3

Interaction of two level atom and electromagnetic field with time varying frequency

3.1 Introduction

A two level atom field system can be used for data storage and for their operations in quantum information processes (QIP). But a controllable atom-field interaction is essential for using it in QIP. There are many models suggesting various methods to control the atom-field interactions [64–67]. After the invention of frequency chirped lasers there are tremendous progresses in the research on atom field interactions where the frequency of the laser field is time dependent. Recent studies show that time dependent field frequency considerably modifies the dynamical evolution of an atom-field system [68–70]. In this chapter we discuss methods to control or manipulate the atom field probability amplitudes during the interaction between a two level atom and quantized electromagnetic field using time dependent field frequency fluctuations. It is to be noted that the sinusoidal field frequency fluctuation can be used for controlling the dynamics of atom field system. A detailed study of the dependence of population inversion on the applied frequency modulation parameters by varying the initial photon distributions are presented in this chapter. As a continuation, the dynamics of interaction between a two level atom
and electromagnetic field with phase shifted sinusoidally varying frequency is also included in this chapter. In the case of phase shifted frequency fluctuations the population inversion behaves as in the case of Fock state atom interaction.

### 3.2 Model and Hamiltonian

We have the Jayness-Cumming Hamiltonian [12], discussed in the previous chapter, corresponding to the interaction between a two level atom and quantized single mode electromagnetic field. Setting $\hbar = 1$ the Hamiltonian is

$$\hat{H} = \nu_0 \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega \hat{\sigma}_z + g (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger),$$

(3.1)

where $\nu_0$ is the frequency of the field and $\omega$ is the transition frequency of the two level atom. The general state of the atom-field system derived using the time dependent Schrödinger equation and the initial condition such that atom is in the excited state at $t = 0$ is,

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} C_n \cos \left( \lambda t \sqrt{n+1} \right) |n\rangle - i \sum_{n=0}^{\infty} C_n \sin \left( \lambda t \sqrt{n+1} \right) |n+1\rangle,$$

(3.2)

where $C_n$ is obtained from the initial photon distribution. These results have been discussed in the previous chapter for the photons initially in Fock state and coherent state. In all these cases the frequency of the interacting field, $\nu_0$, is kept constant; does not have any time dependence. Now let us consider the case with the frequency of the field is also varying with time. In such cases the Hamiltonian and the entire system evolution changes. The fluctuating field frequency can be represented as the combination of a time independent mean frequency, $\nu_0$, and a time dependent function $f(t)$ such that

$$\nu(t) = \nu_0 + f(t).$$

(3.3)
Because of the fluctuations in field frequency, all the parameters involved in the interaction which has a dependency on field frequency also changes with time. From JCM, it is known that the atom field coupling strength $g$ is proportional to the field frequency and inversely proportional to the quantization volume($V$) of the cavity, i.e.,

$$g \propto -\left(\frac{\hbar \omega}{\epsilon_0 V}\right)^{1/2} \left(\frac{d}{\hbar}\right)$$

(3.4)

According to the quantization of electromagnetic field, $V$ is always inversely proportional to the field frequency such that

$$V(t) = \frac{V_0}{1 + f(t)/\nu_0}$$

(3.5)

and now $g$ reads

$$g = g_0 \left(1 + f(t)/\nu_0\right),$$

(3.6)

where $g_0$ is the coupling strength corresponds to the mean frequency $\nu_0$. The time dependent total Hamiltonian after considering the changes in coupling strength due to the field frequency fluctuation is,

$$\hat{H}(t) = \nu_0 \hat{a}^{\dagger} \hat{a} + \frac{1}{2} \omega \hat{\sigma}_z + (1 + f(t)/\nu_0) \left[\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^{\dagger}\right].$$

(3.7)

Now the general state of the two level atom-field system at any arbitrary time $t$ can be represented as

$$|\psi\rangle = \sum_n [C_{e,n}(t)|e, n\rangle + C_{g,n}(t)|g, n\rangle].$$

(3.8)

Here $|e, n\rangle(|g, n\rangle)$ represents the atom in excited(ground) state with $n$ photons and $C_{e,n}(t)(or C_{g,n}(t))$ are the coefficients corresponding to their probabilities. Substituting the Hamiltonian in Eq. (3.7) and the general state given by Eq. (3.8) in the time dependent Schrödinger
interaction of two level atom and electromagnetic field with time varying frequency

equation we get the following infinite set of equations for the evolution of probability amplitudes, $C_{e,n}(t)$ and $C_{g,n+1}(t)$:

\[
\frac{d}{dt} C_{e,n}(t) = -in [\nu_0 + f(t)] C_{e,n}(t) - i \frac{\omega}{2} C_{1,n} - ig_0 \frac{1 + f(t)/\nu_0}{\sqrt{n + 1}} C_{g,n+1}(t), \tag{3.9}
\]

\[
\frac{d}{dt} C_{g,n+1}(t) = -i(n + 1) [\nu_0 + f(t)] C_{g,n+1} + \frac{i}{2} \omega C_{g,n+1} - ig_0 \frac{1 + f(t)/\nu_0}{\sqrt{n + 1}} C_{e,n}(t). \tag{3.10}
\]

In order to make the equations simple and eventually to solve them we define another set of coefficients $M_{e,n}(t)$ and $M_{g,n+1}(t)$ such that

\[
C_{e,n}(t) = \exp \left[ -i (n\nu_0 + \omega/2) t \right] \times \exp \left[ -i n \int_0^t f(t') dt' \right] M_{e,n}(t) \tag{3.11}
\]

\[
C_{g,n+1}(t) = \exp \left[ -i [(n + 1) \nu_0 - \omega/2] t \right] \times \exp \left[ -i(n + 1) \int_0^t f(t') dt' \right] M_{g,n+1}(t). \tag{3.12}
\]

It is important to note that,

\[
|M_{e,n}(t)|^2 = |C_{e,n}(t)|^2; |M_{g,n}(t)|^2 = |C_{g,n}(t)|^2. \tag{3.13}
\]

Substituting Eqs. (3.11) and (3.12) in Eqs. (3.9) and (3.10) we obtain

\[
\frac{d}{dt} M_{e,n}(t) = -ig\sqrt{n + 1} \exp[ -i(n\nu_0 - \omega) t] \times \exp \left( -i \int_0^t f(t') dt' \right) M_{g,n+1} \tag{3.14}
\]

\[
\frac{d}{dt} M_{g,n+1}(t) = -ig\sqrt{n + 1} \exp[ i(n\nu_0 - \omega) t] \times \exp \left( i \int_0^t f(t') dt' \right) M_{1,n} \tag{3.15}
\]

The above set of equations Eqs. (3.14) and (3.15) can be solved to find the nature of evolution of the atom field system. In the previous
Model and Hamiltonian

chapter we have defined the atomic population inversion as the difference in the probability of finding the atom in excited state to ground state; i.e.,

\[ W(t) = \sum_n \left[ |C_{e,n}(t)|^2 - |C_{g,n+1}|^2 \right]. \quad (3.16) \]

Using Eqs. (3.11) and (3.12) and Eq. (3.13), it can be written in the form:

\[ W(t) = \sum_n \left[ |M_{e,n}(t)|^2 - |M_{g,n+1}|^2 \right]. \quad (3.17) \]

In the case where the frequency is constant i.e. \( f(t) = 0 \), the set of Eqs. (3.14) and (3.15) can be re written as

\[
\frac{dM_{e,n}(t)}{dt} = -i g_0 \sqrt{n+1} e^{-i \Delta t} M_{g,n+1}(t), \quad (3.18)
\]

\[
\frac{dM_{g,n+1}(t)}{dt} = -i g_0 \sqrt{n+1} e^{+i \Delta t} M_{e,n}(t), \quad (3.19)
\]

where \( \Delta = \nu_0 - \omega \). We can solve Eqs.(3.18) and (3.19) analytically using the standard techniques of solving coupled differential equations as follows.

Differentiating Eq. (3.18) and substituting Eq. (3.19) gives

\[
\frac{\partial^2}{\partial t^2} M_{g,n+1}(t) - i \Delta \frac{\partial}{\partial t} M_{g,n+1}(t) + g_0^2(n+1) M_{g,n+1}(t) = 0 \quad (3.20)
\]

Assume a solution of the form \( M_{g,n+1}(t) = Ae^{i \theta t} \) and substituting it in Eq. (3.20) we get

\[
\theta^2 - \Delta \theta - g_0^2(n+1) = 0 \quad (3.21)
\]

i.e.,

\[
\theta = \frac{\Delta \pm \sqrt{\Delta^2 + 4 g_0^2(n+1)}}{2} \quad (3.22)
\]

Taking \( \Omega_n^2 = \Delta^2 + 4 g_0^2(n+1) \) the general solution is

\[
M_{g,n+1}(t) = A_+ e^{\frac{1}{2} (\Delta + \Omega_n) t} + A_- e^{\frac{1}{2} (\Delta - \Omega_n) t} \quad (3.23)
\]
Interaction of two level atom and electromagnetic field with
time varying frequency

We have the initial condition such that at time \( t = 0 \), \( M_{g,n+1}(0) = 0 \).
i.e., \( M_{g,n+1}(0) = 0 = A_+ + A_- \) \( \Rightarrow \) \( A_+ = -A_- = A \)(say). Then we get

\[
M_{g,n+1}(t) = A \left\{ e^{i\frac{1}{2}(\Delta + \Omega_n)t} - e^{i\frac{1}{2}(\Delta - \Omega_n)t} \right\}
\]

i.e.,
\[
M_{g,n+1}(t) = A e^{i\frac{1}{2}\Delta t} \left\{ e^{i\frac{1}{2}\Omega_n t} - e^{-i\frac{1}{2}\Omega_n t} \right\}
\]

Using the Eqs. (3.24) and (3.25), the atomic population inversion \( W(t) \) can now be written as,

\[
W(t) = \sum_{n=0}^{\infty} \rho_{nn}(0) \left[ \Delta^2 \Omega_n^2 + \frac{4g_0(n + 1)}{\Omega_n^2} \cos (\Omega_n t) \right], \quad (3.26)
\]

where \( \rho_{nn}(0) \) is obtainable from the initial photon distribution.

3.3 Sinusoidally varying field frequency

We now consider the case with the frequency of the electromagnetic radiation varies sinusoidally with time as given below

\[
\nu(t) = \nu_0 + \Delta \nu \sin(\beta t), \quad (3.27)
\]

where \( \nu_0 \) is the initial mean frequency and \( \Delta \nu \sin(\beta t) \) is the fluctuation with an amplitude \( \Delta \nu \) and a periodicity \( 1/\beta \). Now the coupling strength \( g \) becomes

\[
g = g_0 \left[ 1 + \frac{\Delta \nu \sin(\beta t)}{\nu_0} \right]. \quad (3.28)
\]
With the oscillating field frequency the evolution equations for the probability amplitudes takes the form

\[
\frac{dM_{e,n}(t)}{dt} = -ig_0\sqrt{n + 1} e^{\Delta\nu/\beta[\cos\beta t - 1]} \left[ 1 + \frac{\Delta\nu \sin(\beta t)}{\nu_0} \right] M_{g,n+1}(t) \tag{3.29}
\]

\[
\frac{dM_{g,n+1}(t)}{dt} = -ig_0\sqrt{n + 1} e^{\Delta\nu/\beta[\cos\beta t - 1]} \left[ 1 + \frac{\Delta\nu \sin(\beta t)}{\nu_0} \right] M_{e,n}(t) \tag{3.30}
\]

The above set of equations Eqs. (3.29) and (3.30) are solved numerically using the fourth order Runge-Kutta method for the given initial photon distribution. In the following sections we discuss the interaction for coherent and squeezed field.

### 3.3.1 Interaction with coherent field

Consider the case in which a two level atom interacting with an initial coherent field with time varying frequency. We notice a weak influence of frequency fluctuation in the evolution of population inversion when the amplitude of frequency modulation $\Delta\nu$ and angular frequency $\beta$ is small. For example we choose $\Delta\nu \approx 0.001\nu_0$, and $\beta \approx 0.1g_0$ and the corresponding evolution of atomic population inversion is plotted in Fig. 3.1 as the function of scaled time $\tau = gt$. 

Interaction of two level atom and electromagnetic field with time varying frequency

Figure 3.1: Atomic population inversion against scaled time. Initial coherent field with $\bar{n} = 25$. Field frequency $\nu_0$ is taken to be 10000$g_0$, $\Delta \nu = 0.001\nu_0$ and $\beta = 0.1g_0$. Dotted line shows the field frequency fluctuation.
Here the evolution is similar to the constant frequency case with normal collapses and revivals. But when the amplitude $\Delta \nu$ increases, there are visible changes in the nature of population inversion. In Fig. 3.1 population inversion versus time for both, with frequency fluctuation and without frequency fluctuation, cases are plotted. When there is a frequency variation with a considerable amplitude, we can see from Fig. 3.2 that the revivals are shifted towards to the right. We may say that the system spend more time in the collapse region with a coherent sharing of probability amplitudes between atom and field or in other words, with a zero atomic inversion. In Fig. 3.3 we have plotted the atomic inversion by varying the periodicity of the field frequency fluctuations. Here we observed the quasi periodic oscillations in population inversion with time. It is also noted that,
Interaction of two level atom and electromagnetic field with time varying frequency during the collapse period the atomic inversion is not exactly collapsing to zero but oscillating with a small amplitude. The period of these oscillations are exactly equal to the period of field frequency fluctuations.
Figure 3.3: Population Inversion versus time. Initial coherent field with $\bar{n} = 25$. Field frequency $\nu_0 = 10000g_0$, $\Delta \nu = 10g_0$. Field frequency variation is shown by the dotted line.
3.3.2 Interaction with squeezed field

Quadrature squeezed electromagnetic field has been discussed in the proceeding section 1.4. Squeezed light is the light with minimum uncertainty but the value of uncertainty in different quadratures are not the same. One way in which this can be achieved is to squeeze the uncertainty circle of the vacuum or the coherent state into an ellipse of the same area. The squeezed states for a single mode radiation field may be generated from the vacuum $|0\rangle$ by

$$|\alpha, \xi\rangle = \hat{D}(\alpha)\hat{S}(\xi)|0\rangle.$$  \hspace{1cm} (3.31)

Here $\hat{S}(\xi)$ and $\hat{D}(\alpha)$ are the squeezing and coherent displacement operators respectively and are given by

$$\hat{S}(\xi) = \exp\left[\frac{1}{2}(\xi^*\hat{a}^2 - \xi\hat{a}^\dagger)^2\right],$$  \hspace{1cm} (3.32)

$$\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}),$$  \hspace{1cm} (3.33)

where $\alpha = |\alpha|e^{i\psi}$, $\xi = re^{i\theta}$, $r$ is known as the squeezing parameter and $0 \leq r < \infty$ and $0 \leq \theta < 2\pi$. When $\xi = 0$ we obtain the coherent state. The photon number distribution for a squeezed field is

$$P_n = \frac{(\frac{1}{2}\tanh r)^n}{n!\cosh r} \exp\left[-|\alpha|^2 - \frac{1}{2}\left(\alpha^2e^{i\theta} + \alpha^*e^{-i\theta}\right)\tanh r\right] \times \left|H_n\left(\gamma(e^{i\theta}\sinh 2r)^{\frac{1}{2}}\right)\right|^2,$$  \hspace{1cm} (3.34)

with $\gamma = \alpha \cosh r + \alpha^* \sinh r$ and $H_n$’s are the Hermite polynomials.

Now consider the interaction of quadrature squeezed light with a two level atom. In the interaction the effects of squeezing is maximum when the value $\theta$ is $\pi$ and the effects are minimum for $\theta = 0$. The behaviour of population inversion with time for $\theta = 0$ is almost similar
to that of a coherent field atom interaction. There are series of clear collapses and revivals in population inversion with time. But when the value of $\theta = \pi$ only first two revivals are prominent and clear. The collapse and revival phenomenon is not retained for long time and after the first few there occurs random oscillations in population inversion as shown in Fig. 3.4. When the value of $r$ is increased the random oscillation of population inversion starts early in time; immediately after the first revival itself. These randomness in the time evolution of atomic inversion is due to the squeezing of the interacting field.
Interaction of two level atom and electromagnetic field with time varying frequency

Figure 3.4: Population inversion versus time. Initial squeezed field with $\bar{n} = 25$. Squeezing parameter $r = 0.8$. 

(a): Squeezed field with $\theta = 0$

(b): Squeezed field with $\theta = \pi$
Now we take the case of interaction of atom and squeezed light with time varying frequency. We are considering only the case with $\theta = \pi$ where the effect of squeezing is maximum. The frequency variation is $f(t) = \Delta \nu \sin(\beta t)$ with $\Delta \nu << \nu_0$ and $\beta$ is taken to be very small. For example we choose $\beta = 0.1g_0$ and $\Delta \nu = 0.001\nu_0$. The corresponding evolution of population inversion is shown in Fig. 3.5. In the case of interactions in which field frequency is a constant, we have already noticed that the variation of population inversion is random after the first revival. From Figs. 3.5 and 3.6 it is clear that the randomness in the population inversion is reduced by applying a sinusoidal frequency variation for the squeezed field. Now the collapses and revivals are clear and distinct. Here the variation in population inversion is controlled and collapses and revivals are retained. Also, similar to the behaviour noticed in coherent field case, the occurrence of revivals shift towards the right when the amplitude of fluctuation increases. Thus the revival periodicity has a noticeable dependence on the amplitude($\Delta \nu$) of the field frequency modulation.
Interaction of two level atom and electromagnetic field with time varying frequency

Figure 3.5: Population inversion versus time for two level atom and squeezed field interaction with field frequency fluctuations for $\bar{n} = 25$, $r = 0.8, \theta = \pi$, $\beta = 0.1g_0$. Dotted line shows the field frequency fluctuation.
Figure 3.6: Population inversion versus time for two level atom and squeezed field interaction with field frequency fluctuations. \( \bar{n} = 25, r = 0.8, \theta = \pi, \beta = 0.1g_0 \). Field frequency variation is shown by the dotted line.
Interaction of two level atom and electromagnetic field with time varying frequency

Figure 3.7: Population inversion versus time for two level atom and squeezed field interaction with field frequency fluctuations. Field parameters $\bar{n} = 25$, $r = 0.8, \theta = \pi$, $\nu_0 = 1000g_0$, $\Delta\nu = 20g_0$ (a) $f(t) = 20g_0 \sin(1g_0 t)$ and in (b) $f(t) = 20g_0 \sin(2g_0 t)$. Dotted line shows the field frequency fluctuations.
When we vary the periodicity for frequency fluctuation for a constant amplitude, for example $\beta = g_0, 2g_0$ etc., the population inversion executes a quasi periodic oscillation. Also the collapses are not exactly at zeros and we can see a small amplitude oscillations in the collapse region. The periodicity of these small amplitude oscillations are equal to that of the applied field frequency fluctuations as seen in the Fig. 3.7. We can conclude that the field frequency fluctuation can be used for controlling or manipulating atom field probability amplitudes in an atom field system and this method can be used for realising a controllable atom field interaction in QIP.

### 3.4 Phase shifted frequency modulation

The effect of field frequency variation in the interaction of two level atom in Kerr medium has been studied in a recent publication by Li Wang et al. [70]. Their results shows that the coupling between the atom and photon is enhanced in the Kerr medium and even the atomic population inversion for an initial coherent field behaves like that of an initial Fock state. In this section we suggest another model in linear medium for which the population inversion behaves in a similar manner. We considered the interaction between a two level atom and electromagnetic field in a linear medium where the field frequency has a phase shifted sinusoidal fluctuation. The system has been studied for both coherent and squeezed field and it is noted that the population inversion oscillates sinusoidally just as in the Fock state atom interaction; without collapses and revivals. Phase shifted frequency fluctuations improves the coupling between the two level atom and field.
3.4.1 Sinusoidal frequency variation with a phase difference

Here we consider a field with sinusoidally varying frequency which has a phase difference with the mean frequency $\nu_0$. In such cases, the field frequency at any time $t$ can be written in the form,

$$\nu(t) = \nu_0 + \Delta \nu \sin(\beta t + \phi). \quad (3.35)$$

We have the time evolution equations, Eqs. (3.14) and (3.15) for the coefficients corresponding to the probability amplitudes in section 3.2. Substituting Eq. (3.35) in Eqs. (3.14) and (3.14) the time evolution equations for the probability amplitudes becomes

$$\frac{d}{dt} M_{1,n} = -ig_0 \left( 1 + \frac{\Delta \nu \sin(\beta t + \phi)}{\nu_0} \right) \sqrt{n + 1} e^{-i(\nu_0 - \omega)t} e^{-i \int_0^t \Delta \nu \sin(\beta t' + \phi) dt'} M_{0,n+1} \quad (3.36)$$

$$\frac{d}{dt} M_{0,n+1} = -ig_0 \left( 1 + \frac{\Delta \nu \sin(\beta t + \phi)}{\nu_0} \right) \sqrt{n + 1} e^{i(\nu_0 - \omega)t} e^{-i \int_0^t \Delta \nu \sin(\beta t' + \phi) dt'} M_{1,n} \quad (3.37)$$

The time evolution of the system is now investigated by numerically solving the Eqs. (3.36) and (3.37).
3.4.2 Initial coherent field

Figure 3.8: Population inversion against scaled time. Initial coherent field with $\bar{n} = 25$. Field frequency has phase shifted sinusoidal fluctuations with $\alpha = 20g_0$, $\beta = 0.1g_0$. Dotted line shows the field frequency fluctuations.
In Figs. 3.8 and 3.9 the population inversion is plotted against time for the interaction of two level atom with initial coherent field with a phase shifted sinusoidal frequency modulation. It is to be noted that there are no exact collapses and revivals in population inversion but it oscillates sinusoidally with time. When the value of $\phi$ is equal to $\pi/2$ the evolution of population inversion is identical to the case of Fock field - atom interaction; a sinusoidal oscillation.
Figure 3.9: Population inversion against scaled time. Initial coherent field with $\bar{n} = 25$. Field frequency has phase shifted sinusoidal fluctuations for $\alpha = 20g_0$, $\beta = 0.1g_0$. Dotted line shows the field frequency fluctuations.
3.4.3 Initial squeezed field

Figure 3.10: Population inversion against scaled time. Initial squeezed field with $\bar{n} = 25$, $\theta = \pi$ and $r = 0.8$. Field frequency has phase shifted sinusoidal fluctuations for $\alpha = 20g_0$, $\beta = 0.1g_0$. Field frequency fluctuation is shown by the dotted lines.
As in the case of coherent field atom interaction discussed in the previous section, for the squeezed field also, the population inversion varies like a sinusoidal function with time where the collapses and revivals are insignificant. For the phase $\phi = \pi/2$ the population inversion oscillation is exactly similar to the population inversion variation that occurs in the Fock field atom interactions.
Interaction of two level atom and electromagnetic field with time varying frequency

Figure 3.11: Population inversion against scaled time. Initial squeezed field with $\bar{n} = 25$, $\theta = \pi$ and $r = 0.8$. Field frequency has phase shifted sinusoidal fluctuations for $\alpha = 20g_0$, $\beta = 0.1g_0$. Dotted line shows the field frequency fluctuations.
3.5 Conclusion

We have studied the interaction of a two level atom and squeezed field with time varying frequency. When the quadrature squeezed field interacts with a two level atom, the population inversion oscillates in random with time for squeezing parameter $r > 0.5$ and $\theta = \pi$ without collapses and periodic revivals. However, by applying a sinusoidal variation in the frequency of the field, the randomness in population inversion is reduced and the collapses and periodic revivals are regained. Thus the field frequency modulation manipulates the population inversion in the case of squeezed light atom interaction. Also, the periodicity of revival depends on the amplitude of applied frequency modulation. By varying the periodicity of the applied frequency fluctuation the dynamics of population inversion with time can be manipulated. Two level atom field interaction has an important role in the field of quantum computation. Our results suggest a new method to control and manipulate the population of states in two level atom radiation interaction, which is very essential for quantum information processing.

We have also studied the variation of atomic population inversion with time, when a two level atom interacts with light field, where the light field has a sinusoidal frequency variation with a constant phase. In both coherent field and squeezed field cases, the population inversion variation is completely different from the phase zero frequency modulation case. Variation of phases from 0 to $\pi$ have been considered. It is observed that in the presence of a non zero phase $\phi$, the population inversion oscillates sinusoidally. Also the collapses and revivals gradually disappears when $\phi$ increases from 0 to $\pi/2$. When $\phi = \pi/2$ the evolution of population inversion is identical to the case when two level atom interacts with a Fock state. This is
Interaction of two level atom and electromagnetic field with time varying frequency

the same behaviour of population inversion when two level atom and a frequency varying light interacts in Kerr medium discussed by Li Wang et al[70]. Thus, by applying a phase shifted frequency modulation one can induce sinusoidal oscillations of atomic inversion in linear medium, those normally observed in Kerr medium. We propose this as a method to control the atom field state probability amplitudes in an atom field system. In the field of quantum computation various quantum states can be used for the data storage and this method can be utilized for efficiently handling the data in quantum computation.