 1)e^{-rt} \quad \text{and} \quad I_e(t) = (e^{lt} - 1)e^{-rt} \quad \text{respectively.}
\]

Therefore, the interest charged during \([M, T]\) is given by

\[
IC_{f1} = \int_{M}^{T} C I_c(t) I(t) \, dt \quad (3.1.3.1)
\]

And interest earned per cycle is

\[
IE_{f1} = \int_{0}^{M} C e^{rt} I_e(t) R(t) \, dt \quad (3.1.3.2)
\]

**Case 2**: \(M > T\)

As \(M > T\) vendor can pay in full at the end of permissible delay \(M\), the interest charged

\[
IC_{f2} = 0 \quad (3.1.3.3)
\]
The interest earned per cycle is the interest earned during positive inventory period \([0, T]\) and the interest from the revenue deposited during the time period \([T, M]\). Hence, interest earned per cycle during inflation is

\[
IE_{12} = \int_0^T C I_e(t) R(t) t \, dt + (e^{(I_e(M-T))} - 1) \int_T^M C e^{Rt} R(T) \, t \, dt
\]  

(3.1.3.4)

Hence total cost for both the cases \(K_{if}(T)\) (i.e. for \(i = 1, 2\)) per cycle is given by

\[
K_{if}(T) = OC + CD_f + IHCF_f + IC_f l - IE_{fi}\quad \text{where, } i = 1, 2
\]  

(3.1.3.5)

If there are \(N\)-cycles during a year then \(NT = 1\). Therefore, the total cost \(K_i(T)\) is given by

\[
K_i(T) = K_{if}(T) \sum_{n=0}^{N-1} e^{nRT} = K_{if}(T) \left(\frac{1-e^{RT}}{1-e^{RT}}\right)\quad \text{where, } i = 1, 2
\]  

(3.1.3.6)

The necessary for \(K_i(T)\) to be minimum is \(\frac{\partial K_i(T)}{\partial T} = 0\) and \(\frac{\partial^2 K_i(T)}{\partial T^2} > 0\), where, \(i = 1, 2\)
3.1.4 Numerical example and observation

To validate this model, consider the following parametric values in proper units:

\[ [a, A, l, l_c, l_e] = [20000, 200, 0.12, 0.15, 0.13] \]. With the different set of values of

\[ C = \{10, 20, 30\}, \ R = \{0.03, 0.05, 0.07, 0.08\}, \ \theta = \{0.03, 0.033, 0.035, 0.037\} \] and

\[ M = \{0, \frac{30}{365}, \frac{45}{365}\} \]. The optimum purchase quantity and total inflated cost using

above parametric values is exhibited in table 3.1.1 to 3.1.4.
Table 3.1.1 Optimal solution when deterioration rate varies

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Table 3.1.2  Optimal solution when discounting rate varies

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Table 3.1.3 Optimal solution when allowable credit period varies

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Table 3.1.4 Optimal solution when demand rate varies

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It is observed from table 3.1.1 that increase in deterioration rate increases procurement quantity and total cost due to inflation of the inventory system whereas cycle time decreases for fixed purchase cost C. Increase in purchase cost reduces buying power of the retailer and increases inflated total cost significantly.

While from table 3.1.2 it is observed that when there is an increment in C for fixed r, reduces total inflated cost of an inventory system and purchase quantity. For fixed C, increase in inflation rate; R increases purchase quantity significantly whereas total cost of inventory system due to inflation decreases.

From table 3.1.3 increase in allowable delay period M to settle the account, for fixed C and different inflation rate decreases retailer’s total cost due to inflation because retailer can generate more revenue during specified time. It also increases optimal purchase quantity significantly.

And from last table 3.1.4 increase in demand rate b, decreases total cost due to inflation of the inventory system and cycle time to put new orders. The inflation rate
increases buying power of the retailer and decreases total cost under inflation. In next section we expand this model by allowing shortages.

3.2: An order level inventory system for deteriorating item with trade credit under declining market condition using DCF approach.

3.2.1: Assumptions and Notations

As model 3.2 is extension of model 3.1 all the notations which are used to derive model 3.1 is also used here. But, there is some additional assumption and notations are their which are as follows:

3.2.1.1 Assumption

- The inventory system deals with single item.
- The demand rate is decreasing function of time.
- Replenishment rate is infinite.
- Lead time is zero or negligible.
- Shortages are allowed and completely backlogged.
- The deteriorated units can neither be repaired nor replaced during the cycle time.
3.2.1.2: Notations

\( A \)  
Ordering cost per order

\( C \)  
Unit cost of an item at time \( t = 0 \), $ / unit

\( C_0 \)  
Present value of the inflated price of an item, $ / unit

\( CD \)  
Total cost of deterioration per cycle

\( CD_f \)  
Total cost of deterioration per cycle with inflation

\( IHC \)  
Total inventory holding cost per cycle

\( IHC_f \)  
Total inventory holding cost per cycle with inflation

\( R(t) \)  
\( a(1 - bt) \), Demand rate where \( a > 0 \) is fixed demand and \( 0 \leq b < 1 \) denotes rate of change

\( D(T) \)  
Number of units deteriorated during a cycle time \( T \)

\( f \)  
Inflation rate

\( i \)  
Inventory carrying charge fraction per unit per time unit

\( I_e \)  
Interest earned at time \( t = 0 \), $ / unit time
\[ IE \] total interest earned per cycle

\[ IE_f \] total interest earned per cycle under inflation

\[ I_c \] interest charged at time \( t = 0 \), \$/$/unit time

\[ IC \] total interest charged per cycle

\[ IC_f \] total interest charged per cycle under inflation

\[ r \] discount rate representing the time value of money

\[ R \] \( f - r \), Present value of inflation rate

\[ \theta \] deterioration rate \( 0 \leq \theta \leq 1 \), of the on-hand inventory

\[ \pi_b \] backorder cost at time \( t = 0 \), \$/unit year.

\[ \pi_{b0} \] present value of the inflated backorder cost, \$/unit year.

\[ BC \] total backorder cost per cycle.

\[ BC_f \] total backorder cost per cycle under inflation.

\[ T_1 \] length of the period with positive stock of the item. (a decision variable)

\[ IM \] maximum inventory during positive stock period.

\[ IB \] maximum units backordered during stock out period.
\[ Q \quad (= IM + IB) \], the optimal order quantity; units/ order.

\[ T \quad \text{Cycle time (a decision variable)} \]

### 3.2.2 Mathematical formulation:

Let \( I(t) \) be the inventory level at any instant of time \( t \in [0, T] \). During \([0, T_1]\), the inventory level depletes due to demand and deterioration of items while, there is no loss of units during stock out period \([T_1, T]\) is given by the differential equations:

\[
\frac{dI(t)}{dt} + \theta I(t) = -R(t), \quad 0 \leq t \leq T_1 \tag{3.2.2.1}
\]

\[
\frac{dI(t)}{dt} = -R(t), \quad T_1 \leq t \leq T \tag{3.2.2.2}
\]

with the conditions \( I(0) = IM, I(T_1) = 0 \) and \( I(T') = -IB \).

Using, \( I(T_1) = 0 \) the solution of differential equations (3.2.2.1) and (3.2.2.2) are

\[
I(t) = \frac{a}{\theta} \left( e^{\theta(T_1-t)} - 1 \right) + \frac{ab}{\theta^2} \left( e^{\theta(T_1-t)} - 1 \right) - \frac{abT_1}{\theta} \left( e^{\theta(T_1-t)} \right) + \frac{abT_1}{\theta}, \quad 0 \leq t \leq T_1 \tag{3.2.2.3}
\]

And \( I(t) = a(T_1 - t) + \frac{ab}{2} (t^2 - T_1^2) \), \( T_1 \leq t \leq T \) \tag{3.2.2.4}

using \( I(0) = IM \), the maximum inventory during \([0, T_1]\) is

\[
IM = \frac{a}{\theta} \left( e^{\theta T_1} - 1 \right) + \frac{ab}{\theta^2} \left( e^{\theta T_1} - 1 \right) - \frac{abT_1}{\theta} \left( e^{\theta T_1} \right) \tag{3.2.2.5}
\]
and \( I(T) = -IB \), the maximum backorder units is

\[
IB = a (T - T_1) - \frac{ab}{2} (T^2 - T_1^2),
\]

(3.2.2.6)

Hence, total order size is \( Q = IM + IB \) during a cycle time. The number of units deteriorated during a cycle is given by

\[
DU = IM - R(T_1)T_1
\]

(3.2.2.7)

Under inflation rate \( f \) the item cost \( C \) will be \( Ce^{ft} \) and for a discount rate \( r \) the present value factor is \( e^{-rt} \) for time \( t \). Hence, the present value of the inflated amount of the product at time \( t = 0 \) is given by

\[
C_0 = Ce^{(f-r)t} = Ce^{Rt}, \text{ where } R = f - r
\]

(3.2.2.8)

Now, the different cost component of total cost of inventory system is as follows:

As order is placed initially i.e. at time \( t = 0 \), it will not affected by inflation and hence ordering cost is given by

\[
OC = A
\]

(3.2.2.9)
The present value of \( Q \) – units procured in the beginning of the cycle which is given by \( CQ \) and the present value of the units sold is \( \int_0^{T_1} C_0 R(t) \, dt \). Hence, the present value of cost of deterioration per cycle under inflation is given by

\[
CD_f = CQ - \int_0^{T_1} C_0 R(T) \, dt
\]  
(3.2.2.10)

The inventory holding cost under inflation per cycle is

\[
IHCF_f = i \int_0^{T_1} C_0 I(t) \, dt
\]  
(3.2.2.11)

The backorder cost per cycle under inflation is

\[
BC_f = \int_0^{T-T_1} \pi_b R(t) \, dt = \int_0^{T-T_1} \pi_b e^{R(T+T_1)} R(t) \, dt
\]  
(3.2.2.12)

3.2.3 Theoretical results

The length of the offered credit period creates two cases:

Case 1: Payment is made at or before the total depletion of inventory \( M \leq T_1 \), and

Case 2: Payment is made at after total depletion of inventory \( M > T_1 \).

**Case 1:** \( M \leq T \)

For this case the present value (at \( t = 0 \)) of interest charged rate and interest earned at time \( t \) is
\[ I_c(t) = (e^{I_c t} - 1)e^{-rt} \] and \[ I_e(t) = (e^{I_e t} - 1)e^{-rt} \] respectively.

Therefore, the interest charged during \([M, T_1]\) is given by

\[ IC_{f1} = \int_M^{T_1} CI_c(t)\ I(t) \ dt \quad (3.2.3.1) \]

And interest earned per cycle is

\[ IE_{f1} = \int_0^{M} \ C e^{Rt} I_e(t) \ R(t) \ dt \quad (3.2.3.2) \]

**Case 2: \( M > T_1 \)**

As \( M > T_1 \) retailer can pay in full at the end of permissible delay \( M \), the interest charged

\[ IC_{f2} = 0 \quad (3.2.3.3) \]

The interest earned per cycle is the interest earned during positive inventory period \([0, T_1]\) and the interest from the revenue deposited during the time period \([T_1, M]\). Hence, interest earned per cycle during inflation is

\[ IE_{12} = \int_0^{T_1} C I_e(t) R(t) \ t \ dt + (e^{(I_e (M - T_1))} - 1) \int_{T_1}^{M} C e^{Rt} R(T_1) \ dt \quad (3.2.3.4) \]

Hence total cost for both the cases \( K_{if}(T_1, T) \) (i.e. for \( i = 1, 2 \)) per cycle is given by

\[ K_{if}(T) = OC + CD_f + IHc_f + BC_f + IC_{f} - IE_{fi} \quad (3.2.3.5) \]
If there are $N$-cycles during a year then $NT = 1$. Therefore, the total cost $K_i(T_1, T)$ is given by

$$K_i(T_1, T) = K_{if}(T) \sum_{n=0}^{N-1} e^{nRT} = K_{if}(T) \left( \frac{1-e^{R}}{1-e^{RT}} \right)$$  \hspace{1cm} \text{where } i = 1, 2 \hspace{1cm} (3.2.3.6)$$

The necessary for $K_i(T_1, T)$ to be minimum is

$$\frac{\partial K_i(T_1, T)}{\partial T} = 0, \frac{\partial K_i(T_1, T)}{\partial T_1} = 0 \text{ and } \left( \frac{\partial^2 K_i(T_1, T)}{\partial T^2} \right) \left( \frac{\partial^2 K_i(T_1, T)}{\partial T_1^2} \right) - \left( \frac{\partial^2 K_i(T_1, T)}{\partial T \partial T_1} \right) > 0.$$  

The necessary condition is solved for $T_1$ and $T$ by using mathematical software.

### 3.2.4 Numerical example and observation

To derive this model considers the following parametric values in proper units:

$$[a, A, I, I_c, I_e] = [10000, 0.06, 0.12, 0.15, 0.13].$$  

With the different set of values of

$$C = \{10, 20, 30\}, R = \{0.03, 0.05, 0.07\}, \theta = \{0.15, 0.18, 0.21\}, \pi_b = \{40, 50, 70\}.$$

$$b = \{0.03, 0.04, 0.05\} \text{ and } M = \left\{ 0, \frac{15}{365}, \frac{20}{365} \right\},$$

the optimum purchase quantity and total inflated cost using above parametric values is exhibited in table 3.2.4.1 to 3.2.4.5.
**Table 3.2.4.1** optimal solution with respect to changes in $C, R$ and $A$ (case 1)

\[ b = 0.03, \pi_b = 50, M = \frac{15}{365}, \theta = 0.15 \]

<table>
<thead>
<tr>
<th>$A$</th>
<th>200</th>
<th>300</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$T_1$</td>
<td>$T$</td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
<td>0.122</td>
<td>0.129</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.128</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.135</td>
<td>0.140</td>
</tr>
<tr>
<td>20</td>
<td>0.03</td>
<td>0.086</td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.091</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.094</td>
<td>0.102</td>
</tr>
<tr>
<td>30</td>
<td>0.03</td>
<td>0.069</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.072</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.080</td>
<td>0.089</td>
</tr>
</tbody>
</table>
Table 3.2.4.2 optimal solution with respect to changes in R and M (case 1)

\[ b = 0.03, \pi_b = 50, A = 200, \theta = 0.15, C = 10 \]

<table>
<thead>
<tr>
<th>( M )</th>
<th>0</th>
<th>15/365</th>
<th>20/365</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>( T_1 )</td>
<td>( T )</td>
<td>( Q )</td>
</tr>
<tr>
<td>0.03</td>
<td>0.122</td>
<td>0.129</td>
<td>1283</td>
</tr>
<tr>
<td>0.05</td>
<td>0.127</td>
<td>0.134</td>
<td>1345</td>
</tr>
<tr>
<td>0.07</td>
<td>0.134</td>
<td>0.140</td>
<td>1407</td>
</tr>
</tbody>
</table>

Table 3.2.4.3 optimal solution with respect to changes in R and \( \theta \) (case 1)

\[ b = 0.03, \pi_b = 50, M = \frac{15}{365}, C = 10 \]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0.15</th>
<th>0.18</th>
<th>0.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>( T_1 )</td>
<td>( T )</td>
<td>( Q )</td>
</tr>
<tr>
<td>0.03</td>
<td>0.122</td>
<td>0.129</td>
<td>1294</td>
</tr>
</tbody>
</table>
Table 3.2.4.4 optimal solution with respect to changes in R and $\pi_b$ (case 1)

$$b = 0.03, \pi_b = 50, M = \frac{15}{365}, C = 10$$

<table>
<thead>
<tr>
<th>$C_b$</th>
<th>40</th>
<th>50</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>$T_1$</td>
<td>T</td>
<td>Q</td>
</tr>
<tr>
<td>0.03</td>
<td>0.122</td>
<td>0.129</td>
<td>1284</td>
</tr>
<tr>
<td>0.05</td>
<td>0.127</td>
<td>0.135</td>
<td>1356</td>
</tr>
<tr>
<td>0.07</td>
<td>0.134</td>
<td>0.141</td>
<td>1418</td>
</tr>
</tbody>
</table>

Table 3.2.4.5 optimal solution with respect to changes in R and b (case 1)

$$\theta = 0.15, \pi_b = 50, M = \frac{15}{365}, C = 10$$

<table>
<thead>
<tr>
<th>b</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Q</td>
<td>$K_v$</td>
<td>Q</td>
</tr>
</tbody>
</table>
In table-3.2.1, analysis is done with changes in unit purchase cost, inflation rate and ordering cost. It is observed that increase in inflation rate increases length of positive inventory cycle, total cycle time and procurement quantity but decreases present value of total cost of an inventory system. For fix purchase cost and inflation rate, increase in ordering cost prolongs replenishment cycle time, increases total cost significantly. For fix ordering cost, increase in purchase cost reduces positive stock time in an inventory total cycle time, Purchase units but increases present value of total cost of an inventory system significantly.

In table-3.2.2, effect of credit period and inflation rate is carried out. Increase in delay period reduces present value of total cost of an inventory system because retailer can earn more interest on generated revenue by selling products.
In table-3.2.3, effects of inflation and deterioration rate on decision variables are studied. Increase in deterioration of product decreases purchase capacity of the retailer and increase present value of total quantity.

In table-3.2.4 & 3.2.5 effects of backordered cost is worked out. When there is increment in backorder cost it will increase total cost of the retailer. The decreasing rate of demand reduces purchase quantity and total cost of an inventory system.

**3.3 Conclusions**

From both the models discussed in this chapter we conclude that the total inventory cost decreases with inflation rate while there is increment in buying power of the retailer.