CHAPTER 1
CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION: -

Shells of various shapes to cover large unobstructed areas have been a focus of attention since ancient times. Particularly, thin concrete shells can cover large space and are aesthetically pleasing. Due to complex mathematical equations to define the shapes of these shells, closed formed equations are available only for simple shell forms like cylindrical shells. The analysis of shells is basically divided into two types to find:

a) Membrane forces and
b) Bending forces.

The analysis of membrane forces is comparatively easy; however bending analysis becomes very complicated even for simple cylindrical shells.

Advent of finite element method has given a powerful tool to the designer for analysis and design of these complicated forms. The finite element technique gives both membrane forces and bending forces simultaneously, along with displacement at various nodal positions. However, these techniques are used in the elastic range of the material.

Only recently an interest has been developed in the non-linear elastic behaviour and elasto-plastic behaviour of the shell elements.

Elasto-plastic behaviour with the incremental loading technique and assumed yield criterion traces the load deflection characteristics of the shells to the complete plastification, giving an idea regarding the reserve strength of shells beyond the elastic limit.

Therefore it was thought to pursue the investigation of behaviour of shells, with the consideration of the points mentioned above.

The thesis attempts to investigate the shell behaviour with respect to the following points: -
i) Linear elastic analysis of shells using different types of shell elements like 4 noded and 9 noded plate elements and 8 noded and 20 noded solid elements;

ii) Non-linear elastic analysis of shells for stress-strain relation of concrete using available types of shell elements;

iii) Elasto-plastic behaviour of shells leading to complete plastification in an elemental manner by the application of incremental loading pattern using the stated four types of shell elements.

iv) The behavioural response of shell with the application of steel reinforcement along with the concrete, indicating the increase in the load carrying capacity of shell model with the 4 noded plate elements.

v) A practical shell type problem i.e. natural draught hyperbolic cooling tower of thermal power stations with two different types of supports leading to a new concept of equivalent plate thickness starting from the base. It has been analysed on the elasto-plastic basis.

With a view to undertake in depth study for concrete shell structures a problem of parametric investigation of concrete shell was undertaken. The analysis of the shell was now started with the linear elastic analysis with the following data.

### 1.2 **BASIC DATA:**

(A) All the structures involving the range of parameters were considered to be built in concrete with following data:

\[ E = 0.250 \times 10^8 \text{ KN/m}^2; \quad \nu = 0.15; \quad \gamma = 0.250 \times 10^2 \text{ KN/m}^3 \]

(B) In fig 1.1 the sectional details for the typical cylindrical shell with a circular directrix were presented;
The proposed investigation comprised following parametric details:

Span length ‘L’ was defined by three ratios:

\[ \frac{L}{r} = 1; \quad \frac{L}{r} = 2; \quad \frac{L}{r} = 3 \]

Thus, span lengths considered were, \( L = 7.62 \text{m} \); \( L = 15.24 \text{m} \); \( L = 22.86 \text{m} \)

\( \frac{L}{r} = 1 \) was supposed to represent a short shell, whereas \( \frac{L}{r} = 3 \) was supposed to represent the long shell. Thus, \( \frac{L}{r} = 2 \) was representative of intermediate condition.

1.3 ELEMENT TYPES:

Four types of shell elements were employed for the investigation. These were:

i) 4-noded Kirchhoff Flat plate elements - Zienkiewicz O. C. (2000);

ii) 9-noded Mindlin elements - Pica A. (1980);

iii) 8-noded solid hexahedral elements - Bangash M. Y. H. (1989);


In fig. 1.2 these elements had been typically illustrated:
Salient features regarding characteristics of these elements were presented in chapter no. 3.

1.4 FINITE ELEMENT MODELING:

The shell under consideration had the details such as reference axes, axes of symmetry and boundary condition as shown in fig. 1.3.

Fig. 1.2 Element Types

Fig. 1.3 Details of Axes and Boundary condition for Shell
Due to the available axes of symmetry only one quarter of the shell was considered for the analysis.

The portion of this shell considered in this manner was shown by hatching in the fig. 1.3, along with the boundary conditions.

The four types of the elements mentioned earlier were used to develop the finite element idealization for the entire range of problems considered. The details were as follows:

A) 4-noded Kirchhoff Element: -

The idealization details were presented over fig. 1.4 (a) i, ii, iii

![Diagram of 4-noded Kirchhoff Element]

Fig. 1.4 (a) - i $L/r = 1$
**Idealization Scheme of 4 Noded Plate Elements Used for Shell Analysis**

- **Fig. 1.4 (a) - [ii]**  
  \( L / r = 2 \)

- **Fig. 1.4 (a) - [iii]**  
  \( L / r = 3 \)
B) 9-noded Mindlin Element: - (Lagrangian)

The idealization details were presented over fig. 1.4 (b) i, ii, iii

\[ L / r = 1 \]
Fig. 1.4 (b) - ii  $L/r = 2$

Idealization Scheme of 9 Noded Plate Elements Used for Shell Analysis

Fig. 1.4 (b) - iii  $L/r = 3$
C) 8-noded Solid Hexahedral Element:

The idealization details were presented over fig. 1.4 (c) i, ii, iii.
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**Fig. 1.4 (c) - iii**  \[L / r = 3\]

Idealization Scheme of 8 Noded Solid Elements Used for Shell Analysis
D) 20-noded Solid Hexahedral Element:

The idealization details were presented over fig. 1.4 (d) i, ii, iii

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**Fig. 1.4 (d) - i**  \hspace{1cm}  \( L/r = 1 \)

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**Fig. 1.4 (d) - ii**  \hspace{1cm}  \( L/r = 2 \)

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1.5 **Load Cases:**

The complete range of analyses was performed for the following load cases:

a) Self weight of structure;

b) Live Load, Snow Load etc. over the shell surface;

c) Longitudinal Forces.

In fig. 1.5, these details were schematically shown;
1.6 Types of analysis: -

Following types of analysis were performed:

1.6.1 Linear Elastic Deformation Analysis: -

For the design of the concrete and RCC shells the design code IS 456 permits evaluation of features such as displaced profiles, strains, stresses etc. by considering the total section as made up of gross concrete section even if eventually it would be in the form of an RCC shell. The linear deformation analysis was
performed to serve this purpose. The details derived through rigorous investigations were presented in chapter 4.

1.6.2 Non-linear Elastic Deformation Analysis: -

The linear elastic deformation analysis considered as above was assumed to be valid for the range of loads referred to as working loads. For the load range higher than working loads it would be of interest to examine the change in the structural behaviour by considering the concrete deformation characteristics governed by the non-linear stress strain behaviour established through laboratory investigation of the concrete section. Chapter no. 5 dealt with this kind of investigations.

1.6.3 Elasto-plastic Deformation Analysis: -

The non-linear elastic deformation analysis considered as above had one limitation in sense that at certain load level plasticity got induced. In the idealized system such development of plasticity would begin in some part of the idealized system at a particular load and there by the increase of load had a structural response in the form of elastic behaviour of the unaffected region and plastic behaviour of the affected region. With small increment in load some more segments of the idealization would develop plasticity. This kind of progressive analysis was continued till all the elements in idealization become plastic, meaning thereby till the collapse load. The full details were presented in chapter no.6.

1.6.4 Influence of Reinforcement: -

For the above types of analysis, material was assumed to be homogeneous and isotropic. With the same assumption the analysis was also extended in respect of the influence of the reinforcement provided in the body of the shell. The results of the investigation were presented in chapter no. 7.

1.7 Practical Application: -

The algorithms utilized in the analyses of shell were employed towards demonstration of their applicability to an important practical problem. For this, the Natural Draught Hyperbolic Cooling Tower in RCC was considered. The corresponding details were presented in chapter no. 8.