CHAPTER 8

EOQ models for single vendor and multiple buyers without deterioration and with constant deterioration rate

8.0 Introduction

An integrated inventory policy for single vendor and multiple buyers is developed. The demand function is decrease with respect to time.

In section 8.1 an inventory system is developed for single vendor and N- buyers with the assumption that the inventory system deals with a single item, the demand of an item is decreasing by time t. Shortages of an item at any stage are not allowed and the replenishment rate is instantaneous. i.e. lead – time is zero. A numerical example is given to support our proposed policy. It is established numerically that the integrated approach results in significant decrease in the total cost in compared with the independent decision approach by all the buyers. The sensitivity analysis is carried out for all the parameters.
Section 8.2 is an extension of section 8.1 by allowing deterioration with the same assumption. The effects of deterioration rate and decreasing demand rate on decision variable and objective function are studied for the inventory system for single – vendor multiple buyers. Numerical example is in support of this article. Numerically here also we observed that cost is reduced in integrated policy when it is compared with independent policy of buyer. The effects of deterioration rate and decreasing demand rate on decision variable and objective function are studied.

8.1 A single vendor and multiple buyers integrated inventory policy in demand declining market

8.1.1 Assumptions and Notations

Assumptions and notations used in this chapter is as follows

8.1.1.1 Assumptions

• An inventory system of single – vendor and N buyers is considered.

• The inventory system deals with a single item.

• The demand of an item is decreasing by time t.
• Shortages of an item at any stage are not allowed.

• The replenishment rate is instantaneous. i.e. lead – time is zero.

### 8.1.1.2 Notations

\[ N \]  
Number of buyers in the system

\[ R_i(t) = a_i(1 - b_i t) \]  
Annual demand rate of \( i \)th buyer (units/unit time), where

- \( a_i \) is constant demand,
- \( b_i \) is rate of change of demand with respect to time \( t \). Also, \( a_i > 0 \) and \( 0 < b_i < 1 \) where \( i = 1,2,...,N \)

\[ T \]  
Vendor’s cycle time (decision variable)

\[ n_i \]  
Number of orders during cycle time \( T \) for the \( i \)th buyer (decision variable)

\[ I_v(t) \]  
Inventory level for vendor’s at any time \( t \), \( 0 \leq t \leq T \)

\[ I_{bi}(t) \]  
Inventory level for \( i \)th buyer at any time \( t \), \( 0 \leq t \leq \frac{T}{n_i} \)

\[ I_{mv} \]  
The maximum inventory level of vendor

\[ I_{mbi} \]  
The maximum inventory level of \( i \)th buyer
\( C_v \)  Vendor’s purchase cost per unit ($/unit)

\( C_b \)  Buyer’s purchase cost per unit ($/unit)

\( A_v \)  Vendor’s ordering cost per order cycle ($/cycle)

\( A_b \)  Buyer’s ordering cost per order cycle ($/cycle)

\( I_v \)  Vendor’s inventory carrying charge fraction per unit per time unit ($/annum)

\( I_b \)  Buyer’s inventory carrying charge fraction per unit per time unit ($/annum)

\( K_b \)  Total cost of all Buyer’s per time unit

\( K_v \)  Total cost of vendor per time unit

\( K \)  Total cost for vendor-buyers inventory system when they take decision jointly
8.1.2 Mathematical Formulation

In this section, mathematical model of an integrated inventory system for single vendor and N buyers is developed. Figure 8.1.2.1 represents time-varying inventory status for the vendor and the buyers.

![Vendor-Buyers Inventory system at any time t](image)

**Figure 8.1.2.1**: Vendor-Buyers Inventory system at any time \( t \)

From Fig.8.1.2.1, vendor’s inventory status can be represented by the following differential equation.

\[
\frac{dI_{v}(t)}{dt} = - \sum_{i=1}^{N} a_i (1 - b_i t), \quad 0 \leq t \leq T
\]  

(8.1.2.1)
During any cycle time $T$, each buyer place different order quantity and received replenishment quantity over an interval of time. Hence, for different buyers order quantity $I_{mb}$ and the number of delivery $n_i$ are different. In a given cycle $T$, from the fig. 8.1.2.1, inventory system for $i$th buyer can be expressed in the mathematical form using the following differential equation.

$$\frac{dI_{bi}(t)}{dt} = a_i (1 - b_i t), \quad 0 \leq t \leq \frac{T}{n_i} \quad \text{where, } i = 1, 2 \cdots N \quad (8.1.2.2)$$

Using various boundary conditions $I_v(T) = 0, I_{bi}(T/n_i) = 0$, the solutions of the above differential equations are

$$I_v(t) = \sum_{i=1}^{N} a_i \left[ (T - t) + \frac{b_i}{2} (T^2 - t^2) \right], \quad 0 \leq t \leq T \quad (8.1.2.3)$$

$$I_b(t) = a_i \left[ \frac{T}{n_i} - t \right] + \frac{b_i}{2} \left( \frac{T^2}{n_i^2} - t^2 \right) \quad 0 \leq t \leq \frac{T}{n_i} \quad (8.1.2.4)$$

where, $i = 1, 2 \cdots N$

Using $I_v(0) = I_{mv}$ and $I_b(0) = I_{mbi}$, the purchase quantities for the vendor and the $i$th buyer are

$$I_{mv} = \sum_{i=1}^{N} a_i \left[ T + \frac{b_i T^2}{2} \right] \quad (8.1.2.5)$$

$$I_{mbi} = a_i \left[ \frac{T}{n_i} + \frac{b_i T^2}{2n_i^2} \right] \quad \text{where, } i = 1, 2 \cdots N \quad (8.1.2.6)$$
During cycle time $T$, $i$th buyer’s inventory level is $\int_0^T l_{bi}(t) \, dt$. Therefore, average inventory level for all the buyers per time unit is $\frac{1}{T} \sum_{i=1}^N n_i \int_0^T l_{bi}(t) \, dt$. Hence, inventory holding cost for all $N$ buyers is

$$IHC_b = C_b l_b \frac{1}{T} \sum_{i=1}^N n_i \int_0^T l_{bi}(t) \, dt \quad (8.1.2.7)$$

The ordering cost for $i$th buyer is $n_i A_{bi}$. Hence, the ordering cost for all $N$ buyers per time unit is

$$OC_b = \frac{1}{T} \sum_{i=1}^N n_i A_{bi} \quad (8.1.2.8)$$

Hence, the buyer’s total cost, $K_b$ per time unit

$$K_b = IHC_b + OC_b \quad (8.1.2.9)$$

During the cycle time $[0,T]$ vendor’s average inventory level per time unit is $\frac{1}{T} \int_0^T l_v(t) \, dt$. The vendor’s inventory in the joint two-echelon inventory model is the difference between the vendor-buyers combined inventory and all buyer’s inventory. Therefore, vendor’s holding cost per time unit is

$$IHC_v = C_v l_v \left[ \frac{1}{T} \int_0^T l_v(t) \, dt - \frac{1}{T} \sum_{i=1}^N n_i \int_0^T l_{bi}(t) \, dt \right] \quad (8.1.2.10)$$

Vendor’s ordering cost per time unit is
The vendor’s total cost $K_v$ per time unit is

$$K_v = IHC_v + OC_v$$  \hspace{1cm} (8.1.2.12)\]

The joint total cost of vendor and $N$ buyers in the integrated inventory system $K$ is the sum of equations (8.1.2.9) and (8.1.2.12),

$$K = K_b + K_v$$  \hspace{1cm} (8.1.2.13)\]

Here, joint total cost $K$ is a function of discrete variable $n_i$ and continuous variable $T$,

where, $i = 1,2, \ldots, N$.

### 8.1.2 Computation Algorithm

Here, the objective is to obtain the value of $n_i$, which minimizes joint total cost $K$, where $i = 1,2, \ldots, N$. Since the number of delivery $n_i$, per order cycle $T$ is a discrete variable, the following steps can be carried out to determine value of $n_i$.

**Step1:** The necessary condition to find optimal solution is $\frac{\partial K}{\partial T} = 0$. For each $n_i$, denote order cycle $T$ by notation $T(n_i)$, where $i = 1,2, \ldots, N$. 

\[ OC_v = \frac{A_v}{T} \hspace{1cm} (8.1.2.11) \]
**Step2:** Find the optimal solution of $n_i$, and $T(n_i)$ such that, the following condition must satisfy:

$$K(n_i^* - 1, T(n_i^* - 1)) \geq K(n_i^*, T(n_i^*)) \leq K(n_i^* + 1, T(n_i^* + 1))$$

### 8.1.4 Numerical Example and Sensitivity Analysis

For simplicity here we considered the supply chain with two buyers ($N=2$) with different demands. Other parameter values considered in proper units for numerical analysis are

$$[a_1, a_2, b_1, b_2, Cv, Cb, Iv, Ib, Av, Ab1, Ab2] = [8000, 160000, 0.05, 0.05, 10, 13, 0.15, 0.30, 2000, 200, 200]$$

The numerical analysis of the integrated optimum model and independent policy lead to the following results

**Table 8.1.4.1:** optimal solution of $n_1$ and $n_2$ for integrated policy

<table>
<thead>
<tr>
<th>Integrated</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$T$</th>
<th>$K_b$</th>
<th>$K_v$</th>
<th>$K$</th>
</tr>
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<tbody>
<tr>
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<td>1</td>
<td>3</td>
<td>0.09</td>
<td>32423</td>
<td>29366</td>
<td>61790</td>
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<td>2</td>
<td>2</td>
<td>0.09</td>
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<td>29951</td>
<td>60311</td>
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Table 8.1.4.2: optimal solution of $n_1$ and $n_2$ for independent policy

<table>
<thead>
<tr>
<th>Independent</th>
</tr>
</thead>
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</tr>
<tr>
<td>3</td>
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<td>3&quot;</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

In table 8.1.4.1 and table 8.1.4.2, the optimal solution is exhibited for integrated and independent inventory policy. If both the buyers follow independent policy then
ordering policy is \((n_1 = 3, n_2 = 5)\) with total cost of $60142. If both the buyers agree to
join in the integrated system with ordering policy of \((n_1^* = 2, n_2^* = 3)\) then total integrated
cost is significantly reduce to $59271. The buyer’s cost increase when both the buyers
and vendor agree for joint decision while order cycle time \(T\) decreases. In the integrated
policy, vendor benefits $2113 and buyer looses $1242. Since, integrated strategy is
beneficial to vendor, buyers do not agree for joint decision. To attract the buyers to
cooperate in the integrated system, vendor should offer the buyer a permissible delay in
payment or some proportion of sharing of extra benefits. This integrated policy reduces
the integrated total cost defined as

\[
PETC = \frac{K(n_1, n_2) - K(n_1^*, n_2^*)}{K(n_1, n_2)} \text{ by } 1.54\%
\]

The sensitivity analysis is carried out of for all inventory model parameters for
the percentage changes -20%, -10%, 10% and 20%. Here, Percentage extra total cost

\[
PETC = \frac{K(n_1, n_2) - K(n_1^*, n_2^*)}{K(n_1, n_2)} \text{ where } K(n_1^*, n_2^*) \text{ represents the optimal value of total integrated}
cost } K \text{ and } n_1^* \text{ and } n_2^* \text{ represents optimal number of delivery for both the buyer 1 and}
buyer 2 and } K(n_1, n_2) \text{ represents the optimal value of independent cost } K \text{ and } n_1 \text{ and } n_2
represents optimal number of delivery for both the buyer 1 and buyer 2. The results of sensitivity analysis are given in Table 8.1.4.3 to 8.1.4.12.

**Table 8.1.4.3:** % changes in demand scale parameter $a_1$

<table>
<thead>
<tr>
<th>Parameter $a_1$</th>
<th>% changes</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20%</td>
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<td>3</td>
<td>57040</td>
<td>3</td>
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<td>57894</td>
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</tr>
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<td>3</td>
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<td>0.39</td>
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<td></td>
<td>20%</td>
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<td>61421</td>
<td>3</td>
<td>4</td>
<td>61620</td>
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### Table 8.1.4.4 % changes in demand scale parameter \( a_2 \)

<table>
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<th>( n_2 )</th>
<th>( K )</th>
<th>( n_1^* )</th>
<th>( n_2^* )</th>
<th>( K^* )</th>
<th>PETC(%)</th>
</tr>
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<td>62013</td>
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</tr>
<tr>
<td></td>
<td>20%</td>
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<td>3</td>
<td>62890</td>
<td>3</td>
<td>5</td>
<td>63828</td>
<td>1.49</td>
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### Table 8.1.4.5 % changes in demand rate parameter \( b \)

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<th>( n_2 )</th>
<th>( K )</th>
<th>( n_1^* )</th>
<th>( n_2^* )</th>
<th>( K^* )</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
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<td>( b )</td>
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<td>3</td>
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<td>5</td>
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Table 8.1.4.6 % changes in parameter $C_b$

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<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
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Table 8.1.4.7 % changes in parameter $C_v$

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<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
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### Table 8.1.4.8 % changes in parameter $A_{b1}$

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<th>Parameter</th>
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<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
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<tbody>
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### Table 8.1.4.9 % changes in parameter $A_{b2}$

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<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
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<tbody>
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### Table 8.1.4.10 % changes in parameter $A_v$

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<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
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### Table 8.1.4.11 % changes in parameter $I_b$

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<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_b$</td>
<td>-20%</td>
<td>2</td>
<td>3</td>
<td>55464</td>
<td>3</td>
<td>4</td>
<td>56507</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>2</td>
<td>3</td>
<td>57399</td>
<td>3</td>
<td>4</td>
<td>58049</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>3</td>
<td>4</td>
<td>61018</td>
<td>4</td>
<td>5</td>
<td>62054</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>3</td>
<td>4</td>
<td>62449</td>
<td>4</td>
<td>5</td>
<td>63283</td>
<td>1.32</td>
</tr>
</tbody>
</table>
Table 8.1.4.12 \% changes in parameter $I_v$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>%</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$K$</th>
<th>$n^*_1$</th>
<th>$n^*_2$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% changes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_v$</td>
<td>-20%</td>
<td>3</td>
<td>4</td>
<td>56494</td>
<td>4</td>
<td>5</td>
<td>57153</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>3</td>
<td>4</td>
<td>58043</td>
<td>4</td>
<td>5</td>
<td>59005</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>2</td>
<td>3</td>
<td>60378</td>
<td>3</td>
<td>4</td>
<td>61023</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>2</td>
<td>3</td>
<td>61465</td>
<td>3</td>
<td>4</td>
<td>62459</td>
<td>1.60</td>
</tr>
</tbody>
</table>

From the Table 8.1.4.3 to 8.1.4.12, it is observed that the range of $PETC$ is from 0.32 to 2.23. The value of $PETC$ is very sensitive to all the model parameters. Whereas optimal ordering policy are not sensitive to all the parameters.

Next we generalized the model by applying deterioration to the produced item.

8.2 A deteriorating inventory model for single vendor and multiple buyers in demand declining market

Assumption and Notations are as follows
8.2.1 Assumptions and notations

8.2.1.1 Assumptions

The mathematical model is developed under the following assumptions.

- An inventory system of single – vendor and N buyers is considered.
- The inventory system deals with a single item.
- The demand is decreasing function of time $t$.
- Shortages of an item at any stage are not allowed.
- The replenishment rate is instantaneous. i.e. lead – time is zero.
- The deteriorated units can neither be repaired nor replaced during the cycle time.

8.2.1.2 Notations

The proposed model is derived using the following notations.

\[ N \] Number of buyers in the system

\[ R_i(t) = a_i(1 - b_i t). \] Annual demand rate of $i^{th}$ buyer (units/unit time) where $a_i$ is constant demand, $b_i$ is rate of change of demand with respect to time $t$. Also, $a_i > 0$ and $0 < b_i < 1$ where $i = 1,2 \cdots N$
$T$  
Vendor’s cycle time (decision variable)

$n_i$  
Number of orders during cycle time $T$ for the $i^{th}$ buyer (decision variable)

$\theta$  
Deterioration rate ($0 < \theta < 1$) per time unit ($$/unit)

$I_v(t)$  
Inventory level for vendor’s at any time $t$, $0 \leq t \leq T$

$I_{bi}(t)$  
Inventory level for $i^{th}$ buyer at any time $t$, $0 \leq t \leq \frac{T}{n_i}$

$I_{mv}$  
The maximum inventory level of vendor

$I_{mbi}$  
The maximum inventory level of $i^{th}$ buyer

$C_v$  
Vendor’s purchase cost per unit ($$/unit)

$C_b$  
Buyer’s purchase cost per unit ($$/unit)

$DC_v$  
Vendor’s deteriorating cost per unit ($$/unit)

$DC_b$  
 Buyer’s deteriorating cost per unit ($$/unit)

$I_v$  
Vendor’s inventory carrying charge fraction per unit per time unit

($$/annum)$
$I_b$ Buyer’s inventory carrying charge fraction per unit per time unit
($/annum)$

$K_b$ Total cost of all Buyer’s per time unit

$K_v$ Total cost of vendor per time unit

$K$ Total cost for vendor-buyers inventory system when they take decision jointly

### 8.2.2 Mathematical Algorithm

Let $I_{bi}(t)$ is inventory level for buyer $i, i = 1, 2 \cdots N$ at any instant of time, $0 \leq t \leq \frac{T}{n_i}$ and Let $I_{vi}(t)$ is inventory level for vendor at any instant of time, $0 \leq t \leq T$.

The inventory level depletes due to demand and deterioration of items. The differential equation for vendor and buyer’s are given by

$$\frac{dI_{vi}(t)}{dt} + \theta I_{vi}(t) = -\sum_{i=1}^{N} a_i (1 - b_i t), \quad 0 \leq t \leq T \quad (8.2.2.1)$$

$$\frac{dI_{bi}(t)}{dt} + \theta I_{bi}(t) = -a_i (1 - b_i t), \quad 0 \leq t \leq \frac{T}{n_i} \quad \text{where, } i = 1, 2 \cdots N \quad (8.2.2.2)$$
When N=1 it is simplest case i.e. for single vendor and single buyer.

![Vendor-Buyers Inventory system at any time t](image)

**Figure 8.1.2.1: Vendor-Buyers Inventory system at any time t**

Using various boundary conditions \( I_v(T) = 0, I_{bi}(\frac{T}{n}) = 0 \), the solutions of the above differential equations are

\[
I_v(t) = \sum_{i=1}^{N} a_i \left[ (e^{\theta(T-t)}-1) \left( \frac{1}{\theta} + \frac{b}{\theta^2} \right) - \frac{b}{\theta} (T-t) \right], \quad 0 \leq t \leq T \tag{8.2.2.3}
\]

\[
I_{bi}(t) = a_i \left[ \left( e^{\theta(\frac{T}{n}-t)}-1 \right) \left( \frac{1}{\theta} + \frac{b}{\theta^2} \right) - \frac{b}{\theta} \left( \frac{T}{n_i} - t^2 \right) \right], \quad 0 \leq t \leq \frac{T}{n_i} \tag{8.2.2.4}
\]

where, \( i = 1, 2 \cdots N \)
Using $I_v(0) = I_{mv}$ and $I_{bi}(0) = I_{mbi}$, the purchase quantities for the vendor and the $i$th buyer are

$$I_{mv} = \sum_{i=1}^{N} a_i \left[ \left( e^{\theta T - 1} \right) \left( \frac{1}{\theta} + \frac{b}{\theta^2} \right) - \frac{b}{\theta^2} T \right]$$  \hspace{1cm} (8.2.2.5)$$

$$I_{mbi} = a_i \left[ \left( e^{\frac{T}{n_i} \theta} - 1 \right) \left( \frac{1}{\theta} + \frac{b}{\theta^2} \right) - \frac{b}{\theta^2} \frac{T}{n_i} \right]$$  \hspace{1cm} (8.2.2.6)$$

where, $i = 1, 2 \cdots N$

During cycle time $[0, T]$, inventory level is $\int_0^T I_{bi}(t) \, dt$ for buyer $i$. Therefore, average inventory level for all the buyers per time unit is $\frac{1}{T} \sum_{i=1}^{N} n_i \int_0^T I_{bi}(t) \, dt$. While vendor’s average inventory level per time unit is $\frac{1}{T} \int_0^T I_v(t)$. Hence, yearly inventory holding cost for all $N$ buyers and vendor is given by

$$IHC_b = C_b I_b \frac{1}{T} \sum_{i=1}^{N} n_i \int_0^T I_{bi}(t) \, dt$$  \hspace{1cm} (8.2.2.7)$$

and

$$IHC_v = C_v I_v \left[ \frac{1}{T} \int_0^T I_v(t) \, dt \right] - \frac{1}{T} \sum_{i=1}^{N} n_i \int_0^T I_{bi}(t) \, dt$$  \hspace{1cm} (8.2.2.8)$$

Respectively.

Annual deterioration rate for all buyers and vendor during $[0, T]$ are
\[ DC_b = C_b \theta \frac{1}{T} \sum_{i=1}^{N} n_i \int_{0}^{T} I_{bi}(t) \, dt \]  
(8.2.2.9)

and

\[ DC_v = C_v \theta \left[ \frac{1}{T} \int_{0}^{T} I_v(t) \, dt - \frac{1}{T} \sum_{i=1}^{N} n_i \int_{0}^{T} I_{bi}(t) \, dt \right] \]  
(8.2.2.10)

respectively.

And the setup cost for all buyers and vendor during \([0, T]\) are

\[ OC_b = \frac{1}{T} \sum_{i=1}^{N} n_i A_{bi} \]  
(8.2.2.11)

and

\[ OC_v = \frac{A_v}{T} \]  
(8.2.2.12)

respectively.

Hence, the buyer’s total cost, \(K_b\) per time unit

\[ K_b = IC_b + OC_b + DC_b \]  
(8.2.2.13)

And the vendor’s total cost, \(K_v\) per time unit

\[ K_v = IC_v + OC_v + DC_v \]  
(8.2.2.14)

The integrated cost of vendor and \(N\) buyers in \(K\) is the sum of eq (8.2.2.13) and (8.2.2.14).
Here, joint total cost $K$ is a function of discrete variable $n_i$ and continuous variable $T$,

where, $i = 1, 2 \cdots N$.

**8.2.3 Computation Algorithm:**

Here, the objective is to obtain the value of $n_i$, which minimizes integrated cost $K$, where $i = 1, 2 \cdots N$. Since the number of delivery $n_i$ per order cycle $T$ is a discrete variable, the following steps can be carried out to determine value of $n_i$.

**Step1:** To derive optimal solution, the necessary condition is $\frac{\partial K}{\partial T} = 0$. For each $n_i$, denote order cycle $T$ by notation $T(n_i)$, where $i = 1, 2 \cdots N$.

**Step2:** Find the optimal solution of $n_i$ such that, the following condition must satisfy:

$$K(n_i^* - 1, T(n_i^* - 1)) \geq K(n_i^*, T(n_i^*)) \leq K(n_i^* + 1, T(n_i^* + 1))$$

Such that, $K(n_i^*, T(n_i^*))$ is the optimal value of integrated cost.

**8.2.4 Numerical Example and Sensitivity Analysis:**

To validate the proposed model, let us consider following example by considering two buyers ($N = 2$) with their different demand.

Other parameter values considered in proper units for numerical analysis are
\[ [a_1, a_2, b_1, b_2, C_p, C_b, I_p, I_b, A_p, \theta, A_{b1}, A_{b2}] = \]

\[ [8000, 160000, 0.05, 0.05, 10, 13, 0.15, 0.30, 2000, 0.1, 200, 200] \]

The numerical analysis of the integrated optimum model and independent policy lead to the following results

**Table 8.2.4.1**: optimal solution of \( n_1 \) and \( n_2 \) for integrated policy

<table>
<thead>
<tr>
<th>Integrated</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( T )</th>
<th>( K_b )</th>
<th>( K_v )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.306</td>
<td>6895261</td>
<td>8952</td>
<td>6904214</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.314</td>
<td>6894611</td>
<td>9221</td>
<td>6903833</td>
<td></td>
</tr>
<tr>
<td>2'</td>
<td>3'</td>
<td>0.336</td>
<td>6894112</td>
<td>9671</td>
<td>6903783'</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.353</td>
<td>6894087</td>
<td>9930</td>
<td>6904017</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.354</td>
<td>6894068</td>
<td>9939</td>
<td>6904007</td>
<td></td>
</tr>
</tbody>
</table>
### Table 8.2.4.2: optimal solution of $n_1$ and $n_2$ for independent policy

<table>
<thead>
<tr>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$T$</th>
<th>$K_b$</th>
<th>$K_v$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>0.353</td>
<td>6894087</td>
<td>9930</td>
<td>6904017</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.354</td>
<td>6894068</td>
<td>9939</td>
<td>6904007</td>
</tr>
<tr>
<td>3'</td>
<td>4'</td>
<td>0.371</td>
<td>6893933*</td>
<td>10262</td>
<td>6904196*</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.385</td>
<td>6894010</td>
<td>10491</td>
<td>6904501</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.384</td>
<td>6894097</td>
<td>10716</td>
<td>6904813</td>
</tr>
</tbody>
</table>

In table 8.2.4.1 and table 8.2.4.2, the optimal solution is exhibited for integrated and independent inventory policy. If both buyers and the vendor are agree for joint policy and if they follow integrated policy instead of independent policy then ordering policy is on $(n_1^* = 2, n_2^* = 3)$ instead of $(n_1 = 3, n_2 = 4)$. Optimum cost for both integrated and independent policy is 6903783 and 6904196 respectively. The buyer's cost increase when both the buyers and vendor agree for joint decision while order cycle time $T$ decreases. In the integrated policy, vendor benefits $591 and buyer looses...
$179. Since the vendor is the winner in the integrated policy, it is logical for him to offer
some incentive for the buyers to accept the integrated policy. To attract buyer’s the
vendor should be willing to offer some discount up to certain percentage of his extra
benefit due to the integrated approach and due to this strategy long term relation is
maintained. This integrated policy reduces the integrated total cost defined

\[ PETC = \frac{K(n_1, n_2) - K(n_1^*, n_2^*)}{K(n_1, n_2)} \]

by 0.06 %.

The sensitivity analysis is carried out for all inventory model parameters for the
percentage changes -20%, -10%, 10% and 20%. Here, Percentage extra total cost

\[ PETC = \frac{K(n_1, n_2) - K(n_1^*, n_2^*)}{K(n_1, n_2)} \]

where \( K(n_1^*, n_2^*) \) represents the optimal value of total integrated
cost \( K \) and \( n_1^* \) and \( n_2^* \) represents optimal number of delivery for both the buyer 1 and
buyer 2 and \( K(n_1, n_2) \) represents the optimal value of independent cost \( K \) and \( n_1 \) and \( n_2 \)
represents optimal number of delivery for both the buyer 1 and buyer 2. The results of
sensitivity analysis are given in Table 8.2.4.3 to 8.2.4.13. From the Table 8.2.4.3 to
8.2.4.13, it is observed that the range of \( PETC \) is from 0.00 to 0.012.
### Table 8.2.4.3 % changes in demand scale parameter $a_1$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-20%</td>
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<td>3</td>
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<td>3</td>
<td>4</td>
<td>6444492</td>
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</tr>
<tr>
<td></td>
<td>-10%</td>
<td>2</td>
<td>3</td>
<td>6673928</td>
<td>3</td>
<td>4</td>
<td>6674347</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>2</td>
<td>3</td>
<td>7133633</td>
<td>3</td>
<td>4</td>
<td>7133883</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>20%</td>
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<td>3</td>
<td>7363477</td>
<td>3</td>
<td>4</td>
<td>7363880</td>
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### Table 8.2.4.4 % changes in demand scale parameter $a_2$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$n_1$</th>
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<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2$</td>
<td>-20%</td>
<td>2</td>
<td>2</td>
<td>5984471</td>
<td>3</td>
<td>4</td>
<td>5984845</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>2</td>
<td>3</td>
<td>6444139</td>
<td>3</td>
<td>4</td>
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<td>0.006</td>
</tr>
<tr>
<td></td>
<td>10%</td>
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<td>3</td>
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<td>3</td>
<td>4</td>
<td>7363844</td>
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</tr>
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<td>3</td>
<td>7823021</td>
<td>3</td>
<td>4</td>
<td>7823475</td>
<td>0.006</td>
</tr>
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</table>
### Table 8.2.4.5 % changes in demand rate parameter $b$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>-20%</td>
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<td>3</td>
<td>6452199</td>
<td>3</td>
<td>4</td>
<td>6452702</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>2</td>
<td>3</td>
<td>6678303</td>
<td>3</td>
<td>4</td>
<td>6678759</td>
<td>0.006</td>
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<td>10%</td>
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<td>2</td>
<td>7128137</td>
<td>3</td>
<td>4</td>
<td>7128563</td>
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</tr>
<tr>
<td></td>
<td>20%</td>
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<td>2</td>
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<td>2</td>
<td>2</td>
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</table>

### Table 8.2.4.6 % changes in parameter $c_b$

<table>
<thead>
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<th>Parameter</th>
<th>% changes</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_b$</td>
<td>-20%</td>
<td>2</td>
<td>2</td>
<td>5525385</td>
<td>2</td>
<td>3</td>
<td>5525531</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>2</td>
<td>2</td>
<td>6214618</td>
<td>3</td>
<td>4</td>
<td>6215177</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>2</td>
<td>3</td>
<td>7592892</td>
<td>3</td>
<td>4</td>
<td>7593071</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>2</td>
<td>3</td>
<td>8281990</td>
<td>3</td>
<td>4</td>
<td>8282216</td>
<td>0.003</td>
</tr>
</tbody>
</table>
**Table 8.2.4.7 % changes in parameter $C_v$**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$K$</th>
<th>$n_1^*$</th>
<th>$n_2^*$</th>
<th>$K^*$</th>
<th>PETC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_v$</td>
<td>-20%</td>
<td>2</td>
<td>3</td>
<td>6903017</td>
<td>3</td>
<td>4</td>
<td>6903188</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>2</td>
<td>3</td>
<td>6903405</td>
<td>3</td>
<td>4</td>
<td>6903700</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>2</td>
<td>2</td>
<td>6904116</td>
<td>3</td>
<td>4</td>
<td>6904675</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>2</td>
<td>2</td>
<td>6904393</td>
<td>3</td>
<td>4</td>
<td>6905141</td>
<td>0.011</td>
</tr>
</tbody>
</table>

**Table 8.2.4.8 % changes in parameter $A_{b1}$**

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>$n_2$</th>
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<th>$n_2^*$</th>
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**Table 8.2.4.9** % changes in parameter $A_{b2}$

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<th>PECT(%)</th>
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**Table 8.2.4.10** % changes in parameter $A_v$

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<th>$n_2^*$</th>
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<th>PECT(%)</th>
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<th>$n_2^*$</th>
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<th>PETC(%)</th>
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### Table 8.2.4.12% changes in parameter $I_v$

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<th>$n_2^*$</th>
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<th>PETC(%)</th>
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Table 8.2.4.13 % changes in parameter $\theta$

<table>
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<th>$n_2^*$</th>
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8.3 Conclusion

In this study, a mathematical model for a single vendor-multi buyer’s inventory system with deterioration and without deterioration is developed to study an integrated optimal strategy when demand is decreases by time. It can be seen from numerical analysis that the integrated policy is beneficial to decrease the total cost of an inventory system without considering vendor’s prospective. However cost of all the buyers increases significantly in integrated policy in compared with the independent policy. Therefore, an incentive in the form of quantity discount or cost reduction to induce the
buyer to cooperate must be incorporated into the system to make it more realistic.

Future research on single vendor-multi buyer’s inventory system can be focused considering finite production rate with and without deterioration in the inventory system.