CHAPTER 8

TRANSCENDENTAL PRODUCTION FUNCTION MODEL UNDER NON-NEUTRAL TECHNOLOGICAL PROGRESS

8.1 INTRODUCTION

Technological progress is one of the most important factor which is responsible for economic growth in many sectors of economy. An analysis of technological change in Indian manufacturing sector is important.

Murray Brown [149] classified technological progress into two broad categories namely neutral technological progress and non-neutral technological progress. A neutral change neither saves nor uses labour. It is the one which produces a variation in the production relation itself, but it does not affect the marginal rate of substitution of labour for capital. A non-neutral technological change alters the production function and can be either labour saving (capital using) or capital saving (labour using).

If marginal product of capital increases relative to that of labour, for a given labour capital combination, then a labour saving on capital using change raises the marginal product of labour, relative to that of capital and vice versa.

In a rapidly developing economy there is an impact of the technological improvement and hence it may be necessary to define a new type of production function model which can consider non-neutral technological progress. In this chapter, an attempt is made to explore such a production function model and derive its specific properties. To give an application of the proposed model Cobb-Douglas and Transcendental production function models are considered for this study and it is applied to food product industry in India.
8.2 METHODOLOGY

The non-neutral production function model as defined by Murray Brown/149] is of the form

\[ Q_t(L_t, K_t, \lambda, t) = A_0 e^{\lambda t} F(L_t, K_t) \]  

... (8.1)

where

- \( Q_t \) = Output in the year \( t \)
- \( L_t \) = Labour employed in the year \( t \)
- \( K_t \) = Capital invested in the year \( t \)
- \( t \) = time in years

and

- \( \lambda \) = technology parameter

Here \( F(L_t, K_t) \) is any form of production function like Cobb-Douglas, CES, etc.

The model posed in the equation (8.1) above is a specific form of production function representing the state of technology.

We may consider a more general approach for the above form by rewriting the equation (8.1) as

\[ Q_t(L_t, K_t, \lambda, t) = f(\lambda, t) F(K_t, L_t) \]  

... (8.2)
Here \( f(\lambda,t) \) is a function which can represent the non-neutral technological growth which influences the output. The question here is to consider an appropriate form of the function \( f(\cdot) \). Let us consider the following form

\[
f(\lambda,t) = \frac{A_0 e^{\lambda t}}{1 + c e^{\lambda t}} \quad \text{... (8.3)}
\]

where

\( A_0 \) = scale parameter
\( c \) = shape parameter
\( \lambda \) = technology parameter

\( A_0 > 0, \lambda > 0, c > 0, t = 1, 2, \ldots n \)

The above function \( f(\lambda,t) \) has the following properties

1. \( f(.) \) is a continuous function of \( t \)

2. \( f'(t) = \lambda f \left( 1 - \frac{cf}{A_0} \right) > 0 \) for all \( t \).

Thus \( f'(t) \) is a monotonic function of \( t \)

3. \( f''(t) = \lambda^2 f \left( 1 - \frac{cf}{A_0} \right) \left( 1 - \frac{2cf}{A_0} \right) \)

Thus \( f(.) \) has points of inflexion

when \( f = \frac{A_0}{2c} \) and \( f = \frac{A_0}{c} \)
\[ (4) f(.) = \frac{A_o}{1 + c} \text{ when } t = 0 \]

\[ (5) f(.) = A_o e^{\lambda t} \text{ when } c = 0 \]

\[ (6) \frac{df}{d\lambda} = \frac{ft}{1 + c e^{\lambda t}} \]

Property (4) states that initially there is neutral technological growth and as time goes on, due to technological growth, there is an advancement which in turn is reflected in the production function model by means of equation (8.2). Property (5) suggests that a particular case of this functional form leads to non-neutrality function suggested by Brown.

By introducing this functional form \( f(.) \), now we can rewrite equation (8.2) given by

\[ Q_t(L_t, K_t, \lambda, t) = \frac{A_o e^{\lambda t}}{1 + c e^{\lambda t}} F(L_t, K_t) \quad \ldots (8.4) \]

Thus equation (8.4) defines a more general class of non-neutral production function model. By considering the different forms of \( F(L_t, K_t) \) we can define a class of production function models.

In this study let us consider Cobb-Douglas and Transcendental type of production functions representing the function \( F(L_t, K_t) \). The Cobb-Douglas form is given as below:

\[ F(L_t, K_t) = L_t^\alpha K_t^\beta e^{u_t} \quad \ldots (8.5) \]

and the Transcendental type production function is given by
\[ F(L_t, K_t) = L_t^\alpha K_t^\beta e^{-\alpha L_t} + \beta K_t e^{\alpha L_t} \quad \ldots \quad (8.6) \]

where \( \alpha, \beta, \alpha', \) and \( \beta' \) are the parameters of the model.

Hence substituting equation (8.5) in the equation (8.4), it reduces to the following.

\[ Q_t(L_t, K_t; \lambda, \mu) = \frac{A_\mu e^{\lambda t}}{1 + c e^{\lambda t}} L_t^\alpha K_t^\beta e^{\mu t} \quad \ldots \quad (8.7) \]

Substituting equation (8.6) in the equation (8.4) it reduces to the following.

\[ Q_t(L_t, K_t; \lambda, \mu) = \frac{A_\mu e^{\lambda t}}{1 + c e^{\lambda t}} L_t^\alpha K_t^\beta e^{\alpha L_t} + \beta K_t e^{\mu t} \quad \ldots \quad (8.8) \]

Equations (8.7) and (8.8) are the new functional forms of production function model.

From the equations (8.7) and (8.8) it is observed that

(i) \( \frac{\partial Q_t}{\partial \lambda} = \frac{Q_t}{1 + ce^{\lambda t}} > 0 \) for all \( t \)

and

(ii) \( \frac{\partial \log Q_t}{\partial \log \lambda} = \frac{\lambda t}{1 + ce^{\lambda t}} = \frac{\lambda t}{A_\mu e^{\lambda t}} > 0 \) for all \( t \)
Thus the elasticity of technological growth\textsuperscript{14} remains positive which suggests that it is a more elastic situation as compared to the neutral technological growth.

### 8.3 ESTIMATION PROCEDURE

Equation (8.4) represents a general form of non-neutral production function model. By taking logarithms on both the sides, the later part $F(L_t,K_t)$ can be reduced to linear or log linear form by taking suitable approximation when required. Whereas for the first part $f(\lambda,t)$, the following procedure is considered.

\[ f(\lambda,t) = \frac{A_0 e^{\lambda t}}{1 + c e^{\lambda t}} \]

\[ \ln f(\lambda,t) = \ln A_0 + \lambda t - \ln(1 + ce^{\lambda t}) \quad \ldots (8.9) \]

with the assumption $|ce^{\lambda t}| < 1$, the second degree of approximation of $t$ is considered, hence the equation (8.9) reduces to

\[ \ln f(\lambda,t) = \ln A_0 + \lambda t - \ln \left[ 1 + c \left( 1 + \lambda t + \frac{\lambda^2 t^2}{2} + \ldots \right) \right] \quad \ldots (8.10) \]

again expanding equation (8.10) in terms of logarithm up to second order of $t$

\textbf{14. Elasticity of technological growth (ETG) is defined as the proportional change in output as compared to the proportional change in the technological parameter.}

Symbolically we define it as $\delta = \frac{\partial \log Q_t}{\partial \log \lambda}$

Hence if $\delta > 1$ $\Rightarrow$ more elastic situation as compared to the technological growth and

$\delta < 1$ $\Rightarrow$ less elastic situation as compared to the technological growth
\[ \ln f(\lambda, t) = \ln A_0 + \lambda t - \]
\[
\left[ \frac{e^2}{2} \left( 1 + \lambda t \frac{e^2}{2} + \ldots \right) + \frac{e^3}{3} \left( 1 + \lambda t \frac{e^2}{2} + \ldots \right)^2 + \frac{e^4}{4} \left( 1 + \lambda t \frac{e^2}{2} + \ldots \right)^3 + \ldots \right]
\]
\[
= \ln A_0 - \left[ - \frac{e^2}{2} \left( 1 - c + e^2 - e^3 + \ldots \right) + \lambda t \left( 1 - c + e^2 - e^3 + \ldots \right) \right]
\]
\[
+ \lambda^2 t^2 \left( - \frac{e^2}{2} + \frac{3}{2} c^2 + 2c^4 - \frac{5}{2} e^5 + \ldots \right)
\]
\[
= \ln \left( \frac{A_0}{1 + c} \right) - \ln(1 + c) + \lambda t (1 + c)^{-1} + \lambda^2 t^2 \left( - \frac{c^2}{2(1 + c)^2} \right)
\]
\[
= \ln \left( \frac{A_0}{1 + c} \right) + \frac{\lambda t}{1 + c} - \frac{c\lambda^2 t^2}{2(1 + c)^2}
\]

Therefore \( \ln f(\lambda, t) = \beta_0 + \beta_1 t + \beta_2 t^2 \) \hspace{1cm} (8.11)

where

\[
\beta_0 = \ln \left( \frac{A_0}{1 + c} \right)
\]
\[
\beta_1 = \frac{\lambda}{1 + c}
\]
\[
\beta_2 = \frac{c\lambda^2}{2(1 + c)^2}
\]

Hence the production function models defined in the equations (8.7) and (8.8)
can be written in its logarithmic forms as under
\[ \ln(Q_t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \alpha \ln(L_t) + \beta \ln(K_t) + u_t \] ... (8.13)

\[ \ln(Q_t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \alpha \ln(L_t) + \beta \ln(K_t) + \alpha\lambda t + \beta^\prime K_t + u_t \] ... (8.14)

respectively.

Note that the equations (8.13) and (8.14) are in log linear form hence the estimates of the parameters can be obtained by using OLS method.

To obtain the estimates of the parameters \( A_0, c \) and \( \lambda \), solving the equations in the equation (8.12) the estimates of \( A_0, c, \) and \( \lambda \) can be obtained in terms of \( \beta_0, \beta_1 \) and \( \beta_2 \) as under.

\[
A_0 = e^{\beta_0} \left[ 1 - \frac{2\beta_2}{\beta_1^2} \right]
\]

\[
C = \frac{2\beta_2}{\beta_1^2}
\]

and

\[
\lambda = \frac{\beta_1^2 - 2\beta_2}{\beta_1}
\] (8.15)

the estimates of \( \alpha, \beta, \alpha' \) and \( \beta' \) are obtained directly by OLS method

8.4 DATABASE

The models are considered for food product industry in India. The data are collected from Annual Survey of Industries, New Delhi. The data on Net Value Added is deflated by Index of industrial production, productive capital is deflated by wholesale price index numbers and the data on wages to workers are deflated by consumer price index numbers. The indices of the series are considered by taking 1980-81 as base year.
### Table 8.1

**Transcendental Production Function Model for food product industry**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_o$</td>
<td>165102.3</td>
<td>(0.214093)</td>
</tr>
<tr>
<td>$c$</td>
<td>0.352105*</td>
<td>(4.903502)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.23902*</td>
<td>(4.793226)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.21241*</td>
<td>(2.62)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.564778</td>
<td>(1.19)</td>
</tr>
<tr>
<td>$\alpha'$</td>
<td>0.007129*</td>
<td>(2.26)</td>
</tr>
<tr>
<td>$\beta'$</td>
<td>$1.68 \times 10^{-5}$</td>
<td>(0.2012)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9809</td>
<td></td>
</tr>
<tr>
<td>$DW$</td>
<td>1.406</td>
<td></td>
</tr>
<tr>
<td>D.f.</td>
<td>(6,14)</td>
<td></td>
</tr>
</tbody>
</table>
Table 8.2

Cobb-Douglas Production function model for food product industry

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_o$</td>
<td>82.07454</td>
<td>(0.600219)</td>
</tr>
<tr>
<td>$c$</td>
<td>0.247569*</td>
<td>(2.233415)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.130378*</td>
<td>(3.416271)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.42588</td>
<td>(1.28063)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.536879*</td>
<td>(2.55533)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.974261</td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>D.F.</td>
<td>(4,16)</td>
<td></td>
</tr>
</tbody>
</table>

From the table 8.1, it is observed that while estimating the transcendental production function model for food product industry in India, the parameters $c, \lambda, \alpha$ and $\alpha'$ are significant at 5% level of significance whereas the parameters $A_o, \beta$ and $\beta'$ are insignificant at 5% level of significance. The technology improvement is 23.9% which is significant and value of $R^2$ is 0.9809.

Table 8.2 gives the estimates of the parameters of Cobb-Douglas production function model for food product in India. It shows that the parameters $c, \lambda$ and $\beta$ are found to be significant at 5% level of significance. Whereas $A_o$ and $\alpha$ are found to be insignificant at 5% level of significance and the value of $R^2$ is 0.9742. From the above observation, it may be concluded that the transcendental production function appears to be more appropriate production function model for food product industries.
in India. The actual and estimated values of NVA by both models, CD and transcendental production function models are shown in the table 8.3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>Estimated by Transcendental Production Function</th>
<th>Estimated by Cobb-Douglas Production Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973-74</td>
<td>435.8639</td>
<td>422.6625</td>
<td>447.8924</td>
</tr>
<tr>
<td>75</td>
<td>569.0839</td>
<td>580.9001</td>
<td>520.8728</td>
</tr>
<tr>
<td>76</td>
<td>540.7940</td>
<td>525.7310</td>
<td>538.2706</td>
</tr>
<tr>
<td>77</td>
<td>617.1301</td>
<td>574.7511</td>
<td>594.6160</td>
</tr>
<tr>
<td>78</td>
<td>689.4069</td>
<td>689.1961</td>
<td>688.9791</td>
</tr>
<tr>
<td>79</td>
<td>648.0761</td>
<td>732.6080</td>
<td>741.8453</td>
</tr>
<tr>
<td>80</td>
<td>741.7959</td>
<td>790.6633</td>
<td>788.7596</td>
</tr>
<tr>
<td>81</td>
<td>707.2200</td>
<td>799.8172</td>
<td>796.5605</td>
</tr>
<tr>
<td>82</td>
<td>856.7063</td>
<td>861.9814</td>
<td>858.1488</td>
</tr>
<tr>
<td>83</td>
<td>1053.5820</td>
<td>966.2212</td>
<td>943.3784</td>
</tr>
<tr>
<td>84</td>
<td>1372.1510</td>
<td>1180.6260</td>
<td>1138.2540</td>
</tr>
<tr>
<td>85</td>
<td>1260.3750</td>
<td>1175.7390</td>
<td>1170.0240</td>
</tr>
<tr>
<td>86</td>
<td>1250.1270</td>
<td>1239.2020</td>
<td>1262.2990</td>
</tr>
<tr>
<td>87</td>
<td>1259.2390</td>
<td>1351.1080</td>
<td>1413.8800</td>
</tr>
<tr>
<td>88</td>
<td>1035.3370</td>
<td>1409.636</td>
<td>1465.0080</td>
</tr>
<tr>
<td>89</td>
<td>1580.8730</td>
<td>1489.6810</td>
<td>1591.2190</td>
</tr>
<tr>
<td>90</td>
<td>2023.9000</td>
<td>1898.4950</td>
<td>1763.7510</td>
</tr>
<tr>
<td>91</td>
<td>1821.3550</td>
<td>2051.4980</td>
<td>1922.5190</td>
</tr>
<tr>
<td>92</td>
<td>2088.6960</td>
<td>2013.5320</td>
<td>2030.4230</td>
</tr>
<tr>
<td>93</td>
<td>2213.883</td>
<td>2236.939</td>
<td>2203.525</td>
</tr>
<tr>
<td>94</td>
<td>2944.991</td>
<td>2924.422</td>
<td>3026.861</td>
</tr>
</tbody>
</table>

Graphical presentation of the actual and estimated values of NVA as given by the fitted models are presented in the graphs shown in the fig. 8.1 and fig. 8.2
Fig. 8.1
Graphical presentation of actual and estimated values of NVA by Transcendental Production Function Model for food product industries

Time in years

--- Actual    --- Estimated
Figure 8.2

Graphical presentation of actual and estimated values of NVA by Cobb-Douglas Production Function Model for food product industries.