CHAPTER 7

Integrated Vendor-Buyer inventory model in declining market when the two level trade credit policy is offered

7.0 Introduction

In this chapter, we discussed the co-ordinate inventory model with allowable trade credit in supply chain management where Customer’s demand is sensitive with respect to time and buyer’s price. The models consider the two-level trade credit policy i.e. vendor offers some credit period to buyer and buyer in turn offers partial credit period to the customer. An iterative procedure is developed for the integrated models to determine the buyer’s optimal price and production/order strategy. It is observed that even if buyer’s share is less, the total joint profit of the supply chain increases. To counter balance, the buyer’s loss due to the joint decision, a negotiation scheme is
introduce to distribute the extra profit between the vendor and the buyer. A numerical example and sensitivity analysis are given to validate the proposed problem.

### 7.1 Assumptions and Notations

Assumptions and notations used in this chapter is as follows

#### 7.1.1 Assumptions

- The supply chain consist single-vendor and single-buyer.
- The replenishment rate is instantaneous and lead-time is zero or negligible.
- Shortages are not allowed.
- The production rate is finite and demand rate is decreasing function of time and buyer’s retail price.
- The two-level trade credit policy is considered in which the vendor offers the buyer with a credit period M and the buyer also offers a credit period N to the customers. The customers pay off the buyer when the credit period offered by the buyer is due.
• The buyer deposits the generated revenue in an interest bearing account at the annual interest rate \( I_e \) before the payment due. At the payment time, the buyer settles the accounts against the purchases made to the vendor. The buyer pays interest rate \( I_c \) (with \( I_e < I_c \)) for the unsold stock to the vendor. During the allowable credit period, the vendor has an opportunity interest loss with the annual rate \( I_o \).

### 7.1.2 Notations

Model \( i, i = 1, 2, 3 \).

- \( A_b \) Buyer’s set-up cost per order
- \( A_v \) Vendor’s set-up cost per production run
- \( F \) Fixed process cost for vendor to dealing with each other
- \( I_b \) Buyer’s holding cost rate per unit time excluding interest charges
- \( I_v \) Vendor’s holding cost rate per unit per unit time
- \( C_v \) Vendor’s production cost per unit item
- \( C_{bi} \) The buyer’s procurement cost per unit time in Model \( i \)
\( C_{bd} \)  
Buyer’s purchase cost per unit when the vendor offers the pricing strategy in the negotiation scheme

\[ C_{bd} = C_b - d, \quad 0 < d < C_{bi}, \quad d \text{ denotes discount in unit cost} \]

\( P_{bi} \)  
Buyer’s selling price per unit time in model \( i \) \( (P_{bi} > C_b > C_{bd} > C_v) \)

\( R(t, P_{bi}) \)  
Annual demand rate of the buyer which is a decreasing function of time and the buyer selling price

\[ R(t, P_{bi}) = a(1 - bt) - P_{bi}^\eta \quad \text{where} \quad a > 0 \quad \text{is scale parameter for demand,} \quad 0 < b < 1 \quad \text{denotes the rate of change of demand and} \quad \eta > 1 \quad \text{denotes mark up.} \quad \gamma \quad \text{The ratio of demand rate;} \quad R(t, p) \quad \text{to the production rate} \quad P(t, P_{bi}), \quad \text{where} \quad 0 < \gamma < 1 \]

\( T_{bi} \)  
Replenishment time interval of the buyer in years for Model \( i \)

\[
T_{bi} = \begin{cases} 
T_{bi1}, & \text{if } M \geq N \text{ and } T_{bi} + N \geq M \\
T_{bi2}, & \text{if } M \geq N \text{ and } T_{bi} + N < M \\
T_{bi3}, & \text{if } M \geq N 
\end{cases}
\]

\( Q_i \)  
Procurement quantity for Model \( i \)

\( M \)  
Credit period offered by the vendor to the buyer

\( N \)  
Credit period offered by the buyer to the customer
\[ I_e \] Annual interest rate of deposit for buyer

\[ I_c \] Annual interest charge to be paid per $ in stock per year

\[ I_o \] Annual interest rate for the vendor’s opportunity interest loss due to the delay in payment

\[ n_i \] Number of buyer’s order during the vendor’s production run in Model \( i \)

\[ TPB_i(T_{bi}, P_{bi}) \] total profit per unit time for the buyer (also denoted by \( TPB_i \)) in Model \( i \)

\[
TPB_i \begin{cases} 
T_{PB_{i1}}, & \text{if } M \geq N \text{ and } T_{bi} + N \geq M \\
T_{PB_{i2}}, & \text{if } M \geq N \text{ and } T_{bi} + N < M \\
T_{PB_{i3}}, & \text{if } M \leq N
\end{cases}
\]

\[ TPV_i(T_{bi}, n_i, P_{bi}) \] Total profit per unit time for the vendor (also denoted by \( TPV_i \)), in model \( i \)

\[ TP_i(T_{bi}, n_i, P_{bi}) \] Total profit per unit time by adding \( TPB_i \) and \( TPV_i \)

(also denoted by \( TP_i \))
\[
\alpha = \begin{cases} 
TP_{t1}, & \text{if } M \geq N \text{ and } T_{bl} + N \geq M \\
TP_{t2}, & \text{if } M \geq N \text{ and } T_{bl} + N < M \\
TP_{t3}, & \text{if } M \leq N 
\end{cases}
\]

Extra profit sharing negotiation factor between the vendor and the buyer.

### 7.2 Mathematical formulation

The rate of change of inventory level at any instant of time is governed by the differential equation

\[
\frac{dl(t)}{dt} = a(1 - bt)P^{-\eta}, \ 0 \leq t \leq T \tag{A.1}
\]

with \( l(T) = 0 \) and \( l(0) = Q \). The solution of given differential equation is

\[
l(t) = \left[a(t - T) - \frac{b}{2}(t^2 - T^2)\right]P^{-\eta} \tag{A.2}
\]

Using \( l(0) = Q \), the purchase quantity; Q is given by

\[
Q = \left[-aT + \frac{b}{2}T^2\right]P^{-\eta}. \tag{A.3}
\]

In this section, three models will be developed. First, we derive the vendor's profit model which is common to all the three proposed models.
Vendor’s profit model

The buyer offers $Q_i$-units for $n_i$-times during the vendor’s production run. Consequently, the vendor’s produces $n_iQ_i$-units per production rate $p(t, P_{bi})$ the ratio of demand rate $R(t, P_{bi})$ is $\gamma$, where $0 < \gamma < 1$.

The related cost components of profit per unit time in the vendor’s profit model are as follows.

Set-up cost for the vendor $= \frac{A_v}{n_i T_{bi}}$ (7.2.1)

Process cost to vendor for dealing with buyer $= \frac{n_i F}{n_i T_{bi}} = \frac{F}{T_{bi}}$ (7.2.2)

Following Pan and Yang (2002), inventory holding cost per unit time is

$$= C_v I_v \left[ (n_i - 1)(1 - \gamma) + \gamma \right] a P_{bi}^{-\eta} \frac{n_i T_{bi}}{z} \left( 1 - \frac{2b}{3} n_i T_{bi} \right),$$ (7.2.3)

using (A.3), $Q_i = a P_{bi}^{-\eta} n_i T_{bi} \left( 1 - \frac{b}{2} n_i T_{bi} \right)$ (7.2.4)

Opportunity interest loss per unit time $= \phi_{lo}^M Q_i = a P_{bi}^{-\eta} C_b I_o M \left( 1 - \frac{b}{2} n_i T_{bi} \right)$ (7.2.5)

Revenue generated per unit time for the vendor is

$$\frac{(C_b - C_v) Q_i}{n_i T_{bi}} = (C_b - C_v) a P_{bi}^{-\eta} \left( 1 - \frac{b}{2} T_v \right)$$ (7.2.6)

Therefore the total profit per unit time for the vendor is
$$TPV_i = \text{revenue generated per unit time - total cost per unit time}$$

$$= (C_b - C_v) aP_{b1}^{-\eta} \left(1 - \frac{b}{2} T_v\right) - \frac{A_v}{n_i T_{b1}} - \frac{F}{T_{b1}} - aP_{b1}^{-\eta} C_b I_o M \left(1 - \frac{b}{2} n_i T_{b1}\right)$$

$$- C_v I_v \left[(n_i - 1)(1 - \gamma) + \gamma aP_{b1}^{-\eta} \frac{n_i T_{b1}}{2} \left(1 - \frac{2}{3} n_i T_{b1} \right)\right]$$

$$= \frac{A_b}{T_{b1}}$$

$$\text{Inventory holding cost per time unit} = C_b I_b aP_{b1}^{-\eta} \left(1 - \frac{2}{3} T_{b1}\right).$$

Interest earned and interest charges are to be computed based on the lengths of $T_{b1}, M$ and $N$ as follows. Different conditions may arise due to the scenario of the two-level trade credit policy.

**Case1: $M \geq N$**

**Condition1: Case1: $T_{b1} + N \geq M$ (as shown in fig-7.2.1)**
In the two-level trade credit policy the customer could settle for account when credit period for the buyer is due. In other words, the period for the customer to settle the account with the buyer is $[N, T_{b1} + N]$. The buyer deposits the revenue generated to earn the interest before the credit period offered by the vendor is due. The interest earned per unit time is

$$\frac{P_{b1}l_e}{T_{b1}} \int_0^{M-N} t R(P_{b1}, t) \, dt = \frac{a P_{b1}^{-\eta+1} l_e}{T_{b1}} \left[ \frac{(M-N)^2}{2} - \frac{b(M-N)^3}{6} \right] \quad (7.2.1.1)$$

And interest charged per unit time during $[M, T_{b1} + N]$ is

$$\frac{C h l_e a P_{b1}^{-\eta}}{T_{b1}} \left[ \frac{(T_{b1} + N - M)^2}{2} - \frac{b(T_{b1} - N + M)^3}{6} \right] \quad (7.2.1.2)$$
**Condition 2:** \( T_{b1} + N < M \) (as shown in fig 7.2.2)

**Fig. 7.2.2** Interest earned under \( M \geq N \) and \( T_{b1} < M \)

Here, the customer will settle the account with the buyer when the credit period offered by the buyer is due. The interest earned per unit time during \([N, M]\) by the buyer is

\[
a P_{b1}^{-\eta+1} I_e \left[ M - N - \frac{T_{b1}}{2} - \frac{bT_{b1}^2}{6} - b(M - N - T_{b1})T_{b1} \right].
\]

The interest charges for the buyer is zero.

**Case 2:** \( N \geq M \) (as shown in fig 7.2.3)
The buyer does not have sufficient amount in his account to pay for the purchased units. Hence the interest charged per unit time is

$$a P_{b1}^{-\eta} C_{b} I_{c} \left[ M - N - \frac{T_{b1}}{2} - b T_{b1} - b \left( N - M - \frac{5T_{b1}}{12} \right) \right]$$  \hspace{1cm} (7.2.1.3)

and the interest earned is zero.

Based on the above cases, the buyer’s total profit per unit is as follows.

**Case 1:** $M \geq N$

**Condition 1:** $T_{b1} + N \geq M$

$$TPB_{11} = (P_{b1} - C_{b}) a P_{b1}^{-\eta} \left( 1 - \frac{b}{2} T_{b1} \right) - \frac{A_{b}}{T_{b1}} - C_{b} I_{b} a P_{b1}^{-\eta} \frac{T_{b1}}{2} \left( 1 - \frac{2b}{3} T_{b1} \right)$$

$$- \frac{C_{b} I_{c} a P_{b1}^{-\eta+1}}{T_{b1}} \left( \frac{(T_{b1}+N-M)^2}{2} - \frac{b(T_{b1}+N-M)^3}{6} \right) + \frac{a I_{b} P_{b1}^{-\eta+1} I_{c}}{T_{b1}} \left( \frac{(M-N)^2}{2} - \frac{b(M-N)^3}{6} \right)$$  \hspace{1cm} (7.2.1.4)

**Condition 2:** $T_{b1} + N < M$
\[ TP_{12} = (P_{b1} - C_b)aP_{b1}^{-\eta} \left( 1 - \frac{b}{2} T_{b1} \right) - \frac{A_b}{T_{b1}} - C_b I_b aP_{b1}^{-\eta} T_{b1} \left( 1 - \frac{2b}{3} T_{b1} \right) \]

\[ + \frac{aP_{b1}^{-\eta+1} I_e}{T_{b1}} \left[ M - N - \frac{T_{b1}}{2} - \frac{bT_{b1}^2}{6} - b(M - N - T_{b1}) T_{b1} \right] \text{ (7.2.1.5)} \]

**Case 2: \( N \geq M \)**

\[ TP_{13} = (P_{b1} - C_b)aP_{b1}^{-\eta} \left( 1 - \frac{b}{2} T_{b1} \right) - \frac{A_b}{T_{b1}} - C_b I_b aP_{b1}^{-\eta} T_{b1} \left( 1 - \frac{2b}{3} T_{b1} \right) \]

\[ + aP_{b1}^{-\eta+1} C_b I_c \left[ M - N - \frac{T_{b1}}{2} - bT_{b1} - \left( N - M - \frac{5T_{b1}}{12} \right) \right] \text{ (7.2.1.6)} \]

**7.2.2 Model 2: Integrated vendor-buyer model:**

In integrated policy, the vendor and buyer agrees to determine the optimal production and order policy which maximizes profit jointly. Similar to model 1, the total profit per unit time in model 2 is developed as follows.

**Case 1: \( M \geq N \)**

**Condition 1: \( T_{b2} + N \geq M \)**

\[ TP_{21} = TPB_{21} + TVP_{2} \]

\[ = (P_{b2} - C_v) aP_{b2}^{-\eta} \left( 1 - \frac{b}{2} T_{b2} \right) - \frac{A_b}{T_{b2}} - C_b I_b aP_{b2}^{-\eta} T_{b2} \left( 1 - \frac{2b}{3} T_{b2} \right) - \frac{A_v}{n_{2T_{b2}}} - \frac{F}{T_{b2}} \]

\[ - \frac{C_b I_c aP_{b2}^{-\eta+1}}{T_{b2}} \left[ \left( \frac{T_{b2} + N - M}{2} \right) - \frac{b(T_{b2} + N - M)^3}{6} \right] - \frac{aP_{b2}^{-\eta+1} I_e}{T_{b2}} \left[ \frac{(M - N)^2}{2} - \frac{b(M - N)^3}{6} \right] \]

\[ - C_v I_v \left[ (n_{2} - 1)(1 - \gamma) + \gamma \right] aP_{b2}^{-\eta} \frac{n_{2T_{b2}}}{2} \left( 1 - \frac{2b}{3} n_{2T_{b2}} \right) - aP_{b2}^{-\eta} C_v I_o M \left( 1 - \frac{b}{2} n_{2T_{b2}} \right) \]

\[ (7.2.2.1) \]
Condition 2: $T_{b2} + N < M$

$$TP_{22} = TPB_{22} + TPV_2$$

$$= (P_{b2} - C_v) aP_{b2}^{-\eta} \left(1 - \frac{b}{2} T_{b2}\right) - \frac{A_b}{T_{b2}} - C_b I_b aP_{b2}^{-\eta} T_{b2} \left(1 - \frac{2b}{3} T_{b2}\right) - \frac{A_v}{n_2 T_{b2}} - \frac{F}{T_{b2}}$$

$$- \left[\frac{aP_{b2}^{-\eta+1} I_c}{T_{b2}} \left[M - N - \frac{T_{b2}}{2} - \frac{b T_{b2}^2}{6} - b(N - M - T_{b2} T_{b2})\right] - aP_{b2}^{-\eta} C_b I_o M \left(1 - \frac{b}{2} n_2 T_{b2}\right)$$

$$+ (C_b - C_v) aP_{b2}^{-\eta} \left(1 - \frac{b}{2} n_2 T_{b2}\right) - C_v I_v [(n_2 - 1)(1 - \gamma) + \gamma] aP_{b2}^{-\eta} \left(1 - \frac{2b}{3} n_2 T_{b2}\right)$$

(7.2.2.2)

Case 2: $N \geq M$

$$TP_{23} = TPB_{23} + TPV_3$$

$$= (P_{b2} - C_b) aP_{b2}^{-\eta} \left(1 - \frac{b}{2} T_{b2}\right) - \frac{A_b}{T_{b2}} + (C_b - C_v) aP_{b2}^{-\eta} \left(1 - \frac{b}{2} n_2 T_{b2}\right) - \frac{A_v}{n_2 T_{b2}} - \frac{F}{T_{b2}}$$

$$- C_b I_b aP_{b2}^{-\eta} T_{b2} \left(1 - \frac{2b}{3} T_{b2}\right) - aP_{b2}^{-\eta+1} C_b I_c \left[M - N - \frac{T_{b2}}{2} - bT_{b2} - b(N - M - \frac{5T_{b2}}{12})\right]$$

$$- C_v I_v [(n_2 - 1)(1 - \gamma) + \gamma] aP_{b2}^{-\eta} \left(1 - \frac{2b}{3} n_2 T_{b2}\right) - aP_{b2}^{-\eta} C_b I_o M \left(1 - \frac{b}{2} n_2 T_{b2}\right)$$

(7.2.2.3)

7.2.3 Model 3: Integrated vendor-buyer model with a negotiation factor

Here, we consider that the vendor gives the price discount to the buyer through negotiation to counterbalance for the latter’s loss. The total profit per unit time is formulated as follows.
Case 1: \( M \geq N \)

Condition 1: \( T_{b3} + N \geq M \)

\[
TP_{31} = (P_{b3} - C_v) aP_{b3}^{-\eta} \left(1 - \frac{b}{2} T_{b3}\right) - \frac{A_b}{T_{b3}} - \frac{A_v}{n_3 T_{b3}} - \frac{F}{T_{b3}} - C_{bd} I_b aP_{b3}^{-\eta} \frac{T_{b3}}{2} \left(1 - \frac{2b}{3} T_{b3}\right) - \frac{C_{bd} I_c aP_{b3}^{-\eta+1}}{T_{b3}} \left[\frac{(T_{b3} + N - M)^2}{2} - \frac{b(T_{b3} + N - M)^3}{6}\right] + aP_{b3}^{-\eta+1} I_e \left[\frac{(M - N)^2}{2} - \frac{b(M - N)^3}{6}\right]

- C_v I_v [(n_3 - 1)(1 - \gamma) + \gamma] aP_{b1}^{-\eta} \frac{n_3 T_{b3}}{2} \left(1 - \frac{2b}{3} n_3 T_{b3}\right) - aP_{b3}^{-\eta} C_v I_o M \left(1 - \frac{b}{2} n_3 T_{b3}\right)

(7.2.3.1)

Condition 2: \( T_{b3} + N < M \)

\[
TP_{32} = (P_{b3} - C_v) aP_{b3}^{-\eta} \left(1 - \frac{b}{2} T_{b3}\right) - \frac{A_b}{T_{b3}} - \frac{A_v}{n_3 T_{b3}} - \frac{F}{T_{b3}} - C_{bd} I_b aP_{b3}^{-\eta} \frac{T_{b3}}{2} \left(1 - \frac{2b}{3} T_{b3}\right)

- aP_{b3}^{-\eta+1} I_e \left[M - N - \frac{T_{b3}}{2} - \frac{bT_{b3}^2}{6} - b(M - N - T_{b3}) T_{b3}\right] - aP_{b3}^{-\eta} C_{bd} I_o M \left(1 - \frac{b}{2} n_3 T_{b3}\right)

- C_v I_v [(n_3 - 1)(1 - \gamma) + \gamma] aP_{b3}^{-\eta} \frac{n_3 T_{b3}}{2} \left(1 - \frac{2b}{3} n_3 T_{b3}\right)

(7.2.3.2)

Case 2: \( N > M \)

\[
TP_{33} = (P_{b3} - C_v) aP_{b3}^{-\eta} \left(1 - \frac{b}{2} T_{b3}\right) - \frac{A_b}{T_{b3}} - \frac{A_v}{n_3 T_{b3}} - \frac{F}{T_{b3}} - C_{bd} I_b aP_{b3}^{-\eta} \frac{T_{b3}}{2} \left(1 - \frac{2b}{3} T_{b3}\right)

- aP_{b3}^{-\eta+1} C_{bd} I_c \left[M - N - \frac{T_{b3}}{2} - bT_{b3} - b \left(N - M - \frac{5T_{b3}}{12}\right)\right] - aP_{b3}^{-\eta} C_v I_o M \left(1 - \frac{b}{2} n_3 T_{b3}\right)

- C_v I_v [(n_3 - 1)(1 - \gamma) + \gamma] aP_{b3}^{-\eta} \frac{n_3 T_{b3}}{2} \left(1 - \frac{2b}{3} n_3 T_{b3}\right)

(7.2.3.3)
Assumed \( C_{bd} = C_b - d, \ d > 0 \). For fix \( n_3 \) and \( T_{b3} \), the first derivatives of \( TP_{31}, TP_{32} \) and \( TP_{33} \) with respect to \( d \) are

\[
\frac{\partial TP_{31}}{\partial d} = \frac{1}{2} I_b aP_{b3}^{-\eta} T_{b3} \left( 1 - \frac{2b}{3} T_{b3} \right) + \frac{I_c aP_{b3}^{-\eta}}{T_{b3}} \left[ \frac{(T_{b3} + N - M)^2}{2} - \frac{b(T_{b3} + N - M)^3}{6} \right] \\
+ I_o MaP_{b3}^{-\eta} \left( 1 - \frac{b}{2} n_3 T_{b3} \right) > 0 \tag{7.2.3.4}
\]

\[
\frac{\partial TP_{32}}{\partial d} = \frac{1}{2} I_b aP_{b3}^{-\eta} T_{b3} \left( 1 - \frac{2b}{3} T_{b3} \right) + I_o MaP_{b3}^{-\eta} \left( 1 - \frac{b}{2} n_3 T_{b3} \right) > 0 \tag{7.2.3.5}
\]

And \( \frac{\partial TP_{33}}{\partial d} = \frac{1}{2} I_b aP_{b3}^{-\eta} T_{b3} \left( 1 - \frac{2b}{3} T_{b3} \right) + I_o MaP_{b3}^{-\eta} \left( 1 - \frac{b}{2} n_3 T_{b3} \right) \\
\]

\[
+ I_c aP_{b3}^{-\eta} \left[ M - N - \frac{T_{b3}}{2} - b \left( N - M - \frac{5T_{b3}}{12} \right) \right] > 0 \tag{7.2.3.6}
\]

Equation (7.2.3.4)- (7.2.3.6) suggests that the total profit per unit time will increase when the vendor offers a price discount to the buyer. To compensate the buyer's loss, the price-negotiation factor is introduces based on the difference of total profits from Model 1 and model 3. Define extra profit of vendor and of buyer as follows.

\[
EPV = TP_{33} - TP_{31} \] and \( EPB = TP_{33} - TP_{32} \) respectively.

Consider negotiation relationship \( EPV = \alpha EPB \). When \( \alpha = 0 \), the buyer is beneficial.

When \( \alpha = 1 \), both the players distribute extra profit equally. As the value of \( \alpha \) increases, the vendor will be beneficial in the integrated system.
7.3 Computational algorithm

To find optimal retail price and cycle time for buyer, for model 1 we outline following steps.

**Step1**: let \( M \geq N \). Set \( \frac{\partial TPB_{11}}{\partial T_{b1}} = 0 \), \( \frac{\partial TPB_{11}}{\partial P_{b1}} = 0 \)

Solve these two non-linear equations for a given set of parameters using Maple-10. Designate optimum solution as \((T_{b1}^*, P_{b1}^*)\). If \( T_{b1}^* + N \geq M \) then \( T_{b1}^* \) is optimum cycle time for the buyer and compute total profit per unit time \( TPB_{11} \) for buyer using (7.2.1.4). Otherwise set \( TPB_{11} = 0 \).

**Step2**: Set \( \frac{\partial TPB_{12}}{\partial T_{b1}} = 0 \), \( \frac{\partial TPB_{12}}{\partial P_{b1}} = 0 \). Solve these two non-linear equations for given set of parameters using Maple-10. Designate optimum solution as \((T_{b1}^*, P_{b1}^*)\).

If \( T_{b1}^* + N < M \) then \( T_{b1}^* \) is optimum cycle time which will minimize the buyer's total profit per unit time \( TPB_{12} \). Otherwise set \( TPB_{11} = 0 \).

**Step3**: For \( M < N \), solve \( \frac{\partial TPB_{13}}{\partial T_{b1}} = 0 \), \( \frac{\partial TPB_{13}}{\partial P_{b1}} = 0 \) for a set of parameters using Maple-10. Designate optimum solution as \((T_{b1}^*, P_{b1}^*)\). Compute buyer's total cost profit unit time \( TPB_{13} \) using (7.2.1.6).
Step 4: For optimum value of shipments $n_1 = n_1^*$, substitute $(T_{b_1}^*, P_{b_1}^*)$ in $TPV_1$ satisfying the following condition. $TPV_1(n_1^* - 1) \leq TPV_1(n_1^*) \geq TPV_1(n_1^* + 1)$. Once the optimal solutions are obtained, the total profit per unit time in Model 1 is

$$TP_1(T_{b_1}^*, P_{b_1}^*, n_1^*) = TPB_1(T_{b_1}^*, P_{b_1}^*, n_1^*) + TPV_1(T_{b_1}^*, P_{b_1}^*, n_1^* + 1).$$

Once the optimal solutions are obtained, the total profit per unit time in model 1 is

$$TP_1(n_1^*, P_{b_1}^*, T_{b_1}^*) = TPB_1(n_1^*, P_{b_1}^*, T_{b_1}^*) + TPV_1(n_1^*, P_{b_1}^*, T_{b_1}^*).$$

Solution procedure 2 for model 2

Step 1: Set $n_2 = 1$

Step 2: For $M \geq N$, solve $\frac{\partial TP_{21}}{\partial T_{b_2}} = 0$, $\frac{\partial TP_{21}}{\partial P_{b_2}} = 0$ for $T_{b_2}$ and $P_{b_2}$ using Maple-10 for given set of parameters. If $T_{b_2} + N \geq M$ then compute $TP_{21}$, total profit per unit time using (7.2.2.1). Otherwise set $TP_{21} = 0$.

Step 3: For $M \geq N$, solve $\frac{\partial TP_{22}}{\partial T_{b_2}} = 0$, $\frac{\partial TP_{22}}{\partial P_{b_2}} = 0$ for $T_{b_2}$ and $P_{b_2}$ using Maple-10 for given set of parameters. If $T_{b_2} + N < M$ then compute $TP_{22}$, total profit per unit time using (7.2.2.2). Otherwise set $TP_{22} = 0$. 

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**Step 4:** For \( N > M \) solve, \( \frac{\partial TP_{23}}{\partial T_{b2}} = 0, \frac{\partial TP_{23}}{\partial P_{b2}} = 0 \). For \( T_{b2} \) and \( P_{b2} \) using Maple-10 for given set of parameters.

**Step 5:** increment \( n \) by 1. Continue the steps 2-4 till

\[
TP_2(n^*_2 - 1) \leq TP_2(n^*_2) \geq TP_2(n^*_2 + 1) \quad \text{where} \quad TP_2 = \max\{TP_{21}, TP_{22}, TP_{23}\}
\]

A price negotiation procedure for model 3 is outlined below.

Price negotiation procedure:

**Step1:** Assign positive real numbers to \( \alpha, \epsilon \) and \( d \).

**Step2:** Calculate the maximal profit of \( TPV_1 \) and \( TPB_1 \).

**Step3:** Determine \( n^*_3, T^*_3, P^*_b \) by using solution procedure 2 for model 3. Calculate

\[
TPV_3, TPB_3, EPV \quad \text{and} \quad EPB.
\]

**Step4:** If \(|EPV - \alpha \star EPB| < \epsilon\), the procedure terminates and the optimal solutions are

\[
n^*_3, T^*_3, P^*_b, C^*_u \quad \text{and} \quad TP_3; \quad \text{otherwise go to step 3.}
\]

### 7.4 Numerical example and observations

In this section, we validate the proposed models by a numerical example.

Consider the parametric values as follows:
\( A_b = 100, A_v = 1200, F = 100, I_b = I_v = 0.10, C_b = 5, \)
\( C_v = 2.5, a = 300,000, b = 0.20, \eta = 2, \gamma = 0.8, M = \frac{15}{365}, N = \frac{20}{365}, I_e = 0.09, \)
\( l_c = 0.13, l_o = 0.09, \alpha = 1, d = 2, \epsilon = 0.01 \)

The optimal solutions for the three models with the negotiation factor \( \alpha = 1 \) are exhibited in table 7.4.1.

**Table 7.4.1:** The optimal solutions for three models

<table>
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<tr>
<th>Model ( i )</th>
<th>( i=1 )</th>
<th>( i=2 )</th>
<th>( i=3 )</th>
</tr>
</thead>
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<tr>
<td>( C_{bi}^* )</td>
<td>5</td>
<td>5</td>
<td>3.47</td>
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<tr>
<td>( n_{i}^* )</td>
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<td>1</td>
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<td>( T_{bi}^* )</td>
<td>0.30</td>
<td>0.36</td>
<td>0.39</td>
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<tr>
<td>( P_{bi}^* )</td>
<td>9.95</td>
<td>5.31</td>
<td>5.17</td>
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<tr>
<td>( TP_{bi} )</td>
<td>14189</td>
<td>3199</td>
<td>18399</td>
</tr>
<tr>
<td>( TP_{vi} )</td>
<td>4159</td>
<td>20805</td>
<td>3721</td>
</tr>
<tr>
<td>( TP_{i} )</td>
<td>18348</td>
<td>25004</td>
<td>25791</td>
</tr>
<tr>
<td>( PET_{P_{i}} )</td>
<td>0</td>
<td>36.27%</td>
<td>40.56%</td>
</tr>
</tbody>
</table>

\[ PET_{P_{i}} = \left( \frac{TP_{i}}{TP_{i} - 1} \right) \times 100, \quad i = 1, 2, 3 \]
From the table 7.4.1, it is observed that when buyer agrees for joint decision, the total profit of the supply chain increases by 36.27%. The buyer’s retail price decreases by 46.63% and results loss of $10,990. When takes joint decision instead of sole decision maker. The vendor benefits by $16646. Obviously, buyer will hesitate to opt for joint decision. To entice the buyer for integrated policy, the vendor will adopt price discount promotional tool. By giving 30% discount in unit purchase cost (model 3), the total profit per unit time of the supply chain will increase by 40.56%. Under negotiation scheme, extra profit of the supply chain $7443 is to be distributed equally among buyer and the vendor. The concavity of the total joint profit for model 1 and model 3 are shown in fig 7.4.1 and fig 7.4.2 respectively.

Fig 7.4.1 Concavity of total joint profit for model 1
Table 7.4.2 shows the sensitive analysis of the optimal solution with respect to the model parameters.

**Table 7.4.2: Sensitive analysis**

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<th>Parameters</th>
<th>% changes</th>
<th>$P_b$</th>
<th>$n_3$</th>
<th>$C_b$</th>
<th>$T_b$</th>
<th>$T_P$</th>
<th>$T_{P_2}(PET_{P_2} %)$</th>
<th>$T_{P_3}(PET_{P_3} %)$</th>
<th>MR(%)</th>
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MR = \frac{P_{b3}^* - C_{b3}}{P_{b3}} \times 100

7.4.5 Conclusions:

This article studies the ongoing scenario of decreasing demand due to recession period prevailing globally. It is observed from Table 7.4.2 that the model is most sensitive with respect to parameter \( \eta \) (price sensitive parameter) and \( \alpha \) negotiation factor. It is also seen that total profit increment is almost linear whether we increase or decrease the credit period. In all other parameter total profit is increased but it is almost same increment in profit factor.