CHAPTER - 1

INTRODUCTION

1.1 PRODUCTION AND PRODUCTION FUNCTIONS

The theory of production is an effort to explain the principles by which a business firm decides how much of each kind of inputs (i.e. labour, capital, raw material, energy consumption, etc.) are required to produce the required output. The production theory is often called marginal productivity theory in reference to the decision rules that represent necessary conditions for the achievement of maximum profit.

Production function is a unique technological relationship between inputs and output within a production unit, a production unit can be a firm, industry or the national economy. Thus the production function expresses the way in which outputs are produced by inputs, and the way inputs co-operate with each other in varying proportion between outputs and inputs and between the inputs themselves are determined by the technology that rules at any given time. The technology is embedded in the production function and can be expressed in terms of it.

According to Heathfield, "the production function is the core concept in the economic theory of production. The production processes are as the means of transforming certain inputs into certain outputs. It is clearly absolutely critical to know, how much output can be produced with certain combinations of inputs and what, if any alternatives there are to producing particular outputs in particular ways. The

production function is an attempt for defining these alternatives." As defined by Walters' "production function is a technological relationship confronting a firm. It is the entrepreneur who chooses factor proportions and output levels". The function may have linear or non-linear form depending upon the hypothesised relationship between the inputs and the output and between one input and another.

In production of one output there can be many inputs such as land, labour, capital, material, etc. which finally can be classified into two major inputs namely labour and capital. In case of one output and two inputs the relationship between output and input can be expressed mathematically as under:

\[ Y = F(L,K) \]  \( \ldots (1.1) \)

Where \( Y \) is output and \( L \) and \( K \) are amount of input factors of production. It is customary (in general) to consider these inputs as labour and capital respectively. If equation (1.1) provides largest output (\( Y \)) that can be produced for given input factors, it is known as efficiency frontier. By means of production function one can determine the change in the output resulting from increase in each of the various factors of production. The change in output from one additional unit of a particular factor of production is measured by marginal productivity of that factor of production.

1.2 SOME BASIC CONCEPTS

1.2.1 ELASTICITY OF SUBSTITUTION

Elasticity of substitution measures the substitutability between the inputs, namely labour and capital. Knowledge of the value of the elasticity of substitution in the industrial sectors can be useful for policy makers for changing the market signals to

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ensure greater labour absorption. The elasticity of substitution is a technical parameter characterising a production function. ACMS described it as “Estimation of Elasticity of Substitution implies an analysis of production function and its translation into an estimation form”. This suggests the correlation between labour productivity and real wage rate. The higher value of elasticity of substitution implies the effect of change of real wage rate on the labour productivity is higher.

The ratio of marginal product of capital to the marginal product of labour is the marginal rate of substitution of labour for capital and by marginal productivity theory, it is equal to the per unit rental of capital relative to the wage rate. The elasticity of substitution is denoted by ‘σ’ and it is defined as under:

\[
\sigma = \frac{L_d \frac{K}{L}}{dR / R}
\]

... (1.2)

where \( R = \frac{MP_l}{MP_k} \)

\( MP_l \) = Marginal Productivity of Labour

\( MP_k \) = Marginal Productivity of capital

The elasticity of substitution as defined in the formula given in the equation (1.2) relates the proportional change in the relative factor inputs to a proportional change in the marginal rate of substitution between labour and capital.

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For VES production function model $F(L,K) = \Phi\left(\frac{K}{L}\right)$ which can generate a class of such type of production function. For such a form in general we can obtain the elasticity of substitution given by $\sigma = \psi\left(\frac{K}{L}\right)$, so that we can generate a series for such elasticities pertaining to each pair of observations on K and L.

![Isoproduct Curve (Isoquant)](image)

1.2.2 MARGINAL RATE OF TECHNICAL SUBSTITUTION (MRTS)

For the factors say A and B, the MRTS of factor B for A is explained by the slope of the isoproduct curve, that is $\partial B/\partial A$ and it is shown in the Fig. 1.1.

From the Fig. 1.1, it can be seen that MRTS of B for A is falling. MRTS of B for A (MRTS$_{BA}$) is that amount of factor B for which the production is ready to sacrifice to get an additional amount of factor A. As explained by Singh, Parashar and Singh the MRTS of factor B for A is the ratio of marginal physical product of factor A and marginal physical product of factor B.

6. *It may be noted that Isoquant are continuous functions which are downward sloping and convex to the origin.*

MRTS_{BA} = \frac{\text{Marginal Physical Product of factor A}}{\text{Marginal Physical Product of factor B}}

1.2.3 TECHNOLOGICAL CHANGE

Technological progress is one of the most important factors responsible for the economic growth in many sectors of the economy. The concept of production function and technological progress are closely related. According to Golder\textsuperscript{8} “production function corresponds to a specific level of technological knowledge and any change in this leads to a shift in production functions”.

There are two general types of technological change neutral and non-neutral. According to J.R. Hicks\textsuperscript{9} “A neutral change neither saves nor uses labour (capital) it is one which produces variation in the production relation itself but does not affect the marginal rate of substitution of labour for capital (capital for labour). A non-neutral technological change alters the production function and can be either labour saving (i.e. capital using) or capital saving (i.e. labour using)”. According to Murray Brown\textsuperscript{10} “A neutral change alters the production function but does not affect the marginal rate of substitution. A non-neutral change does affect the marginal rate of substitution. If the marginal product of capital increases relative to that of labour, for given labour-capital combination, then a labour saving or capital using change raises the marginal product of labour relative to that of capital and vice versa.

An increase in capital intensity is always labour saving and decrease is capital saving. An increase in capital intensity increases output if labour is growing more


slowly than capital. An increase would reduce output if capital is slower growing factor”.

A non-neutral technological change is depicted by variation in the ratio of two elasticities of production that is $\alpha$ changes relative to $\beta$. Factor saving or factor using technological gains are indicated by the direction of the change of the ratio $\beta/\alpha$. If $\beta$ raises relative to $\alpha$, then a capital using technological change has occurred. Prof. Tinbergen introduced an additional source of change in labour (and capital) productivity which he calls raise in efficiency. This efficiency component is specified by an exponent $e^u$ so that the production function becomes

$$Q = A_\alpha e^u F(L,K) \quad \ldots \quad (1.3)$$

where $F(L,K)$ is any well defined form of production function model.

In this study a new non-neutrality function has been introduced as under:

$$f(\lambda, \mu) = \frac{A_\alpha e^u}{1 + ce^u} \quad (1.4)$$

So that the production function will be of the form

$$Q = \frac{A_\alpha e^u}{1 + ce^u} F(L,K) \quad \ldots \quad (1.5)$$

Different forms of production function models for $F(L,K)$ can be considered and they can be inserted in the above equation (1.5) to generate new class of production function models which considers the desired technological changes.

1.2.4 PRODUCTIVITY RATIOS

Productivity does not have any clear-cut definition, there are as many expressions and interpretations as there are number of users. Productivity is broadly expressed
as overall efficiency with which any industry perform, here productivity is in terms of industrial efficiency. The ratio of the output of the commodity to the input of the factor is the measure of productivity in relation to the particular factor of production. The choice of the factor depends upon the purpose of inquiry, but generally productivity is measured in terms of labour and capital. Productivity means the physical volume of output attained per worker or per man-hour or capital. Thus productivity is defined as the ratio of output to the corresponding input of labour or capital. According to Sir Ewart Smith and Dr. R. Beeching, measurement of productivity consists in determining the output relative to the total effort expanded. Thus productivity means the volume of output achieved in a given period is a relation to the sum of direct and indirect effort expanded in its production.

The first empirical attempt to measure total factor productivity was made by Jan Tinbergen in the year 1942. The concept of TFP was further elaborated by John Kendrick in the year 1951. Later on Robert Solow explicitly used a production function framework, which helped to establish TFP as an operational concept. Kendrick defines the partial productivities as the ratio of value added to the corresponding input. That is labour productivity is defined as the ratio of output to the corresponding labour input.

\[ i.e. \ LP_t = \frac{Q_t}{L_t} \quad \ldots (1.6) \]

where \( LP_t \) = Labour Productivity for the period \( t \)

\( Q_t \) = Output for the period \( t \)

\( L_t \) = Labour Input for the period \( t \)

and its index is defined as

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\[ I_t(LP) = \frac{LP_t}{LP_0} \times 100 \]

where \( LP_0 \) = Labour Productivity in the base year

\[ I_t(LP) = \text{Index of Labour Productivity for the period } t \]

Similarly other partial productivity ratios can be calculated.

The total factor productivity is computed by the formula given by Kendrick which is as under:

\[ TFP_t = \frac{Q_t}{\alpha N_t + \beta F_t}, \quad t = 0,1,2, \ldots \]

where \( \alpha = \frac{W_o}{N_o} \) and \( \beta = \frac{Q_o - W_o}{F_o} \)

where \( \alpha \) and \( \beta \) are the marginal productivities which represents the base year wage rate and the base year return on per rupee of the fixed capital for the respective industrial sector as a whole. Where \( Q_o, W_o, N_o \) and \( F_o \) are NVA, wages to workers, number of workers and fixed capital respectively in the base year.

Thus the concept of productivity is of considerable importance as productivity is an index of economic welfare. Productivity is more important than production because the difference in the economic conditions of advanced and underdeveloped countries are due more to the differences in productivity among them rather than to their volume of output. An improvement in productivity can result in extra production without a corresponding increase in either plant or equipments. Thus productivity measure is an important tool of economic and social analysis.
1.3 VARIOUS FORMS OF PRODUCTION FUNCTIONS

There can be many mathematical models representing the production functions. Here an attempt is made to visualise some specific standard production function models and their basic properties are highlighted.

1.3.1 COBB-DOUGLAS PRODUCTION FUNCTION

In the area of production function the first remarkable contribution was given by Cobb C.W. and P.H.Douglas/43/. They have proposed a production function in which the relationship between labour and capital inputs and output was explained, which later on became popular as Cobb-Douglas Production function model.

The general form of Cobb-Douglas production function model is

\[ Q = A L^\alpha K^\beta \]  

where \( Q \) = output

\( L \) = Labour input

\( K \) = Capital input

\( A, \alpha \) and \( \beta \) are positive parameters.

Some properties of Cobb-Douglas production function model are as under:

(i) In unrestricted form, the CD production function model for two input factors can be written as

\[ Q = A L^\alpha K^\beta \]

where \( A, \alpha \) and \( \beta \) are constants to be determined.
The marginal product of labour is denoted by $MP_L$ and it is defined as under:

$$MP_L = \frac{\partial Q}{\partial L}$$

$$= \alpha A L^{\alpha-1} K^\beta$$

$$= \frac{\alpha Q}{L}$$

Hence

$$\alpha = \frac{L}{Q} \frac{\partial Q}{\partial L}$$

Similarly,

$$\beta = \frac{K}{Q} \frac{\partial Q}{\partial K} \quad \left\{ \right.$$ ...

(1.10)

$\alpha$ is called partial elasticity of production with respect to labour. It denotes the percentage change in output attributable to percentage change in labour input keeping capital as constant. Similarly $\beta$ is the partial elasticity of production with respect to capital inputs. $\alpha$ and $\beta$ represent individually the percentage change in output for percentage change in labour and capital. The two coefficients taken together measures the total percentage change in output for a given percentage change in labour and capital and $\alpha + \beta$ is known as degree of homogeneity of CD production function model. For CD production function model the returns to scale are characterised as under:

$$\alpha + \beta < 1 : \text{diseconomy of scale or decreasing returns to scale}$$

$$\alpha + \beta = 1 : \text{constant return to scale}$$

$$\alpha + \beta > 1 : \text{economy of scale or increasing return to scale.}$$

(ii) The unitary elasticity of substitution is a famous property of the Cobb-Douglas production function model. It guarantees that relative income shares of capital labour are constant for any changes in the relative supplies
of capital and labour. It provides a rational for the 'relative' constancy of factor shares observed in developed economy.

(iii) If the inputs are increased by \( \lambda \) times, the total output will also be increased by \( \lambda \) times.

i.e. if the input \( L \) and \( K \) of CD production function model are multiplied by \( \lambda \) then

\[
Q' = A(\lambda L)^\alpha (\lambda K)^\beta
\]
\[
= \lambda^{\alpha+\beta} A L^\alpha K^\beta
\]
\[
= \lambda^{\alpha+\beta} Q
\]

If \( \alpha+\beta = 1 \) that is production under constant returns to scale is operating,

\[
Q' = \lambda Q
\]

(iv) Labour and capital both are essential factors of production if any one of them is zero the output is zero.

(v) The CD production function is homogeneous of degree \( \alpha+\beta \), if \( \alpha+\beta=1 \) then it is homogeneous of degree one.

This can be proved with the help of Euler's theorem. The CD production function model is

\[
Q = A L^\alpha K^\beta
\]

Differentiating both the sides with respect to 'L.'
\[ \frac{\partial Q}{\partial L} = \alpha Q \]

\[ L \frac{\partial Q}{\partial L} = \alpha Q \] \hspace{1cm} \cdots (1.12)

Again partially differentiating with respect to \( K \)

\[ \frac{\partial Q}{\partial K} = \beta Q \]

Therefore \[ K \frac{\partial Q}{\partial L} = \beta Q \] \hspace{1cm} \cdots (1.13)

Now by Euler’s theorem

\[ L \frac{\partial Q}{\partial L} + K \frac{\partial Q}{\partial K} = \alpha Q + \beta Q \] \hspace{1cm} \cdots (1.14)

\[ = (\alpha + \beta) Q \]

If \( \alpha + \beta \) is one the Euler’s theorem is satisfied for CRT C.

(vi) In the linear and homogeneous production function \( Q = A L^\alpha K^\beta \), \( \alpha \) and \( \beta \) represent the labour share and capital share to the total output respectively.

### 1.3.2 CES PRODUCTION FUNCTION

In Cobb-Douglas production function the elasticity of substitution is unity everywhere. However, there are situations where the elasticity of substitution may be constant but not necessarily unity. In such situations Constant Elasticity of Substitution (CES) production function model was derived by two different groups of researchers. One consists of the researchers, K.J. Arrow, Chenery, B.C. Minhas and R.M. Solow (ACMS) and the other group has M Brawn and De Cani as the researchers. Although their derivations were different but the results were the same, ACMS suggested the form of CES production function model as under: 

\[ Q = A L^\alpha K^\beta \]
\[ Q = A_o \left[ \delta K^{-\rho} + (1-\delta)L^{-\rho} \right]^{-\nu/\rho} \] \hspace{1cm} \ldots (1.15)

\( A_o > 0, \, 0 < \delta < 1, \, \rho > -1 \)

where

\( A_o = \) Technical Efficiency Coefficient

\( \rho = \) Substitution parameter

\( \nu = \) degree of homogeneity

\( 1-\delta = \) labour intensity coefficient

The elasticity of substitution for the CES production function model defined above is given by

\[ \sigma = \frac{1}{1+\rho} \]

The other forms of CES production function models with their respective elasticity of substitution are given below

(i) \[ Q = A_o \left[ \delta_1 L^{-\rho} + \delta_2 K^{-\rho} \right]^{-\nu/\rho}; \quad \sigma = \frac{1}{1+\rho} \hspace{1cm} \ldots (1.16) \]

(ii) \[ Q = A_o \left[ \delta_1 L^{-\rho'} + (1-\delta)K^{-\rho'} \right]^{-\nu/\rho'}; \quad \sigma = \frac{1}{1-\rho'} \hspace{1cm} \ldots (1.17) \]

(iii) \[ Q = A_o \left[ \frac{L^\rho K^\rho}{\delta_1 L^{-\rho} + (1-\delta)K^{-\rho}} \right]^{-\nu/\rho}; \quad \sigma = \frac{1}{1-\rho} \hspace{1cm} \ldots (1.18) \]
1.3.3 VES PRODUCTION FUNCTIONS

The elasticity of substitution can be considered as linear function of the capital-labour ratio. In other words 'α' becomes a function of the per capita capital investment, that is 'α' varies with the capital-labour ratio, hence the study of Variable Elasticity of Substitution (VES) production function model are needed. Sato and Hoffman/178/ have developed VES production function model dropping the assumption of Constant Elasticity of Substitution. Lu and Fletcher/135/ also assumed that the elasticity of substitution depends upon the input ratio. Zellner and Revenkar/225/ have given the generalised approach to VES production function model. Kadiyala etc./114/ have also given the same approach as Zellner and Revenkar with Cobb-Douglas and CES as special case. Lovell/134/, Ferguson/66/, Plasmans/161/ etc. have also largely contributed in this area. Revenkar/171/ gave the form of VES production function model as under:

\[ Q = A_o K^{\alpha(1-\delta p)} \left[ L + (p-1)K \right]^{\alpha p \delta} \]  \hspace{1cm} (1.19)

where \( A_o > 0, \alpha > 0, 0 < \delta < 1, 0 \leq p \delta \leq 1 \)

The elasticity of substitution is obtained as:

\[ \sigma = 1 + \left( \frac{p-1}{1-p\delta} \right) \frac{K}{L} \]  \hspace{1cm} (1.20)

Here Revenkar observed that 'α' varies linearly with capital labour ratio around the intercept term unity. For 'α' to be positive in the empirically relevant range of K/L it required to have the following condition to be satisfied.

\[ \frac{L}{K} > \frac{1-p}{1-p\delta} \]
Another form of VES production function model was suggested by Revenkar /171/, Wolkowitz/213/ and the others which is explained as under:

\[ Q = A \cdot e^{\lambda t} \left[ (1 + \beta)KL^\beta + \alpha L^{1+\beta} \right]^{1+\beta} \]  

... (1.21)

The elasticity of substitution for the above VES production function model was derived as under:

\[ \sigma = 1 + \frac{\alpha}{\beta} \frac{L}{K} \]  

... (1.22)

\[ \sigma = 1 + \frac{\alpha}{\beta} k^{-1}, \quad k = \frac{K}{L} \]

\[ \sigma \geq 1 \]

\[ \frac{d\sigma}{dk} \geq 0 \]  

as \( \alpha \geq 0 \)

Ferguson/64/ defines the form of VES production function model as under:

\[ Q = A K^\alpha L^\mu e^{\mu k} \]  

... (1.23)

where \( k = \frac{K}{L} \)

for which the elasticity of substitution was derived as under:

\[ \sigma = 1 - \frac{\mu k}{(\alpha + \mu k)^2 - \alpha} \]  

... (1.24)

In the above VES model defined by Ferguson if the assumption of constant return to scale is dropped, then the model reduces to
\[ Q = A_0 K^\alpha L^\beta e^{\mu k} \] ... (1.25)

For this model the elasticity of substitution is given as under:

\[ \sigma = \frac{1}{\frac{\alpha}{\alpha + \mu k} - \frac{\mu k}{\beta - \mu k}} \] ... (1.26)

1.4 REVIEW OF LITERATURE

There are many researchers who have attempted to do sizable and significant work in the areas related to this study. In this section, it is my humble approach to make an attempt to present here some essential research work done by them in the form of review of literature.

1.4.1 PRODUCTION FUNCTIONS

Dutta/56/ has used the cross-section data for the year 1946-47 for the Indian manufacturing sector. He has considered Cobb-Douglas production function model and found the constant return to scale (CRS). Murty and Sastry/151/ attempted the same as Dutta/56/ for the industrial sector as a whole as well as some of the group of the industries for the year 1951-52. They have observed that the sum of elasticity of substitution with respect to capital and labour was different from unity indicating constant returns to scale. Dutta and Majumdar/57/ have also arrived at the similar conclusions on the basis of time series data for the Indian manufacturing sector for the decade 1951-61.

Yeh/219/ has considered 29 industries for the years 1953-58. In his study, it was observed that 17 industries which were enjoying increasing returns to scale
(IRS), two industries were having CRS and the remaining ten industries were having decreasing returns to scale (DRS). Similar variations were observed by Diwan and Gujarati/50/ at the 29 individual industries level, Jaiswal/104/ has used time series data for the industrial sector as a whole during the years 1946-58 to estimate Cobb-Douglas production function model for three input factors namely, capital, labour and raw materials and observed that a constant returns to scale exists. Rama Rao and Anjuneyula/166/ have estimated Cobb-Douglas production function for cotton textile industry using time series data and they have observed a decreasing returns to scale.

Narsimha and Fabrey/153/ have obtained estimates of returns to scale for 28 industries during the years 1946-58. They have considered Cobb-Douglas, CES and homothetic isouquant and observed a constant returns to scale.

Banerji/18/ has considered Cobb-Douglas production function model using both time series and cross-section data for jute and textile industry (1946-63), sugar industry (1946-52), and paper and bicycle industries (1946-58). Cotton and jute textile industry showed CRS and paper and bicycle industries showed IRS. Similar facts have been observed by Gupta and Patel/88/ for sugar industry during the period 1946-66 and Acharya and Nair/1/ for cement industry during the year 1959-71.

Mehta/142/ has studied individual industries considering Cobb-Douglas and CES production function model during the years 1953-65 and observed CRS in most of the cases except four industries.

Subramaniyan/204/ has made an attempt to study the interstate comparison for sugar industry. He has considered Cobb-Douglas production function model during the years 1953-69 and observed IRS for all India and Tamil Nadu state and CRS in Bihar, Maharashtra, Uttar Pradesh and Andhra Pradesh states.
Agarwal/3/ has estimated Cobb-Douglas production function model for 20 industries for the years 1967-71 (for 7 industries) and 1975-80 (for 13 industries) using ASI data. The estimated value of the degree of returns to scale parameters as obtained by the sum of coefficients of labour, capital and raw materials turn out to be very close to unity for all 20 industries.

Sen Gupta/186/ has estimated CES production function model for seven industries during the years 1948-58 to test the hypothesis of elasticity of substitution ($\sigma$) equal to zero and one separately and found no consistent results in these industries.

Diwan and Gujarati/50/ have estimated CES production functions model for 28 Indian industries for the time series data during the years 1946-58. They have observed that only the starch industry and cement industry are having the elasticity of substitution greater than one and for the remaining industries, it is observed that $\sigma < 1$.

Venkatswami/212/ has concluded that elasticity of substitution equals to unity in 21 out of 28 industries under study during the years 1948-67.

Sankar/179/ has used CES production function with modification and the coefficients for 15 industries during the years 1946-58 are found to be significant. He has also observed that 50% estimates were close to unity.

Banerji/16/ has considered five different forms of CES production function models to estimate elasticity of substitution for the Indian industries during the years 1946-58. Mehta/143/ has also considered five different models of CES production functions models to study the elasticity of substitution for 27 large scale Indian industries using time series data.

Kazi/117/ has studied elasticity of substitution for nine industries considering cross section data for the years 1960, 1961 and 1962. He applied OLS method
and instrumental variable technique to the CES production function model and compared the bias resulting from OLS method, the estimates vary between zero and one. The estimates of $\sigma$ calculated using CES and VES production function models showed that there is an upward bias in the elasticity of substitution by the CES production function model. The distribution of $\sigma$ by VES production function showed that 75% of them were below unity.

Another study was made by Kazi [117,1] using the cross-section data for the years 1973, 1974 and 1975 considering CES and VES production function models.

Reddy and Rao [169] observed the neutral technical progress in Indian industries during the years 1946-57. They have considered Solow's method using $Q= A(t) F(L,K)$.

Sen Gupta [186] observed that neutral technical change is not significant in cement industry but it is significant in iron and steel industry and sugar industry. Venkatswami [212] observed that technical change varies among industries while fitting Cobb-Douglas and CES production function models.

Banerji [18,19] found that there is no technical progress in cotton textile and jute textile industries whereas it is observed in sugar industry, paper industry and bicycle industry.

Rai and Mishra [162] have fitted Cobb-Douglas, CES and VES production function models with technology parameter for Bihar state and All India data during the years 1946-71 and concluded that the elasticity of substitution for Cobb-Douglas production function model is significantly different from unity and CES production function is not merely a virtue of elimination technique rather due to capital labour ratio and VES production function model is more appropriate for both the industrial sectors.
Rajakshmi/165/ has considered VES production function model to estimate elasticity of substitution for industrial sector of Rajasthan and all India data during the years 1960-75 and concluded that capital using technology is observed in Rajasthan state.

Singh and Singhal/193/ have considered CD and CES production function models for the manufacturing industries of Punjab state using ASI data during the years 1967-80. They have observed that the output growth in these sectors has been achieved through the increased factor inputs and not through the technical progress and economies of scale.

Lee and Tyler/129/ have made an attempt to consider a stochastic frontier Cobb-Douglas production function model to a cross-section of Brazilian enterprise data. They have made the comparison between OLS estimates and stochastic frontier maximum likelihood estimates (SFMLE) for inter-firm Cobb-Douglas production function model. They have observed that on the stochastic frontier the intercept term, indicating the level of technical efficiency is greater than in the OLS, also by SFMLE the output elasticity for capital is higher and that for labour is lower than by OLSE.

Broeck etc./28/ made the comparison between two different techniques for estimating deterministic and stochastic frontiers for cross-section data for 28 Swedish dairy plants for the years 1964-1973. They have considered Linear Programming (LP), Maximum Likelihood (ML) and with Composed Error (ML-CE) methods. They concluded that the parameters estimated from the composed error model are more stable than the others.

Another study made by Meeusen and Broeck/41/ was about efficiency estimation from Cobb-Douglas production function model with composed error. They have considered the data from 1962 French Census of manufacturing industries.
have arrived at the conclusion that on an average sectoral efficiency for the industries at issue which lies between 0.70 and 0.94.

Greene/80/ has described a method of estimating the frontier production function model using the translog functional form. As an application, he has considered the time series data for the U.S. manufacturing sector during the years 1947-71. Earlier Berndt and Wood/25/ estimated a translog cost function using the same data. They assumed the constant return to scale in their study but no restriction was placed on partial elasticities of substitution.

Bhavani/27/ has selected four metal product industries to study the technical efficiency in a frontier model. The data for the selected industries were collected from the Census of Small Scale Industrial Units 1973. He has concluded that the selected metal product industries belonging to modern small scale sector are reasonably efficient in production and hence contribute to the growth objective.

Page Jr./156/ has applied a four factor frontier translog production function to measure technical inefficiency in four Indian industries during the years 1979-81. He has examined the relationship between firm size and relative levels of technical efficiency - and hence total factor productivity. He has concluded that out of four industries only one industry showed consistent correlation between the firm size and technical efficiency, for other industries there was no systematic relationship between firm size and relative efficiency.

Stevenson/203/ has made an attempt to test the empirical significance of the generalised stochastic frontier model by two different methods OLS and ALS (Aigner, Lovell and Schmidt/6/). The generalised frontier parameter estimates are derived for US primary metal industry (SIC 33) 1975-58 data. He has observed that generalised ALS method gives more significant estimates.
Jondrow et al. have proposed a method of separating the error term of the stochastic frontier model into the components for each observation. They have considered half normal and exponential distributions for the random error term.

Ramaswami has considered Cobb-Douglas production frontier for the four manufacturing industries in India. He has concluded that estimates indicate lower intra-industry variations in technical variation in technical efficiency, and small-scale firms operate in a strong competitive environment. He has also suggested that the scope for output gains with existing input quantities and combination is rather limited.

Zellner et al. have considered the specification and estimation of Cobb-Douglas production function model. They have used maximum likelihood estimation and Bayesian analysis for the CD production function model.

Solow has studied technical change in the US using aggregate production function during the years 1909-1949. He has suggested a simple way of segregating shifts of the aggregate production function from movements along it. He has observed a neutral technical change during that period. He has also observed that production function showed increasing behaviour at an AGR of about 1% per year for the first half and 2% per year for the second half.

Revenkar has studied theoretical properties of various forms of VES production function models.

Zellner and Richard have used Bayesian analysis approach to the US economy formulated and estimated by Morishina and Saito. They have concluded that Bayesian approach gives better results as compared to classical least squares and two stage least squares.

Richmond has estimated average efficiency for Norwegian manufacturing industries in the year 1963. He has assumed Cobb-Douglas production function model.
and observed that the estimated average efficiency levels range from about 75% to about 96% whereas the estimate for total manufacturing was about 87%.

Golder/74/ has studied the economic efficiency in small scale washing soap industry in India for the year 1972. He has also calculated estimates of partial productivity measures, relative efficiency and capital intensity and concluded that tiny washing soap units are inefficient as compared with relative bigger units within small scale washing soap industries.

Aigner etc./5/ have suggested a new approach to the frontier production function. They have considered the data on the US primary metal industry (SIC 33) consisting of observations across 28 states for the years 1957-58. Earlier these data set were analysed by Aigner and Chu/4/ who have suggested the estimation of the parameters by mathematical programming methods. Hildbrand and Liu/98/ also have analysed the same data.

Love/134/ has derived one form of CES and two forms of VES production function models and they were considered for the US manufacturing sector over the period 1947-68. They have estimated the entire parameter set for each function, which produces economically and statistically acceptable results. He has observed that the average value of the elasticity of substitution was about 0.47, which is significantly less than unity. Other research workers ACMS/11, (1909-49), Ferguson/65, (1929-63), Kendrick and Sato/121, (1919-60) and Kravis/127, (1900-57) all have found the elasticity of substitution for CES production function model and it lies between 0.57 and 0.67 for various definitions of the aggregate US economy. He has also observed that from both VES production function models the elasticity of substitution varies significantly with movements in capital-labour ratio.
Kmenta/125/ has estimated the CES production function model under non-constant returns to scale. He has also utilised simultaneous equation method with uniform prices and non-uniform prices and the estimates are obtained.

Kumbhakar/128/ has considered the generalised production frontier for 42 US class 1 railroads for the period 1951-75. He has considered specification and estimation of technical and allocative inefficiency for the above data. The whole data were divided into five groups and the mean cost of technical inefficiency over the subperiods were observed as 12.15%, 7.79%, 9.38%, 10.56% and 8.23% respectively. He has also observed that the degree of allocative inefficiency in labour is less than that of fuel.

Olson etc./154/ have estimated stochastic frontier production function model by Monte Carlo method, and compared it with the stochastic frontier production function model of the type introduced by Aigner, Lovell and Schmidt/6/ and Meeusen and Van den Broeck/141/. They have concluded that MLE tended to outperform COLS in sample size larger than 400 whereas COLS tended to outperform MLE in sample size of less than 400.

Bauer/23/ has studied a brief review work in the area of Econometric estimation of frontiers. The measure of efficiency derived by Farrell/63/ is widely used. The econometric approach to estimating frontiers, which represents technology along with two-part composed error term was first proposed by Aigner, Lovell and Schmidt/6/ Meeusen and Van den Broeck/141/, and Batlese and Cosra/20/. Some research workers have used one sided distributions. Aigner, Lovell and Schmidt/6/ have used the half normal and exponential distribution. Stevenson/203/ has used truncated normal distribution and the two parameter Gamma distribution was proposed by Greene/81/ other composed-error distribution could be constructed following Greene's/81/ methodology.
Aigner, Amemiya and Piorier/5/ have estimated the parameters of a discontinuous density function by MLE. They have produced a "theory of outliers" for the frontier function. They concluded that the error term might be represented by normal distribution whereas technological differences among firm may be represented by the negative truncated normal.

Green and David Mayes/78/ have examined technical inefficiency of manufacturing industry in the United Kingdom for 151 industries for the year 1977. They have used translog stochastic frontier production function model and decomposed the error term into two components, one measuring inefficiency and other unobservable random factors. They have observed that the results obtained were having similar characteristics to those for the United States, Japan, and Australia. They have also concluded that in about half the cases, the residual from the stochastic frontier could not be decomposed. Kadiyala/114/ has extended the CES production function to a class of production function which are homogeneous and which possess the property that the elasticity of substitution varies with the input ratio.

Zellner and Revenkar/225/ have introduced generalised production function and estimated for the US transport equipment industry in the year 1957. The parameters were estimated by the maximum likelihood method.

Houthakkar/103/ has studied the mathematical properties of the Pareto distribution and Cobb-Douglas production function model in activity analysis.

Timmer/206/ has used linear programming techniques to estimate a frontier Cobb-Douglas production function model for US agriculture for the years 1960 to 1967. He has observed that the average state was only 7.6% away from the frontier in the 98% LP model. He has also observed from technical efficiency at the state level that it may be possible for substantial productive inefficiencies to exist within.
states with few observed differences between states. The significance of the allocative inefficiencies suggests that it has small welfare impact in a competitive setting.

Some condition for collapsing the production function, properties of the index function with corresponding price index were studied by Solow/199, 200/. He has also presented a linear programming model for the production function.

The Bayesian estimation of CES production function with its application to the cross-section data of Indian textile industry for the year 1960-61 was studied by Chetty and Sankar/37/.

After the pioneer work of Farrell/63/, the serious consideration has been given to the possibility of estimating frontiers in an effort to bridge the gap between theory and empirical work. Farrell’s approach has been extended and applied by Aigner/7/, Fare and Lovell/61/, Forsund and Hjalmarsson/71/, Aigner and Chu/4/ were the first to follow Farrell’s suggestion. They have specified a homogeneous Cobb-Douglas production frontier and required all observations to be on or beneath the frontier. Forsund and Jansen/70/; Forsund and Hjalmarsson/71/ have relaxed the restrictive homogeneous Cobb-Douglas form. Aigner/7/ was the first to propose explicitly non-stochastic statistical production frontier model. which has been considered further by Richmond/173/; Schmidt/183/ and Greene/79/. Richmond/173/ has apparently first noted the correlation in OLS estimates. Schmidt proposed that under certain circumstances least squares may be an appropriate estimation methodology even for a frontier production function with one sided disturbance but statistical properties of estimators remain uncertain. Nonstochastic production frontier is also discussed in Chen and Tang/36/, Sharma/188/, Ramaswamy/168/, Sinha and Sharma/190/ and others. Sharma and Sinha/190/ have estimated and compared technical and allocative inefficiency of private and public enterprises. Aigner, Lovell and Schmidt/6/ and Meeusen & Van den Broeck/141/ proposed a new specification for frontier production function models. Stochastic production frontier and inefficiency is also
nicely discussed in Olson, Schmidt and Waldman [154], Sharma [188], Bhavani [27], Ramaswamy [168], Shiyani and Deb [192] and others.

Caselli [33, A] has studied technological revolution through generalised production function for US. He has observed that increase in wage inequality is associated with increased inequality in capital labour ratios. As an overview, in the history of technology indices, the model may have interpretative power for other technological resolutions, including de-skilling ones.

Reinhard, Lovell and Thijssen [170, A] have estimated technical and environmental efficiency of a panel of Dutch dairy farms. A stochastic translog production frontier was specified to estimate the output-oriented technical efficiency. Environmental efficiency was estimated as the input oriented technical efficiency of a single input and they have concluded that intensive dairy farms were both technically and environmentally more efficient than extensive farms.

1.4.2. PRODUCTIVITY ANALYSIS

Singh [195] has studied the productivity trends for the Indian industries during the years 1951-63. He has observed that labour productivity has an increasing trend whereas capital productivity has decreasing. Similar behaviour is observed by Shivamaggi etc. [191] for seven industries during the years 1951-61.

Rajkrishna and Mehta [163] have considered 27 industries using CMI data during the years 1946-58 and ASI data for the period 1959-66. They have concluded that the overall productivity shows a declining trend.

Banerji [16] has calculated both the partial productivity measures namely labour productivity and capital productivity for the years 1946-64 by using the simple ratio of net value added to the respective inputs and observed that labour productivity is rising through capital deepening.

Ommen/155/ has considered broad industry group for the years 1959-69. He has estimated output elasticities and productivity changes using Cobb-Douglas production function model and concluded that the elasticity of substitution of output to labour input is significant. He has also observed that there is an upward productivity trend in most of the industries.

Mukherjee/148/ has made comparative study of productivity in large scale industries of Bihar state as compared to all India. He has considered CMI data during the years 1950-58 and ASI data during the years 1959-67. He has observed that the index of total factor productivity is influenced by the capital productivity at both state and national levels.

Rao/166,A/ has considered mining and registered manufacturing cross section data for the years 1960-61, 1970-71 and 1979-80 to calculate output input ratio (productivity ratio) and observed a steady decline.

Sastry/180/ has studied the trend in the average productivity of labour and capital in the cotton textile industry in India.

Banerji/19/ has constructed the average productivity ratios of labour and capital for the years 1946-64, the same was estimated by Jaiswal and Jani/106/ for the years 1949-66. Reddy and Rao/169/ have observed a declining trend in TFP computed by Solow's measure of the TFP for the years 1946-57.

Hashim and Dani/90/ have observed that Solow's index of TFP has an AGR of about 3% per year and contribution of TFP growth to output growth was around 50%. Sinha and Sawmney/196/ have observed increasing behaviour in Kendrick's index of TFP over the period 1950-63 for five major industries in India. Jani and Jaiswal/105/ have observed a declining trend in Kendrick's measure of TFP for 14 major industries for the cross section data for the years 1951-52, 1956-57, 1961-62 and 1971-72.
Rajkrishna and Mehta/163/ have observed a declining behaviour in total productivity over the period 1946-64.

Mehta/143/ has observed a declining behaviour in TFP by both, Kendrick’s and Solow’s measure of TFP for the years 1953-65. He has also calculated the partial productivity measures.

Sastry/180/ has studied TFP for cotton textile industry for the years 1949-70. He has used three measures of TFP namely Kendrick’s, Solow’s and Domar’s measures of TFP for all India and also for two specific regions. He has observed an increasing behaviour upto the year 1961 and after this year a declining trend was observed. Sastry had used energy consumption instead of capital.

Singh, Kochak and Malhotra/194/ have calculated labour productivity and capital productivity for paper and paper board industry in India. They have observed that labour productivity is related with capital intensity i.e. capital per labour unit. This was expected, considering the law of substitution between factors of production.

Ball, Gollop, Hawke, and Swinand/15/ have studied agricultural productivity growth in U.S. at both sector and state level. They have concluded that the smooth persistently positive trend was observed for farm sector productivity growth which makes considerable variation across states and region. They have also concluded that farm productivity growth is a function of productivity trends in the individual states.

Bernard and Jones/24/ have estimated labour productivity, total factor productivity for 14 industrialized countries during the years 1970-87. They have also introduced a new measure called multifactor productivity and Total Technological Productivity (TTP).
Fare, Grosskopf, Norris and Zhang/62/ have analysed productivity in 17 OECD (Organization for Economic Co-operation and Development) countries over the period 1979-88. A non-parametric programming method (activity analysis) was used to multiquest productivity indices, they have estimated two measures namely technical change and efficiency change. They have observed that US productivity growth was slightly higher than the average and that was due to technical change. They have also observed that Japan’s productivity growth was highest in the sample, and that was due to efficiency change.

Park and Kwon/157/ have studied TFP for 28 Korean manufacturing sector during the period 1966-89. They have observed market imperfection, the existence of scale economies and bias in the traditional TFP measure. They have also observed that a rapid growth of output is possible with negative TFP growth.

 Flaig and Steiner/69/ have estimated traditional and adjusted measure of TFP growth for West German manufacturing two digit industries during the years 1961-85. They have observed aggregation bias in productivity growth. They have also observed that there has been neither a long term productivity slowdown nor a structural break in TFP growth at the industry level within the estimation period

1.5 SOME BASIC DEFINITIONS

We shall discuss here some conceptual definitions of the basic terms used for data analysis as per the norms given by CMI or ASI published by CSO, Government of India/76/.

(l) FIXED CAPITAL

Fixed capital represents the depreciated value of fixed assets owned by the factory as on the closing day of the accounting year. Fixed assets are those which...
have a normal productive life of more than one year. Fixed capital used, covers all types of assets, new or used or own constructed, deployed for production, transportation, living or recreational facilities, hospitals, schools etc. for factory personnel. It includes the fixed assets of the head office allocable to the factory and also the full value of assets taken on hire-purchase basis (whether fully paid or not) excluding interest element. It excludes intangible assets and assets solely used for post manufacturing activities such as sale, storage, distribution etc.

(ii) PHYSICAL WORKING CAPITAL

Physical working capital is defined to include all physical inventories owned, held or controlled by the factory as on the closing day of the accounting year, such as the materials, fuels and lubricants, stores, etc. that enter into products manufactured by the factory itself or supplied by the factory to others for processing. Physical working capital also includes the stock of materials, fuels and storage etc. purchased exclusively for resale, semi-finished goods and work in progress on account of others and goods made by the factory which are ready for sale at the end of accounting year. However, it does not include the stock of the materials, fuels, stores, etc. supplied by the others to the factory for processing. Finished goods processed by others from raw materials supplied by the factory and held by them are included and finished goods processed by the factory from raw material supplied by the others are excluded.

(iii) INVESTED CAPITAL

Invested capital is the total of fixed capital and physical working capital as defined above.
(iv) WORKING CAPITAL

Working capital is the sum total of the physical working capital already defined above and the cash deposits on hand and at bank and the net balance of amount receivable over amounts payable at the end of the accounting year. Working capital, however, excludes unused overdraft facility, fixed deposits irrespective of duration, advances for acquisition of fixed assets, loans and advances by proprietors and partners irrespective of their purpose and duration long-term loans including interest thereon and investments.

(v) PRODUCTIVE CAPITAL

Productive capital is the total of fixed capital working capital as defined above.

(vi) NUMBER OF WORKERS

The number of workers are defined to include all persons employed directly or through any agency whether for wages or not and engaged in any manufacturing process or in cleaning of any part of the machinery or premises used for manufacturing process or in any other kind of work incidental to or connected with the manufacturing process or the subject of manufacturing process. Labour engaged in the repair and maintenance or production of fixed assets for factory’s own use or labour employed for generating electricity or producing coal, gas are included. However, persons holding position of supervision or management or employed in administrative office, store keeping section and welfare section, sales department as also those engaged in the purchase of raw materials etc. and in production of the fixed assets for the factory and watch and ward staff are excluded.
(vii) WAGES TO WORKERS

Wages are defined to include all remuneration capable of being expressed in monetary terms and also payable more or less regularly in each pay period to workers (defined above) as compensation for work done during the accounting year. It includes

(a) direct wages and salaries (i.e. basic wages/ salaries, payment of overtime, dearness, compensatory, house rent and other allowances).

(b) remuneration for period not worked (i.e. basic wages, salaries, and allowances payable for leave period, paid holiday, lay-off payments and compensation for unemployment, if not paid from the sources other than employers),

(c) intervals (i.e. incentive bonuses, good attendance bonuses, productive bonuses, profit sharing bonuses, festival or year end bonuses, etc.).

It excludes lay-off payment which are made from trust or other special funds setup exclusively for this purpose, i.e. payments not made by the employer. It also excludes imputed values of benefits in kind, employer’s contribution to old age benefits and other social security charges, direct expenditure on maternity benefits and other group benefits. Travelling and other expenditure incurred for business purpose and reimbursed by the employer are excluded. The wages are expressed in terms of gross value i.e. before deduction for fines, damages, taxes, provident fund, employee’s state insurance contribution etc.

(viii) NET VALUE ADDED

Net Value Added (NVA) is the increment to the value of goods and services that is contributed by the factory and is obtained by deducting the value of total inputs and depreciation from value of output. It is also referred to as value added by manufacturer (VAM).
1.6 DATABASE

Data are collected from different issues of All India Survey of Industries, Factory Sector, Summary and Results. Vol. I published by Central Statistical Organisation, Government of India/76/ and Chandhok's/35/ volume of “Indian Database : The Economy” published by McGraw Hill International Book Co., New York. The basic factors are Net Value Added by manufacturer, input factors like labour employed as number of workers or wages to workers, and capital employed as invested capital, fixed capital or productive capital. The results are obtained on the basis of constant prices series. For this purpose net value added are deflated by index of industrial production. The wages to workers are deflated by consumer price index number and fixed capital, working capital and productive capital are deflated by wholesale price index numbers. All these indices are considered 1980-81 as the base year and they are published by CSO, Ministry of Industry, Government of India/77/.

1.7 METHODOLOGY

In almost all the chapters the solution to the estimation of production function model are obtained by reducing the model into either linear or log-linear form by taking logarithm and suitable approximation when needed, and applied the OLS method. For estimation of CD and VES production function models, log linear equations are obtained by taking logarithms on both the sides, and the estimates of the parameters of these production function models can be obtained by using OLS method. For estimating the CES production function model, the method suggested by Kmenta is adopted, that is first taking the logarithms on both the sides and then utilising Taylor's series expansion around $\rho = 0$ and upto the second order of approximation a log-linear equation is obtained. Then the OLS method gives the estimates of the parameters of log-linear equation which are the function of parameters of CES production function model. By solving the parameters of log-linear equation which are the function of
the parameters of CES production function model, the parameters of CES production function model can be obtained.

The growth of partial and total factor productivity measures have been estimated from the regression of the following standard type, that is with time as independent variable and the concerned variable as the dependent variable.\textsuperscript{12}

\[
\log Y = \log a + t \log b
\]

i.e. \(Y = ab^t\) \hspace{1cm} (1.27)

where \(b\) measures the growth rate.

1.8 OUTLINE OF STUDY

The research study undertaken by me contains a preface in which a brief outline about the work carried out, is presented along with some preliminary remarks about the subject matter.

The entire work is divided into TEN chapters. A summary of the same is as under:

**Chapter-1** describes a brief academic study about the production function models. A general outline of the review of literature is carried out to give a bird eye view of the work executed in this area. The specific forms of neutral and non-neutral types of production function models are discussed and some preliminary ideas for estimation of the parameters involved in these models are indicated. Some basic concepts and definitions as well as relevant terms that are used in the study are explained. Reference is also given for the database corresponding to the specific applications for the models used in the respective studies.

In this manner, this chapter provides some preliminary introductory remarks about the entire study carried out in this thesis.

Chapter-2 contains the study of production function model for Basic Metal & Alloys Industries, Other Manufacturing Industries and Food Product Industries in India. Here non-neutral forms of Cobb-Douglas, CES and VES production function models are used. The respective parameters for these production function models are estimated and the statistical tests of significance are also considered for these models.

Chapter-3 considers the non-neutral Cobb-Douglas, CES and VES production function models for Chemical and Chemical Product and Non-Electrical Machinery Industries in India. The respective parameters of these production function models are estimated and tested statistically. The corresponding variable elasticity of substitution is also calculated.

Chapter-4 discusses the non-neutral CES production function model and it is estimated for the Textile Product and Leather & Leather & Fur product Industries in India. For fitted model, the estimates of the parameters are tested statistically.

Chapter-5 contains non-neutral VES production function model for all India, all industries factory sector data during the years 1959-60 to 1982-83. The results for the whole time series (1959-83) show that neither CD nor VES production function is well fitted. This may be due to the phenomena of auto-correlation, but for the years 1959-69, VES production function model is well fitted, the corresponding variable elasticity of substitution is also calculated. For the remaining years 1970-83, CD production function model is found to be a better fit.
Chapter-6 considers the total factor productivity, partial productivity namely labour productivity and capital productivity ratios, they are computed for All India during the years 1980-81 to 1991-92. An alternative approach to Kendrick’s measure of TFP is suggested here and the projections are made for the next five years on the basis of the suggested model.

Chapter-7 discusses about Kendrick’s measure of TFP and partial productivity measures are computed for the years 1973-74 to 1992-93. An alternative approach is also suggested for Kendrick’s measure of TFP and the projections are made for the next five years.

Chapter-8 considers an alternative form of the function to Murray-Brown’s non-neutrality function \( f(\lambda,t) = e^{\lambda t} \) and its theoretical properties are highlighted.

Chapter-9 describes the modified non-neutral CES production function model for Indian Industries. In this chapter, the new non-neutrality function suggested in Chapter-8 has been utilised with CES production function model. The model is considered for All India all industries factory sector data for the years 1973-74 to 1993-94. The respective parameters are estimated and tested statistically, the actual and estimated values of Net Value Added by manufacturer are tabulated. The Projections of NVA for the next five years are made on the basis of this model.

Chapter-10 contains a non-neutral VES production function model under constant returns to scale and non-constant returns to scale with new suggested non-neutrality function given in Chapter-8 for Gujarat State factory sector data during the years 1973-74 to 1992-93, the corresponding value of elasticity of substitution under CRTS and non-CRTS is calculated. The estimated values of NVA are also tabulated.