CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION: -

The field of finite element analysis of shells and shell structures has been very widely researched consequently enormous literature was available regarding various aspects of their behaviour. It would be impossible to cover all such publications; therefore some selected segments of the literature were presented herein by the way providing the literature survey. The important aspect in such publications was concisely presented in the form of the abstracts of the subject matter presented in such publications. Hence, it was opined that the listing of the abstracts of the selected segment of the literature should serve the purpose of literature review adequately. In this chapter, the material was presented in this manner wherein sequential arrangement as per the date of publication had been reproduced.

2.2 Review on Cylindrical Shells: -

2.2.1 Analysis of Axisymmetric Shells by the Direct Stiffness Method

Grafton P. and Strome D. R. (1963) had developed a method for the structural analysis of shells of revolution, composed of materials with orthotropic properties. The development was based on the direct stiffness method. A truncated cone element was introduced to take advantage of symmetry. Derivations of the stiffness and stress matrices for the truncated cone element were given. Several examples were solved on the digital computing machine using a program that was based on the truncated cone element. The results were compared to other theoretical results, and the correlation was excellent. Extension of the technique to handle linear un-symmetric deformation and nonlinear symmetric deformation was discussed.

The principle changes involved, in applying this technique to shells of revolution with rotationally symmetric loading were a change in nodal concept and the development of stiffness and stress matrices for a different type of element.
Nodal circles on the shell replaced the nodal points previously used. The displacements of the shell could then be approximated from three components of displacement (two components of linear displacement and a rotation) at each nodal point. Due to rotational symmetry, the advantage of this modification was replacement of a large number of nodal points, each having six degrees of freedom in general case, by a relatively smaller number of nodal circles, each having three degrees of freedom.

The direct stiffness method was developed to meet the needs for analysis of complex structures, too complicated to be handled either by simple engineering techniques or the classical continuum mechanics approach. It was based on the concept that the actual structure can be idealized as a set of finite elements connecting nodal points. It was assumed that the displacements everywhere in structure can be described in terms of the displacements of these nodal points and the actual loading of the structure can be replaced by a set of equivalent loads at nodal points; equivalent in the sense that the work done during any incremental deformation approximates the work done by actual loading. Compatibility of deformation was satisfied precisely only at the nodal points and approximately along other elemental boundaries.

For each element, relationships could be derived based on the elastic properties of the element and reasonable approximation of the interior deformation of element, giving forces at the node points (equivalent to the distributed forces on the element boundaries) in terms of the displacement at the node points. These algebraic equations written in the matrix form were,

\[ \{F\} = [k] \{\delta\} \]

where, \(\{F\}\) and \(\{\delta\}\) were column matrices of forces and displacements at nodes and \([k]\) was stiffness matrix for the element.

Main purpose of this paper was to present the development of analysis techniques for a shell of revolution, with capability of accounting for the orthotropic elastic properties. The development was based on techniques which had
proved useful in the analysis of complex redundant structures, and which were readily automated by the use of computer. Based on the direct stiffness approach to the formulation of stress and deformation problems these techniques had been applied directly to the analysis of isotropic shell structures.

For shells of revolution with axisymmetric loading, the modifications developed here, which take advantage of rotational symmetry, offer significant saving in the time and labour of analysis.

2.2.2 Direct Stiffness Method Analysis of Shells of Revolution Utilizing Curved Elements

A doubly curved axisymmetric shell element was constructed by Joe R. (1966) for use in the analysis of shells by the direct stiffness method. The new element was expected to remedy difficulties experienced with conical shell element. The curved element provided a highly accurate approximation to the shapes of shells of revolution with arbitrarily curved meridians. Numerical illustrations of shape approximations were given for spherical and elliptical shells. The Direct stiffness method equations were derived and calculations were given for the case of an internally pressurized spherical shell. Comparisons of these results with those provided by the conical element were presented and discussed.

The structural analysis of shells of revolution by means of digital computation had become extensive in the past several years. A method, which had proved very useful in practical applications, was the direct stiffness method with a finite element shell representation. The principle finite element procedure made use of a conical shell element and approximated shell of revolution by means of an assemblage of conical frustums.

It had been found to yield excellent results for edge bending problems. With very small elements, it had yielded acceptable solutions for certain problems of pressure-loaded shells of revolution, whereas for other, but similar problems, it had failed to provide satisfactory results.
The difficulties encountered had raised the question of practicability of finite element method for analysis of shell structures. Sensitivity of shell behavior to structural discontinuities such as inter-elemental junctures, and to smoothness of variation of radii of curvature, was real basis for this question. It was necessary to determine how closely the finite element assemblage must approximate the continuous shell surface in order to provide accurate solutions to stress and deflection analysis problems.

The construction of a doubly curved shell element herein was based on the requirements that the assemblage of elements forms a smooth surface whose slope was continuous and whose slopes and radii at the element ends match those of the actual shell. Between element ends, which were nodal circles, the surface composed of the elements may depart from the true shell surface.

2.2.3 Analysis of Thick and Thin Shell Structures by Curved Finite Elements

Soharabuddin A. (1970) presented a general formulation for the curved, arbitrary shape of thick shell finite elements along with a simplified form for axisymmetric situations. A number of examples ranging from thin to thick shell applications were given, which include a cooling tower, water tanks, an idealized arch dam and an actual arch dam with deformable foundation.

A new process using curved, thick shell finite elements was developed overcoming the previous approximations to the geometry of the structure and the neglect of shear deformation.

A general formulation for a curved, arbitrary shape of shell was developed as well as a simplified form suitable for axisymmetric situations.

Several illustrated examples ranging from thin to thick shell applications were given to assess the accuracy of solution attainable. These examples include a cooling tower, tanks, and an idealized dam for which many alternative solutions were used.
The usefulness of the development in the context of arch dams where a thick shell situation exists, led in practice to a fuller discussion of problems of foundation deformation, etc, so that practical application became possible and economical.

2.2.4 A Refined Curved Element for Thin Shells of Revolution

A refined axisymmetric curved finite element for analysis of thin elasto-plastic shells of revolution was described by Popov E. P. (1971). The improved element was obtained by employing cubic polynomials in terms of local Cartesian coordinates for the assumed in-plane and out-of-plane displacements. This introduced into the solution two internal degrees of freedom in the cord direction of each element. These internal degrees of freedom were removed by static condensation before assembling the individual element stiffness matrices, and were subsequently recovered after the nodal displacements were obtained.

On comparison with the previous formulation, this procedure greatly improved the accuracy of the solution especially with regards to the in-plane stress resultants at discontinuities in the meridional curvature and the inter-element equilibrium of forces. The later fact made it possible to analyse shells with a discontinuous meridional slope. In using this element, improvement in the convergence of the elastic-plastic solutions had also been observed. Several examples illustrated the quality of solutions. The reported study was limited to axisymmetric loadings and boundary conditions.

2.2.5 Mixed Finite Element Method for Axisymmetric Shells

A mixed variational formulation was used as basis for developing a mixed finite element method for axisymmetric shells. The independent unknowns of the method were the axial and radial displacement components, the rotation of the normal to the middle surface and the meridional bending stress couple. The basic element was a frustum of curved meridian. General advantages of the mixed
method were presented, one of which was the possibility of using piece-wise linear functions of the meridional arc length to represent the basic unknowns.

Test results were presented by Elias Z. M. (1972) for plate bending, transverse shear deformation, membrane behavior, edge-zone bending, bending near the junction of two shells, convergence of the method and accuracy of middle surface curvature interpolation. Shell geometries considered were cylindrical, conical, spherical and ellipsoidal. Good results were obtained which should increase interest in the relatively less known and less tested mixed method as compared to the stiffness method. The mixed method to be presented here was based on a variational formulation of the thin shell theory in which the independent and free fields were the displacements of the middle surface and the stress couple.

Previous finite element methods dealing with a frustum shell element use a stiffness formulation, in which the nodal unknowns were, in the axisymmetric case, the two displacement components and the rotation in the meridional plane. The axisymmetric shell problem being mathematically one-dimensional was treated more efficiently, using a frustum as basic shell element instead of a triangular or a quadrilateral element.

2.2.6 A Field-Consistent Three-Noded Quadratic Curved Axisymmetric Shell Element

A curved three-noded quadratic isoparametric axisymmetric thick shell element was developed. Field-consistency interpretations allowed various configurations of the element to be designed so as to satisfy the specific problem needs. Typical applications demonstrated the versatility and accuracy of this element in its different problem-specific forms. The first approach made by Rameshbabu C. (1986) to the finite element solution of axisymmetric shells was based on the use of simple conical frustum elements to model the curved surface.

The two-noded linear element was the simplest flat element that can be developed for applications to axisymmetric shells. However, it showed a little improvement in accuracy compared to the conical frusta elements and required fine meshes at the end closures of axisymmetric shells near the axis of revolution.
Another problem with this element was that due to its flat or straight-line description, the ‘\(w/R\)’ contribution to the meridional strain, where ‘\(w\)’ was normal displacement and ‘\(R\)’ was principle radius in meridional plane, was dropped. This led to errors in doubly curved shells, which undergo predominantly membrane action, as in the internally pressurized sphere.

Mohr presented a curved three-noded quadratic isoparametric thick shell element. However, some difficulties were experienced in the use of reduced integration strategy, necessitating the introduction of a thickness-related penalty factor for the shear energy terms and the omission of ‘\(w/R\)’ term from meridional strain.

In this paper, it was stated that a field-consistent strategy as a more elegant basis for re-deriving a curved three-noded quadratic isoparametric thick shell element with transverse shears strain. The used element was very accurate both in terms of displacement and stress predictions.

Field consistency theory was a conceptual scheme recently introduced into the literature, which helped to understand and evaluate the problems of shear locking, membrane locking, parasitic shear etc. It was recognized that more familiar principles on which finite elements were designed; for example, completeness of polynomials for shape functions, compatibility of these across inter-element boundaries, the ability of elements to permit strain-free rigid-body motions and to recover constant strains were not sufficient to explain the very poor behavior of several elements in many practical engineering situations.

The field consistency approach identified these problems as those that need a description of its behavior by more than one field variable and whose strain fields were derived from these several field variables. In many situations these strain fields had to be constrained in particular ways. The use of simple independent low order interpolations for field variables resulted in ‘spurious constraints’ that cause ‘locking’. By insuring that the interpolations for strain fields were redistributed or integrated selectively, it was possible to produce elements that will have only ‘true constraints’ and thus be free of locking. We should explore the role that a field-
consistent way of strain smoothing will play to achieve a flawless quadratic shell element.

The behavior of a thick axisymmetric shell (including effects of transverse shear) could be described adequately by three independent field variables, tangential displacement $u$, radial displacement $w$, section rotation $\beta$, based on a general curvilinear system.

### 2.2.7 A Field-Consistent Two-Noded Curved Axisymmetric Shell Element

The efficiency of a field-consistent two-noded linear curved axisymmetric shear-flexible shell element was shown to be due to the removal of both shear and membrane locking. Typical applications illustrated how the field-consistent representation dramatically improves performance, and allowed greater flexibility in tailoring element design to satisfy specific problem needs.

A very efficient two-noded straight element included shear deformation effects. This was made possible by use of a single point numerical integration for shear strain energy thereby relieving the shear locking effect. Since, the element was used as a linear segment; problem of membrane locking was not encountered. A curved quadratic shear flexible element was attempted by Rameshbabu C. and Pratap G. (1986), but again; apparently due to membrane locking, contribution to meridional membrane strain was omitted. This could not be without error in applications to curved shells of revolution.

The field-consistency approach was a better procedure compared to the reduced integration technique, as it freed restriction on integration rules and allowed use of any uniform integration rule depending on specific problem requirements. The efficiency of a field-consistent two-noded linear curved axisymmetric shear-flexible shell element was shown to be due to the removal of both shear and membrane locking. Typical applications illustrated how the field-consistent representation dramatically improved performance, and allowed greater flexibility in tailoring element design to satisfy specific problem needs.
A shear-flexible curved axisymmetric shell element was a tedious job in structural mechanics, which needed a description of its continuum behavior by three independent variables, the tangential displacement $u$, radial displacement $w$, and section rotation $\beta$, based on a curvilinear system. The extensional, flexural and shear strain associated with this could be written as:

$$\{\varepsilon\} = \{\varepsilon_s\ \varepsilon_o\ \chi_s\ \chi_s\ \gamma\}$$

2.2.8 A Simple Shell Element formulation for Large-Scale Elasto-plastic Analysis

Nonlinear structural mechanics problems raised in a wide variety of disciplines, from automotive crash dynamics to earthquake engineering to nuclear power plant analysis. An important aspect of many of these problems involved the study of shell structures undergoing potentially large displacement but small strains. In addition, material nonlinearity may play a major role. Due to the complexity of solution strategies for nonlinear problems, these analyses could require large amounts of computer time to achieve a solution. Thus, practical developments for nonlinear analysis must balance accuracy and robustness against computational speed. Engelmann B. E., et al. (1989), described a recently developed four-node quadrilateral shell element with good accuracy properties. The element combined many components from other successful mixed formulations in a three-field variational framework, and was formulated in a large-displacement small-strain context. Emphasis had been placed on the development of a simple, low order formulation with good accuracy in a wide range of loading situations. Implementation of complex constitutive models was straightforward in this formulation. In addition, this four-node flat shell element was suitable for most widely used contact algorithms. These characteristics made the present formulation amenable to implementation in a general purpose analysis environment.

The paper opened with a description of the shell kinematics and governing equations. This was followed by a detailed description of the finite element formulation with particular emphasis on the fields chosen for in-place and
transverse variables. Finally, several numerical examples were given to illustrate the performance of the proposed element on both linear and nonlinear problems.

2.2.9 Curved Mindlin Beam and Axisymmetric Shell Elements – A New Approach

Two sets of strain definitions for analysis of curved Mindlin beams and axisymmetric shells were examined. Their conventional finite element implementations were shown to be unsuitable. The parameter used as a nodal degree of freedom in the first conventional method to describe the cross section rotation was in general a discontinuous function and has no clear physical meaning.

The second conventional method had relied on an incorrect differentiation for its successful use in finite element analysis. A new approach, which overcame these problems, was presented. Examples were given to illustrate the incorrect results that could be obtained from the two conventional methods and to show that the new approach provided accurate solutions to general curved beam and axisymmetric shell problems.

The simple 3-node isotropic curved Mindlin beam and axisymmetric shell elements had been studied for research work by Day R. A. (1990) and had been directed towards the problem of membrane and shear force locking. In this, strain definitions for beam element and their implementations into finite element method had been examined.

Two conventional sets of strain definitions for curved Mindlin beam and axisymmetric shell elements were found unsuitable for implementation in finite element method of analysis.

2.2.10 Layer wise Theory for Discretely Stiffened Laminated Cylindrical Shells

The Layer wise Shell Theory was used to model discretely stiffened laminated composite cylindrical shells for stress, vibration, pre-buckling and post-buckling analysis. The layer wise theory reduced a three-dimensional problem to a two-dimensional problem by expanding the three-dimensional displacement field as a function of a surface-wise two dimensional displacement field and a one-
dimensional interpolation through the shell thickness. Any required degree of accuracy could be obtained by an appropriate, independent selection of the one-dimensional interpolation functions through the thickness and the two-dimensional interpolation of the variables on the surface. Using a layer wise format, discrete axial and circumferential stiffeners were modelled as two-dimensional beam elements.

Similar displacement fields were prescribed by Kassegne S. K. (1992) for both the stiffener and shell elements. The contribution of the stiffeners to the membrane stretching, bending and twisting stiffnesses of the laminated shell was accounted for by forcing compatibility of strains and equilibrium of forces between the stiffeners and the shell skin. The layer wise theory was also used to model initial imperfections of the unstressed configuration. A finite element scheme of the layer wise model was developed and applied here to investigate the effect of imperfections on the response of laminated cylindrical shells.

Using a finite element model of the layer wise theory for shells and shell stiffener elements, the accuracy and reliability of the elements was investigated through a wide variety of examples. The examples included laminated stiffened and un-stiffened plates and shells and stand-alone beams under different types of external destabilizing loads. Finally, the study identified the particular types of problems where the layer wise elements possessed a clear advantage and superiority over the conventional equivalent single-layer models.

2.2.11 Finite Elastoplastic Deformation of Membrane Shells

Under restriction of an isotropic elastic response of deformed lattice, developed a covariant theory of finite elasto-plasticity in principal axes of a pair of deformation tensors. In material description, the tensor pair consisted of the plastic deformation tensor and the total deformation Cauchy-Green tensor. Ibrahimbegovic A. (1996) applied the proposed theory to elasto-plastic membrane shells, whose references and current configurations could be arbitrary space-curved surfaces.

Pressure-insensitive Von-Mises yield criterion with isotropic hardening and a quadratic form of the strain energy function given in terms of elastic principal stretches were considered as a model problem. Through an explicit enforcement of the plane stress
condition we arrived at a reduced two-dimensional problem representation, which was set in membrane tangent plane.

Numerical implementation of the presented theory relied crucially on the operator split methodology to simplify the state update computation. Presented a set of numerical examples in order to illustrate the performance of the presented methodology and indicated possible applications in area of sheet metal forming.

2.2.12 Alternative Lower Bounds Analysis of Elastic Thin Shells of Revolution

Itaru M. presented an alternative lower bound to the elastic buckling collapse of thin shells of revolution, in comparison with results from geometrically non-linear elastic analysis. The numerical finite element method was based on axisymmetric rotational shell elements whose strain-displacement relations were described by Koiter’s small finite deflection theory, with displacements expanded circumferentially using a Fourier series. Itaru M. et al. (1996), compared the reduced stiffness linear analysis, based on the buckling equation without incremental linear in-plane energy components corresponding to the lowest Eigen mode (for a particular cylindrical shell under external pressure).

Second, the non-linear astatic (quasi-static) elastic analysis to clamped spherical caps under uniform external pressure was carried out in order to compare the results from a reduced stiffness analysis from viewpoints of not only buckling loads, but also total potential energy. It argued that the astatic buckling loads may relate to reductions due to a specific imperfection effect on elastic buckling collapses.

2.2.13 Finite Elasto-Plastic Deformation of Membrane Shells

Under restriction of an isotropic elastic response of deformed lattice, develops a covariant theory of finite elasto-plasticity in principal axes of a pair of deformation tensors. In material description, the tensor pair consists of the plastic deformation tensor and the total deformation Cauchy-Green tensor.

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2.2.14 Non-linear Finite Element Analysis of Composite Shells by Total Lagrangian Decomposition

A total Lagrangian finite element scheme for arbitrarily large displacements and rotations was applied to a wide range of shell geometries. The scheme decomposed the deformation into stretches and rigid-body rotations, examining the deformed state with respect to an orthogonal, rigidly translated and rotated triad located at the point of interest on the deformed structure.

The stresses and strains, which were resolved along the axes of this triad, were employed by James M. G. Jr. (1997) in the algorithm. Local and layer-wise thickness stretching and shear warping functions were used to model the three-dimensional behavior of shells. These functions were developed through the use of the constitutive equations, certain stress and displacement continuity requirements at ply interfaces and laminate surfaces, and the behavior of shell reference surface.

Two finite elements were employed in the analysis: an eight-noded, 36 degree-of-freedom (DOF) element, and a four-noded, 44 DOF elements. The 36 DOF element, which was not a compatible element with respect to the derivatives of the in-plane deformations proves adequate for moderate rotation problems, but failed in modeling very large rotation problems. The use of the 44 DOF elements provided dramatically improved results in the large rotation problem.
2.2.15 A New Approach for Finite Element Analysis of Sandwich Shells

A new finite element approach to sandwich shells was proposed. It used existing shell finite elements formulated for homogeneous shells. The “Sandwich” nature of the problem was hidden from the main finite element program. Based on the several assumptions the proposed homogenization procedure calculated stress increments in a homogeneous fictitious material, called “equivalent”, which correspond to the strain increments in the equivalent material points. The stresses in the equivalent material were calculated by Tabiei A. (1999) based on the stress and strain fields in the sandwich layers, which were determined from the incoming strain field for the equivalent material.

2.2.16 Subdivision Surfaces-A New Paradigm for Thin Shell Finite Element Analysis

A new paradigm for thin-shell finite-element analysis based on the use of subdivision surfaces for: i) describing the geometry of the shell in its undeformed configuration, and ii) generating smooth interpolated displacement fields possessing the bounded energy within the strict framework of the Kirchhoff-Love theory of the thin shells. The particular subdivision strategy adopted by Cirak F. et al. (2000), was in Loop’s scheme, with extensions such as required to account for creases and displacement boundary conditions. The displacement fields obtained by subdivision have a finite Kirchhoff-Love energy. The resulting finite elements contain three nodes and element integrals were computed by one point quadrature.

The displacement field of the shell was interpolated from nodal displacements only. In particular, no nodal rotations were used in the interpolation. The interpolation scheme induced by subdivision was non-local, i.e., the displacement field over one element depends on the nodal displacements of the element nodes and all nodes of immediately neighboring elements. However, the use of subdivision surfaces ensured that all the local displacement fields thus constructed combine conformingly to define one single limit surface.
2.2.17 Free Vibration Analysis of Cylindrical Shells Partly Supported on the Edges

In this study the free vibration analysis of open circular cylindrical shells partly supported on the straight edges was presented. The geometry of the shell was defined by the subtended angle, the length, and the shell thickness. The method of solution implemented here was a modified version of the Rayleigh-Ritz method.

Kandasamy S. (2002) introduced a set of displacement grid points on the middle surface and each grid point has five degrees of freedom, viz., three translational components along the cylindrical coordinates and two rotational components of the normal to the middle surface.

The number of grid points for the analysis depends upon the orders of the polynomials chosen for the displacement components and as a consequence considerably high-order polynomials were used for this purpose.

The constitutive equations were derived from the three-dimensional strain-displacement relations in the cylindrical coordinate system for the first order shear deformable cylindrical shell. To validate the method, a convergence study was carried out for the case in which the shell was assumed to be partly supported on the straight edges. Further, the numerical results obtained from the present method were compared with the ones obtained from a commercially available finite element computer code.

2.2.18 Parametric Study on the Buckling of Thin Steel Cylindrical Shells under Elevated Local Axial Compression Stresses

It was well known that the buckling strength of a thin cylindrical shell under uniform axial compression was highly sensitive to imperfections in the shell wall. The buckling behaviour increased considerably in complexity when non-uniform stress distributions were also considered. Thus cases of local high compression had not received much attention in the literature developed by Minjie C et al. (2003), so far even though they were of great practical relevance.

Furthermore, with changing shell geometry, loading patterns and boundary conditions the buckling phenomena observed vary considerably both quantitatively and qualitatively. This paper presented some results from a comprehensive
parametric study exploring the full range of practically relevant geometries for metal cylindrical silo shells with different boundary conditions and subjected to elevated local axial compressive actions of varying magnitude and extent. In the parametric study, influences of geometric imperfections and material non-linearity on the buckling strength of the shell were also considered.

Some simple rules corresponding to different types of analyses and different buckling phenomena were presented for future implementation into the appropriate design standards.

2.2.19 Mixed Finite Elements for the Elasto-plastic Analysis Of 2D Continua

Antonio B. et al. (2004), aimed to formulate assumed stress finite elements for the analysis of elasto-plastic structures. The interpolations of the displacement and stress fields, typical of the elastic version of the mixed elements, were enriched with the FEM representation of the plastic strain field. The formulation of the elasto-plastic problem of the element was then established, consistently, with respect to its variational framework based on the weak enforcement of the compatibility condition. It led to a minimization problem of a quadratic functional subjected to a linearized form of the plastic admissibility constraints.

2.2.20 A Triangular Finite Element Applied to the Geometrically Exact Analysis of Elastic-Plastic Shells

Following a pure Lagrangian description, the geometrically-exact finite strain shell models were reviewed by Pimenta P. M. et al. (2004), and extended to the case of elastic-plastic shells. Isotropic hardening plasticity was considered here. We derived the plastic stress response with a multiplicative decomposition of the deformation gradient, based on the concept of a natural configuration. The elastic domain was defined by a logarithmic material and a Von Mises yield function, while a radial return method was developed for integration of the rate stresses.

The model was implemented over the triangular shell finite element and was briefly assessed by means of some numerical examples.
2.2.21 An Enhanced Strain 3D Element for Large Deformation Elasto-Plastic Thin-Shell Applications

In this work a previously proposed solid-shell finite element, entirely based on the Enhanced Assumed Strain (EAS) formulation, was extended in order to account for large deformation elasto-plastic thin-shell problems. An optimal number of 12 enhanced (internal) variables were employed by Valente R. A. F. et al. (2004), leading to a computationally efficient performance when compared to other 3D or solid-shell enhanced elements.

This low number of enhanced variables was sufficient to (directly) eliminate both volumetric and transverse shear locking, the first one arising, for instance, in the fully plastic range, whilst the last appears for small thickness’ values. The enhanced formulation comprised an additive split of the Green-Lagrange material strain tensor, turning the inclusion of nonlinear kinematics a straightforward task. Finally, some shell-type numerical benchmarks were carried out with the present formulation, and good results were obtained, compared to well-established formulations in the literature.

2.2.22 Nonlinear Limit State Analysis of Laminated Plates by P-Version of F.E.M.

A $p$-version finite element model based on degenerate shell element was proposed for the analysis of orthotropic laminated plates. In the nonlinear formulation of the model developed by Woo K. S. et al. (2005), the total Lagrangian formulation was adopted with large deflections and moderate rotations. The material model was based on the Huber-Mises yield criterion and Prandtl-Reuss flow rule in accordance with the theory of strain hardening yield function.

The model was also based on extension of equivalent-single layer laminate theory (ESL theory) with shear deformation leading to continuous shear strains at the interface of two layers. The validity of the proposed $p$-version finite element model was demonstrated through several comparative points of view in terms of ultimate load, convergence characteristics, nonlinear effect, and shape of plastic zone.
2.2.23 A Finite-Strain Quadrilateral Shell Element Based on Discrete Kirchhoff - Love Constraints

Pedro M. A. et al. (2005) improved the 16 degrees-of-freedom quadrilateral shell element based on point wise Kirchhoff - Love constraints and introduced a consistent large strain formulation for this element. The model was based on classical shell kinematics combined with continuum constitutive laws.

The resulting element was valid for large rotations and displacements. The degrees of freedom were the displacements at the corner nodes and one rotation at each mid-side node. The formulation was free of enhancements; it was almost fully integrated and was found to be immune to locking or unstable modes. The patch test was satisfied.

In addition, the formulation was simple and amenable to efficient incorporation in large-scale codes as no internal degrees of freedom were employed, and the overall calculations were very efficient. Results were presented for linear and non-linear problems.

2.2.24 Analysis of Finite Strain Anisotropic Elastoplastic Fracture in Thin Plates and Shells

A finite element methodology for analyzing fracture in thin shells in the large strain elasto-plastic regime was presented. The post localization constitutive model was based on a cohesive surface dissipation mechanism. Pedro M. A. (2006) employed a Kirchhoff - Love shell model and the corresponding discretization by finite elements and made use of the extended finite element technique in the implicit form of mid-surface displacement and director field discontinuities.

Applications showing the possibilities of this technique were shown, and the effect of plastic anisotropy in the crack pattern was numerically inspected.

2.2.25 Nonlinear Dynamic Analysis of Thin Shells Using a Finite Element with Drilling Degrees of Freedom
The good performance of the mixed degenerate shell finite element with drilling degrees of freedom (DDSE) in linear and nonlinear static analysis problems was in the basis of the present work concerned with its extension to nonlinear dynamic regime. The 9-noded element (DDSE9) had exhibit in a previous work the best performance among the other elements of the DDSE family. It was, consequently, chosen by Djermane M. et al. (2007), to experience the nonlinear dynamic behaviour in this paper. Both geometrical and material nonlinearities were taken into account and the different integration schemes were considered in the present investigation.

The results of the benchmarks treated by the DDSE proved its accuracy and efficiency when compared to those available in the literature. This conclusion would enhance the main advantage of this kind of elements which remained their suitability to model many engineering problems without any numerical “tricks” or adjustments.

2.3 **Review on Hyperbolic Cooling Towers: -**

2.3.1 *Towers Supported on Columns*

Stresses due to the individual effects of forces, moments and deformations at the base of a hyperboloid cooling tower were presented. With the assumption that the column-spacing was sufficiently small to permit taking an average stiffness per unit length, forces and moments in a loaded cooling tower, supported on columns, were calculated for different column stiffness and different load distributions in the circumferential direction.

It was shown by Salman H. (1970) that all base displacements had finite values, membrane (not bending) stiffness of the column was important and column effect was noticeable when bending moments were significant.

2.3.2 *Analysis and Design of Hyperbolic Cooling Towers*

The analysis and design of a given hyperbolic cooling tower for a nuclear power plant were reviewed. Analysis was performed using available computer programs. Various loading conditions on the tower such as wind, earthquake,
thermal, and self-weight, were considered and their effects were combined and compared.

A number of wind-load distributions, used in Britain, Belgium, and the US, were taken for the analysis by Gurfinkel G. (1972); and the corresponding results were compared. The effects of various conditions of base restraint on the results were also studied. The design of the reinforcement for the shell was given with details of the actual reinforcement layout. It was concluded that wind loading was the governing design condition for the hyperbolic towers.

Also, it was suggested that ultimate design methods be used with adequate load and capacity-reduction factors. To prevent widespread cracking of the concrete, the design should call for reinforcement bars placed in double layers in both faces of the shell.

2.3.3 Stability and Dynamic Analyses of Cooling Towers

The behaviour of natural draft cooling tower wind pressure was investigated. Buckling loads of the towers of different meridional curvatures and shell thicknesses were computed and compared.

The results showed that:

(1) An increase in stiffness of the structure with an increase in meridional curvature; and

(2) Changes of buckling load caused by changes in shell thickness was approximately proportional.

A dynamic analysis of the tower was also presented by Changhua Y. (1973).

The response of the structure was increased by the dynamic effect. The amount of increase depends upon the wind time history.

2.3.4 Stability of Hyperboloid Shells

An analytical and experimental investigation was carried out to determine the buckling loads of hyperboloid shells with different geometries subjected to the axisymmetric loadings of external pressure and axial compression. Sander’s thin
shell equations were used in conjunction with the finite element method to determine the bifurcation buckling load of the shell.

The experimental program conducted by Veronda D. R. (1975), yielded data on the instability behaviour of hyperboloid shells subjected to combined loadings. Moulded PVC specimens were used in the experiments. Shell specimens were:

1. Clamped on both ends; and
2. Clamped on one end and free on the other end.

The experimental data were found to be in good agreement with the analysis for all types of loading conditions.

2.3.5 Stability of Pressurized Hyperboloid Shells

The data obtained were compared with predicted results using a linear finite element stability analysis was and a corresponding nonlinear analysis was wherein the effects of geometric nonlinearities were included. Experimental buckling loads were in good agreement with linear theory predictions, and the geometric nonlinearities were found to have little effect on the calculated critical loads of the hyperboloids tested. The experimental results were also compared to analytical data for cylindrical shells.

The ratio of experimental results to analytical predictions was far lower for cylindrical shells than hyperboloid shells. These results obtained by Veronda D. R. and Weingarten V. I. (1975) indicated that hyperboloid shells have a lower sensitivity to geometric imperfections than cylindrical shells.

The experimental data for internally pressurized hyperboloid shells under axial load indicated that the axial buckling value asymptotically approached a constant value when the additional load carried by the internal pressure was subtracted. Sanders thin shell (strain-displacement) equations were used to develop finite element models for both the linear and nonlinear analyses.

2.3.6 Shell Elements for Cooling Tower Analysis
Intending to achieve optimum finite element modelling of column supported cooling towers for seismic response studies according to the distributions of dominating bending and membrane stresses and intending to model the vulnerable shell-column region using discrete column elements and quadrilateral shell elements, a set of elements was adopted, modified or extended.

The set included:

(1) A 16 Degrees of Freedom column element;

(2) A 48 Degrees of Freedom doubly curved quadrilateral general shell element;

(3) A 42 Degrees of Freedom doubly curved general-membrane transition element;

(4) A 21 Degrees of Freedom and 39 Degrees of Freedom doubly curved triangular membrane filler elements; and

(5) A 28 Degrees of Freedom doubly curved quadrilateral membrane element.

Examples were given to evaluate a single type, combined types, and the whole set of elements with results in good agreement with alternative solutions.

These elements used by Yang T. Y. (1983) for accurate and efficient free vibration analysis of column supported cooling towers. The modelling may be used in seismic response analysis of cooling towers for obtaining detail stress distribution in the vulnerable shell-column region.

This model may be extended to include material nonlinearity in the shell-column region for the seismic response studies.

### 2.3.7 Finite element techniques for the analysis of cooling tower shells with geometric imperfections

Two finite element methods for analysing geometrically imperfect cooling tower shells were presented. In the first the geometry of the imperfection was modelled by the elements; in the second the imperfection was represented by an equivalent load on the shell. Axisymmetric and general shell elements had been considered.
Results were given by Moy S. S. J. (1983), which showed that the first approximation to the equivalent load was sufficiently accurate and that it was possible to represent local imperfections by axisymmetric imperfections which required less computation.

It was also shown that axisymmetric elements should be used wherever possible, because of their greater efficiency, following the geometry of an axisymmetric imperfection but representing local imperfections by equivalent loads.

2.3.8 Simulation of Static and Kinetic Buckling of Un-stiffened and Stiffened Cooling Tower Shells

Natural draught cooling tower shells were loaded mainly by their dead weight and by the wind, which may both cause buckling failure. The present paper compared various numerical procedures to investigate the stability behaviour of cooling tower shells. These were a complete nonlinear analysis, a linear Eigen value analysis was for a stationary non-axisymmetric wind load, and a linear Eigen value analysis was for a wind load, approximated to be axisymmetric.

The aim was to evaluate whether a geometrically nonlinear analysis can be replaced by a time-saving classical buckling analysis, probably even for an axisymmetric state of stress. The third procedure, as the most conservative, but a very effective one, would be applied to investigate the mechanical influence of ring stiffeners on the buckling behaviour of cooling tower shells.

The kinetic instability phenomena would also be examined by Eckstein U. et al. (1987). The structural improvement resulting from ring stiffeners would be quantified and summarized in design recommendations.

2.3.9 Non-Linear Buckling Analysis of Hyperbolic Cooling Tower Shell with Ring-Stiffeners

It was concerned with a numerical solution of hyperbolic cooling tower shell, a class of full nonlinear problems in solid mechanics of considerable interest in engineering applications. In this analysis, the post-buckling analysis of cooling
tower shell with discrete fixed support and under the action of wind loads and dead load was studied.

The influences of ring-stiffener on instability load were also discussed by Loo Wen-da (1989). In addition, a new solution procedure for nonlinear problems which was the combination of load increment iteration with modified R-C length method was suggested.

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2.3.10 Buckling Analysis of Cooling Tower Shells Subjected to Non-Axisymmetric Wind Pressures

Safety against buckling under wind loading was an important criterion in the design of cooling tower shells. A rigorous bifurcation analysis of a cooling tower shell subjected to actual non-axisymmetric wind pressure distribution using quadrilateral or triangular elements was time consuming. An attempt by Srinivasarao P. et al. (1991), at such an analysis using ring elements in the form of a frustum of a cone with only four degrees of freedom per nodal line had yielded encouraging results.

The coupling of harmonics that occurred in a bifurcation analysis using a Fourier series has been found to be only of minor disadvantage in the computational efforts.

A computer program had been developed using this alternative method. No simplifying assumptions such as equivalent axisymmetric pressure or stress had been made for wind pressure effects. The results obtained from the present analysis were compared with the results available in the literature.

2.3.11 Finite Element Analysis of Column Supported Hyperbolic Cooling Towers Using Semi-Loof Shell and Beam Elements

In most of the early works related to the analysis of hyperbolic cooling towers, under either dead or the wind loads, only the tower shell was considered in the analysis and a continuous boundary condition in the form of fixity of the base of the shell was considered. However, the tower shell was supported by columns in
the form of A-frames. In order to consider realistic boundary conditions, it was essential to consider the supporting columns in the analysis along with the shell.

The present study conducted by Karwasiddappa M. et al. (1998) consider this problem and an attempt has been made to represent the tower shell by semi-loof shell elements and the supporting columns by semi-loof beam elements. The column ends were assumed to be fixed at their bases. The analysis had been carried out for only the dead load. Hoop forces had been found to have altered significantly in the lower portion of the shell near the column-shell junction. Moreover, this proposed model gives a better physical representation of a column supported hyperbolic cooling tower.

2.3.12 Self-weight Stresses in Hyperbolic Cooling Towers of General Shape

Explicit analytical results were developed by Zingoni A. (1999), for self-weight membrane stresses in hyperbolic cooling towers of general shape, for a number of different patterns of shell-thickness variation along a meridian of the shell of revolution.

The very general nature of the results made them particularly useful in obtaining quick estimates of dead-weight stresses for structures of this type, and any desired parametric studies may readily be undertaken, permitting the designer to choose the most optimum shell-thickness variation for a given combination of geometric parameters of the hyperboloid of revolution.

The results were intended for use during the preliminary phase of the design process. Other equally important effects (such as wind loading and thermal gradients) were best taken into account numerically using the finite element method, and will therefore not be treated in this paper. Some numerical results for self-weight stresses were then obtained on the basis of special but very practical cases, and an analysis of these results reveals interesting design implications.

2.3.13 Non-Linear Analysis of a RC Cooling Tower with Geometrical Imperfections and a Technological Cut-Out
The analysis of nonlinear deformation and ultimate load should be a vital element of the checking-design of cooling towers. This issue was discussed in the paper with respect to an existing cooling tower with large geometrical imperfections and a cut-out made for gas discharge.

A brief theoretical background was given by Waszczyszyn Z. et al. (2000), with special attention paid to the finite element model and constitutive relations for reinforced concrete.

The numerical analysis dealt with three cases:

1. Perfect hyperboloid shell of revolution,
2. Actual shell with measured geometrical imperfections of the shell mid-surface,
3. Imperfect shell with a cut-out.

Two types of nonlinearities were considered:
(a) Material nonlinearity only,
(b) Material and geometrical nonlinearities.

The computations were carried out for a cooling tower shell pinned at the bottom contour. The dead load and wind pressure were applied to the shell. Finite element code ANKA was used as the main tool for the numerical analysis. The results of computations for a perfect hyperboloid shell were compared with the results obtained using the DIANA package.

It was noted that a considerably softer response corresponds to larger deformations and advanced cracking. The geometrical imperfections only slightly influence the response of the shell. The cut-out introduced into the imperfect shell does not significantly change the shell deformations either.

2.3.14 Reliability of Natural-Draught Hyperbolic Cooling Towers Using a 3D Finite Element Analysis Coupled with Probabilistic Methods

Reliability analyses of structural components aimed at estimating the probability that the external loading exceeds the structure resistance. In industrial cases, when finite element calculations were needed, the failure of the structure could not be defined by an explicit function of the random variables. Defaux G.
and Heinfling G. (2000) dealt with a feasibility study of the reliability analysis of a typical natural-draught cooling tower subjected to different combined actions.

A 3D finite element analysis was coupled with probabilistic methods. The mechanical deterministic model was validated by direct comparison of the corresponding results with others obtained by different finite element codes. Different coupling methods were used for the reliability analysis.

Sensitivity analyses were also performed in order to evaluate the influence of the probabilistic definition of the random variables. The results emphasized a good agreement of the different methods. They allowed defining the best modelling options to use in industrial frameworks by establishing a parallel between the precision of the reliability results, the accuracy of the model and the cost performances.

2.3.15 Dynamic Response of RCC Cooling Tower Shell Considering Supporting Systems

RCC cooling towers were used for many kinds of industrial and power plants. These were huge structures and also thin shell structures. RCC cooling towers were subjected to its self-weight and the dynamic load such as an earthquake motion and a wind effects.

Especially, dynamic analyses of these structures were important factor to design RCC cooling tower structures. In this paper, dynamic behaviour of RCC cooling tower shell under an earthquake loading was analyzed by use of FEM focused on the column arrangement systems.

To reduce the computational efforts to solve such problems, the technique for a parallel computing was applied by Takashi Hara (2002). To solve the nonlinear dynamic response of a huge cooling tower with local deviation, the element by element (EBE) parallel approach was adopted by using of PC cluster. From the numerical analyses, the effects of the combination of RCC shell and column systems were examined.
In the next chapter, linear elastic deformation of cylindrical shells, using different finite elements, was presented.