CHAPTER 3

\( \tau^* \)-GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

3.1 INTRODUCTION

In 1970, Levine studied the concept of generalized closed sets and a class of topological spaces called \( T_{1/2} \) – space. Using generalized closed sets, Dunham (1982) introduced the concept of generalized closure operator \( \text{cl}^* \) and obtained a class of topology, namely \( \tau^* \) topology.

3.2 \( \tau^* \)-GENERALIZED CLOSED SETS

In this section, a new class of set called \( \tau^* \)-generalized closed set is introduced and some of its properties are studied.

**Definition 3.2.1:** A subset \( A \) of a topological space \( X \) is called \( \tau^* \)-generalized closed set (briefly \( \tau^*-g \)-closed) if \( \text{cl}^*(A) \subseteq G \) whenever \( A \subseteq G \) and \( G \) is \( \tau^* \)-open.

**Theorem 3.2.2:** A subset \( A \) of \( X \) is \( \tau^*-g \)-closed if and only if \( \text{cl}^*(A) - A \) contains no non-empty closed set in \( X \).

**Proof:** Let \( A \) be a \( \tau^*-g \)-closed set. Suppose \( F \) is a non-empty closed set such that \( F \subseteq \text{cl}^*(A) - A \). Then \( F \subseteq \text{cl}^*(A) \cap A^c \), since \( \text{cl}^*(A) - A = \text{cl}^*(A) \cap A^c \). Therefore \( F \subseteq \text{cl}^*(A) \) and \( F \subseteq A^c \). Since \( F^c \) is open, it is \( \tau^* \)-open. Now, by the
definition of $\tau^*$-g-closed set, $\text{cl}^*(A) \subseteq F^c$. That is $F \subseteq [\text{cl}^*(A)]^c$. Hence $F \subseteq \text{cl}^*(A) \cap [\text{cl}^*(A)]^c = \emptyset$. That is $F = \emptyset$, which is a contradiction. Thus $\text{cl}^*(A) - A$ contains no non-empty closed set in $X$.

Conversely, assume that $\text{cl}^*(A) - A$ contains no non-empty closed set. Let $A \subseteq G$, where $G$ is $\tau^*$-open. Suppose that $\text{cl}^*(A)$ is not contained in $G$, then $\text{cl}^*(A) \cap G^c$ is a non-empty closed subset of $\text{cl}^*(A) - A$, which is a contradiction. Therefore $\text{cl}^*(A) \subseteq G$ and hence $A$ is $\tau^*$-g-closed.

**Corollary 3.2.3:** A subset $A$ of $X$ is $\tau^*$-g-closed if and only if $\text{cl}^*(A) - A$ contains no non-empty $\tau^*$-closed set in $X$.

**Proof:** The proof follows from the Theorem 3.2.2 and the fact that every closed set in $X$ is $\tau^*$-closed set in $X$.

**Corollary 3.2.4:** A subset $A$ of $X$ is $\tau^*$-g-closed if and only if $\text{cl}^*(A) - A$ contains no non-empty $g$-closed set in $X$.

**Proof:** The proof follows from the Theorem 3.2.2 and the fact that every closed set in $X$ is $g$-closed set in $X$.

**Theorem 3.2.5:** If a subset $A$ of $X$ is $\tau^*$-g-closed and $A \subseteq B \subseteq \text{cl}^*(A)$, then $B$ is $\tau^*$-g-closed set in $X$.

**Proof:** Let $A$ be a $\tau^*$-g-closed set such that $A \subseteq B \subseteq \text{cl}^*(A)$. Suppose $U$ is a $\tau^*$-open set of $X$ such that $B \subseteq U$. By assumption, we have $\text{cl}^*(A) \subseteq U$. Now $\text{cl}^*(A) \subseteq \text{cl}^*(B) \subseteq \text{cl}^*\{\text{cl}^*(A)\} = \text{cl}^*(A) \subseteq U$. That is $\text{cl}^*(B) \subseteq U$, $U$ is $\tau^*$-open. Therefore $B$ is $\tau^*$-g-closed set in $X$.

**Remark 3.2.6:** The converse of the above theorem need not be true as seen from the following example.
Example 3.2.7: Consider the topological space \((X, \tau)\), where \(X = \{a, b, c\}\) and the topology \(\tau = \{X, \emptyset, \{a\}, \{a, b\}\}\). Let \(A = \{c\}\) and \(B = \{a, c\}\). Then \(A\) and \(B\) are \(\tau^*\)-g-closed sets in \((X, \tau)\) such that \(A \subseteq B\). Also \(cl^* (A) = \{c\}\). Therefore \(A \subseteq B \subsetneq cl^*(A)\).

**Theorem 3.2.8:** Let \(A\) be a \(\tau^*\)-g-closed in \(X\). Then \(A\) is g-closed if and only if \(cl^*(A) - A\) is \(\tau^*\)-open.

**Proof:** Let \(A\) be a \(\tau^*\)-g-closed set in \(X\). Suppose \(A\) is g-closed in \(X\), then \(cl^*(A) = A\), which implies \(cl^*(A) - A = \emptyset\) which is \(\tau^*\)-open in \(X\). Conversely, suppose \(cl^*(A) - A\) is \(\tau^*\)-open in \(X\). Since \(A\) is \(\tau^*\)-g-closed, by the Theorem 3.2.2, \(cl^*(A) - A\) contains no non-empty \(\tau^*\)-closed set in \(X\). Then \(cl^*(A) - A = \emptyset\). Hence \(A\) is g-closed.

**Theorem 3.2.9:** For a point \(x \in X\), the set \(X - \{x\}\) is \(\tau^*\)-g-closed or \(\tau^*\)-open.

**Proof:** Suppose \(X - \{x\}\) is not \(\tau^*\)-open. Then \(X\) is the only \(\tau^*\)-open set containing \(X - \{x\}\). This implies \(cl^*(X - \{x\}) \subseteq X\). Hence \(X - \{x\}\) is a \(\tau^*\)-g-closed in \(X\).

**Theorem 3.2.10:** Every closed set in a topological space \(X\) is \(\tau^*\)-g-closed.

**Proof:** Let \(A\) be a closed set. Suppose \(G\) is a \(\tau^*\)-open set in \(X\) such that \(A \subseteq G\). By assumption, \(cl(A) = A \subseteq G\). But \(cl^*(A) \subseteq cl(A)\). Thus, we have \(cl^*(A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is \(\tau^*\)-open. Therefore \(A\) is \(\tau^*\)-g-closed.

**Remark 3.2.11:** The converse of the above theorem need not be true as seen from the following example.
Example 3.2.12: Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a, c\}\}$. In this topological space $(X, \tau)$, the set $\{a, b\}$ is $\tau^*$-g-closed but not closed.

Theorem 3.2.13: Every g-closed set in $X$ is $\tau^*$-g-closed.

Proof: Let $A$ be a g-closed set. By definition, $\operatorname{cl}(A) \subseteq G$, whenever $A \subseteq G$ and $G$ is open in $X$. But $\operatorname{cl}^*(A) \subseteq \operatorname{cl}(A)$ and every open set is $\tau^*$-open. Therefore $\operatorname{cl}^*(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is $\tau^*$-open in $X$. Hence $A$ is $\tau^*$-g-closed.

Remark 3.2.14: The converse of the above theorem need not be true as seen from the following example.

Example 3.2.15: Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a\}\}$. In this topological space $(X, \tau)$, the set $\{a\}$ is $\tau^*$-g-closed but not g-closed.

Theorem 3.2.16: Every strongly g-closed set in $X$ is a $\tau^*$-g-closed set.

Proof: Let $A$ be a strongly g-closed set. By definition, $\operatorname{cl}(A) \subseteq G$, whenever $A \subseteq G$ and $G$ is g-open in $X$. But $\operatorname{cl}^*(A) \subseteq \operatorname{cl}(A)$ and $G$ is $\tau^*$-open. Therefore $\operatorname{cl}^*(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is $\tau^*$-open in $X$. Hence $A$ is $\tau^*$-g-closed.

Remark 3.2.17: The converse of the above theorem need not be true as seen from the following example.

Example 3.2.18: Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{c\}, \{a, b\}\}$. In this topological space $(X, \tau)$, the set $\{a\}$ is $\tau^*$-g-closed but not strongly g-closed.
**Remark 3.2.19:** From the above discussion, we get Figure 3.1, which shows the position of $\tau^*-g$-closed set.

\begin{figure}[h]
\centering
\begin{tikzpicture}
  \node (closed) {closed set};
  \node (sgclosed) [below of=closed] {strongly $g$-closed set};
  \node (gclosed) [below of=sgclosed] {$g$-closed set};
  \node (tau-gclosed) [below of=gclosed] {$\tau^*$-$g$-closed set};
  \draw [->] (closed) -- (sgclosed);
  \draw [->] (sgclosed) -- (gclosed);
  \draw [->] (gclosed) -- (tau-gclosed);
\end{tikzpicture}
\caption{Position of $\tau^*$-$g$-closed set}
\end{figure}

**Remark 3.2.20:** In the Figure 3.1, none of the implications can be reversed.

**Remark 3.2.21:** The following examples show that $\tau^*$-$g$-closed set is independent from sp-closed set.

**Example 3.2.22:** Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi, \{a\}\}$. In this topological space $(X, \tau)$, the sets $\{a, b\}$ is $\tau^*$-$g$-closed but not sp-closed.

**Example 3.2.23:** Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi, \{a, b\}\}$. In this topological space $(X, \tau)$, the sets $\{a\}$ is sp-closed but not $\tau^*$-$g$-closed.

**Remark 3.2.24:** The following examples show that $\tau^*$-$g$-closed set is independent from sg-closed set.

**Example 3.2.25:** Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi\}$. In this topological space $(X, \tau)$, the set $\{b\}$ is $\tau^*$-$g$-closed but not sg-closed.
Example 3.2.26: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. In this topological space $(X, \tau)$, the sets $\{a\}$ is sg-closed but not $\tau^*$-g-closed.

Remark 3.2.27: The following examples show that $\tau^*$-g-closed set is independent from $\alpha$-closed set.

Example 3.2.28: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}\}$. In this topological space $(X, \tau)$, the set $\{a, c\}$ is $\tau^*$-g-closed but not $\alpha$-closed.

Example 3.2.29: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. In this topological space $(X, \tau)$, the set $\{b\}$ is $\alpha$-closed set but not a $\tau^*$-g-closed set.

Remark 3.2.30: The following examples show that $\tau^*$-g-closed set is independent from pre-closed set.

Example 3.2.31: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}\}$. In this topological space $(X, \tau)$, the set $\{a, b\}$ is $\tau^*$-g-closed but not pre-closed.

Example 3.2.32: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{b\}, \{a, b\}\}$. In this topological space $(X, \tau)$, the set $\{a\}$ is pre-closed but not $\tau^*$-g-closed.

Remark 3.2.33: The following examples show that $\tau^*$-g-closed set is independent from gs-closed set.
Example 3.2.34: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset\}$. In this topological space $(X, \tau)$, the set $\{b\}$ is $\tau^*$-g-closed but not gs-closed.

Example 3.2.35: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. In this topological space $(X, \tau)$, the set $\{b\}$ is gs-closed but not $\tau^*$-g-closed.

Remark 3.2.36: The following examples show that $\tau^*$-g-closed set is independent from gsp-closed set.

Example 3.2.37: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. In this topological space $(X, \tau)$, the set $\{a, b\}$ is gsp-closed but not $\tau^*$-g-closed.

Example 3.2.38: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}\}$. In this topological space $(X, \tau)$, the set $\{a\}$ is $\tau^*$-g-closed but not gsp-closed.

Remark 3.2.39: The following example shows that $\tau^*$-g-closed set need not be $\alpha g$-closed set.

Example 3.2.40: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \emptyset, \{a\}\}$. In this topological space $(X, \tau)$, the set $\{a\}$ is $\tau^*$-g-closed but not $\alpha g$-closed.

Remark 3.2.41: The following example shows that $\tau^*$-g-closed set need not be $g\alpha_\ell$-closed set.
Example 3.2.4: Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. In this topological space $(X, \tau)$, the set $\{b\}$ is $\tau$-g-closed but not $g\alpha$-closed.

Remark 3.2.43: From the above discussion, we obtain the following Figure 3.2.

![Figure 3.2 Isolations of $\tau^*$-g-closed set](image)

Theorem 3.2.44: For any two sets $A$ and $B$, $\text{cl}^*(A \cup B) = \text{cl}^*(A) \cup \text{cl}^*(B)$

Proof: Since $A \subseteq A \cup B$, we have $\text{cl}^*(A) \subseteq \text{cl}^*(A \cup B)$ and since $B \subseteq A \cup B$, we have $\text{cl}^*(B) \subseteq \text{cl}^*(A \cup B)$. Therefore $\text{cl}^*(A) \cup \text{cl}^*(B) \subseteq \text{cl}^*(A \cup B)$. Also, $\text{cl}^*(A)$ and $\text{cl}^*(B)$ are the $g$-closed sets. Therefore $\text{cl}^*(A) \cup \text{cl}^*(B)$ is also a $g$-closed set. Again, $A \subseteq \text{cl}^*(A)$ and $B \subseteq \text{cl}^*(B)$ implies $A \cup B \subseteq \text{cl}^*(A) \cup \text{cl}^*(B)$. Thus, $\text{cl}^*(A) \cup \text{cl}^*(B)$ is a $g$-closed set containing $A \cup B$. Since $\text{cl}^*(A \cup B)$ is the smallest $g$-closed set containing $A \cup B$, we have $\text{cl}^*(A \cup B) \subseteq \text{cl}^*(A) \cup \text{cl}^*(B)$. Thus, $\text{cl}^*(A \cup B) = \text{cl}^*(A) \cup \text{cl}^*(B)$.

Theorem 3.2.45: Union of two $\tau^*$-g-closed sets in $X$ is a $\tau^*$-g-closed set in $X$.

Proof: Let $A$ and $B$ be two $\tau^*$-g-closed sets. Suppose $A \cup B \subseteq G$, where $G$ is $\tau^*$-open. By the assumption, $\text{cl}^*(A) \cup \text{cl}^*(B) \subseteq G$. But by Theorem 3.2.44,
\( \text{cl}^\ast(A) \cup \text{cl}^\ast(B) = \text{cl}^\ast(A \cup B) \). Therefore \( \text{cl}^\ast(A \cup B) \subseteq G \). Hence, by the definition, \( A \cup B \) is a \( \tau^* \)-g-closed set.

**Theorem 3.2.46:** Intersection of \( \tau^* \)-g-closed set and g-closed set is a \( \tau^* \)-g-closed set.

**Proof:** Let \( A \) and \( B \) be g-closed set and \( \tau^* \)-g-closed set in a topological space \( X \). Suppose \( A \cup B \subseteq G \), where \( G \) is a g-open set in \( X \). Since \( A \) is g-closed, by Theorem 3.2.13, it is \( \tau^* \)-g-closed. Therefore \( \text{cl}^\ast(A) \subseteq G \). Since \( A \cap B \subseteq G \), \( \text{cl}^\ast(A \cap B) \subseteq G \). This implies \( A \cap B \) is \( \tau^* \)-g-closed set in \( X \).

**Definition 3.2.47:** A subset \( A \) of a topological space \( X \) is called \( \tau^* \)-generalized open set (briefly \( \tau^* \)-g-open) if its complement \( A^c \) is a \( \tau^* \)-generalized closed set.

**Theorem 3.2.48:** Every singleton point set in a topological space \( X \) is either \( \tau^* \)-g-open or \( \tau^* \)-open.

**Proof:** Let \( X \) be a topological space. Let \( x \in X \). We prove \( \{x\} \) is either \( \tau^* \)-g-open or \( \tau^* \)-open, that is \( X - \{x\} \) is either \( \tau^* \)-g-closed or \( \tau^* \)-open, which follows from Theorem 3.2.9.

**Theorem 3.2.49:** If \( \text{int} (A) \subseteq B \subseteq A \) and if \( A \) is \( \tau^* \)-g-open, then \( B \) is \( \tau^* \)-g-open.

**Proof:** \( A^c \subseteq B^c \subseteq \text{cl}(A^c) \) and since \( A^c \) is \( \tau^* \)-g-closed, it follows from Theorem 3.2.5 and the fact that \( \text{cl}^\ast(A^c) \subseteq \text{cl}(A^c) \), \( B^c \) is \( \tau^* \)-g-closed. Thus \( B \) is \( \tau^* \)-g-open.
3.3 \( \tau^*-T_g \) SPACE IN TOPOLOGICAL SPACES

In this section, a new topological space namely \( \tau^*-T_g \) space is introduced and some of its properties are studied.

**Definition 3.3.1:** A topological space \((X, \tau^*)\) is called \( \tau^*-T_g \) space if every \( \tau^*-g \)-closed set in \( X \) is \( g \)-closed in \( X \).

**Theorem 3.3.2:** If \( X \) is a \( T_{1/2} \) space, then it is a \( \tau^*-T_g \) space.

**Proof:** Let \( X \) be a \( T_{1/2} \) space. Since every \( g \)-closed set is \( \tau^*-g \)-closed, every closed set is \( \tau^*-g \)-closed and \( X \) is a \( T_{1/2} \) space, it follows that \( X \) is a \( \tau^*-T_g \) space.

**Remark 3.3.3:** The converse of the above theorem need not be true as seen from the following example.

**Example 3.3.4:** Let \( X = \{a, b, c\} \) be a topological space with topology \( \tau = \{\emptyset, X, \{b, c\}\} \). Then \((X, \tau)\) is a \( \tau^*-T_g \) space. However, it is not a \( T_{1/2} \) space, since the set \( \{a, c\} \) in \((X, \tau)\) is a \( g \)-closed set, but not a closed set.

**Theorem 3.3.5:** If \( X \) is a \( T_s \)-space, then it is a \( \tau^*-T_g \) space.

**Proof:** Let \( X \) be a \( T_s \)-space. Let \( A \) be a \( g \)-closed set in \( X \). By definition of \( T_s \)-space, \( A \) is strongly \( g \)-closed in \( X \). By Theorem 3.2.16, \( A \) is \( \tau^*-g \)-closed in \( X \). It proves that \( X \) is a \( \tau^*-T_g \) space.

**Remark 3.3.6:** The converse of the above theorem need not be true as seen from the following example.
Example 3.3.7: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then $(X, \tau)$ is a $\tau^*_g$-space and it is not a $T_\varepsilon$-space, since the set $\{a, b\}$ in $(X, \tau)$ is a g-closed set but not a strongly g-closed set.

Theorem 3.3.8: If $X$ is a pre-regular $T_{1/2}$ space, then it is a $\tau^*_g$-space.

Proof: Let $X$ be a pre-regular $T_{1/2}$ space. Suppose $A$ is a gpr-closed set in $X$. Then $pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open. Pre-closure implies semi pre-closure which is a subset of $cl^*$. Also, regular open implies $\tau^*$-open. Thus, we have $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tau^*$-open. This shows that $A$ is a $\tau^*_g$-closed set in $X$. Since $A$ is a gpr-closed set in $X$ and since $X$ is a pre-regular $T_{1/2}$ space, we have $A$ is pre-closed in $X$. By hypothesis, $A$ is g-closed in $X$. Hence $X$ is a $\tau^*_g$-space.

Remark 3.3.9: The spaces $\tau^*_g$ and semi-$T_{1/2}$ are independent as seen from the following examples.

Example 3.3.10: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\emptyset, X, \{a, b\}\}$. Then $(X, \tau)$ is a $\tau^*_g$-space. On the contrary, it is not a semi-$T_{1/2}$ space, since the set $\{a, c\}$ in $(X, \tau)$ is sg-closed, but not semi closed.

Example 3.3.11: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\emptyset, X, \{c\}\}$. Then $(X, \tau)$ is a semi-$T_{1/2}$ space. But it is not a $\tau^*_g$-space, since the set $\{c\}$ in $(X, \tau)$ is $\tau^*_g$-closed, but not g-closed.

Remark 3.3.12: The spaces $\tau^*_g$ and $T_d$ are independent as seen from the following examples.
Example 3.3.13: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Then the space $(X, \tau)$ is $\tau^*-T_g$ space. However, it is not a $T_d$ space, since the set $\{b\}$ in $(X, \tau)$ is $g_s$-closed, but not $g$-closed.

Example 3.3.14: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\phi, X, \{a\}\}$. Then $(X, \tau)$ is a $T_d$ space. On the other hand, it is not $\tau^*-T_g$ space, since the set $\{a\}$ in $(X, \tau)$ is $\tau^*$-g-closed, but not $g$-closed.

Remark 3.3.15: $\tau^*-T_g$ space and $\alpha$-space are independent as seen from the following examples.

Example 3.3.16: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Then $(X, \tau)$ is a $\tau^*-T_g$ space. However, it is not an $\alpha$-space, since the set $\{b\}$ in $(X, \tau)$ is $\alpha$-closed, but not closed.

Example 3.3.17: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\phi, X, \{c\}\}$. Then $(X, \tau)$ is an $\alpha$-space. On the other hand, it is not a $\tau^*-T_g$ space, since the set $\{c\}$ in $(X, \tau)$ is $\tau^*$-g-closed, but not g-closed.

Remark 3.3.18: The spaces $\tau^*-T_g$ and $T_p$ are independent as seen from the following examples.

Example 3.3.19: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Then $(X, \tau)$ is a $\tau^*-T_g$ space. But it is not a $T_p$ space, since the set $\{a, b\}$ in $(X, \tau)$ is strongly $g$-closed, but not closed.

Example 3.3.20: Let $X = \{a, b, c\}$ be a topological space with topology $\tau = \{\phi, X, \{c\}\}$. Then $(X, \tau)$ is a $T_p$ space. However, it is not a $\tau^*-T_g$ space, since the set $\{c\}$ in $(X, \tau)$ is $\tau^*$-g-closed, but not g-closed in $X$. 
Remark 3.3.21: Visualization of the above discussion is shown in the Figure 3.3.

\[
\begin{array}{ccc}
T_{1/2}\text{-space} & T_c\text{-space} & \text{Semi } T_{1/2}\text{-space} \\
\alpha\text{-space} & T^*g\text{-space} & \text{Pre-regular } T_{1/2}\text{-space} \\
T_d\text{-space} & & T_f\text{-space}
\end{array}
\]

Figure 3.3 Separation axioms on $\tau^*-T_g$-space

3.4 CONCLUSION

In this chapter, $\tau^*$-generalized closed set in topological spaces has been defined and some of its properties are studied. Comparison of this set with few other sets in topological spaces is also done. Basic properties of $\tau^*$-generalized open set are also studied. $\tau^*-T_g$ space is defined and studied. In future, study of $\tau^*$-generalized closed sets can be extended on bitopology and fuzzy topology.