CHAPTER 6

\( \tau^* \)-SEMI GENERALIZED CONTINUOUS FUNCTIONS AND
\( \tau^* \)-GENERALIZED SEMI CONTINUOUS FUNCTIONS IN
TOPOLOGICAL SPACES

6.1 INTRODUCTION

Bhattacharyya & Lahiri (1987) introduced semi generalized closed set and Arya & Nour (1990) introduced generalized semi closed set in topological space. In this chapter, we define two functions in topological spaces, namely \( \tau^* \)- semi generalized continuous function and \( \tau^* \)-generalized semi continuous function and study some of their properties.

6.2 \( \tau^* \)-SEMI GENERALIZED CONTINUOUS FUNCTION

In this section, a new class of function namely \( \tau^* \)-semi generalized continuous function in topological spaces is introduced and some of its properties and relationship with some existing functions are studied.

Definition 6.2.1: A function \( f : X \to Y \) from a topological space \( X \) into a topological space \( Y \) is called \( \tau^* \)-semi generalized continuous function (briefly \( \tau^* \)-sg-continuous) if the inverse image of every sg-closed set in \( Y \) is \( \tau^* \)-g-closed in \( X \).

Theorem 6.2.2: Let \( f : X \to Y \) be a function from a topological space \((X, \tau^*)\) into a topological space \((Y, \sigma^*)\).
(i) The following statements are equivalent

(a) \( f \) is \( \tau^* \)-sg-continuous

(b) The inverse image of each sg-open set in \( Y \) is \( \tau^* \)-g-open in \( X \)

(ii) If \( f : X \to Y \) is a \( \tau^* \)-sg-continuous function, then \( f(\text{cl}_\tau^*(A)) \subseteq \text{cl}(f(A)) \) for every subset \( A \) of \( X \).

**Proof:**

(i) Assume that \( f : X \to Y \) is \( \tau^* \)-sg-continuous. Suppose \( G \) is sg-open in \( Y \). Then \( G^C \) is sg-closed in \( Y \). By the assumption, \( f^{-1}(G^C) \) is \( \tau^* \)-g-closed in \( X \). But \( f^{-1}(G^C) = X - f^{-1}(G) \). Thus \( X - f^{-1}(G) \) is \( \tau^* \)-g-closed in \( X \) and so \( f^{-1}(G) \) is \( \tau^* \)-g-open in \( X \). Therefore (a) \( \implies \) (b).

Conversely assume that the inverse image of each sg-open set in \( Y \) is \( \tau^* \)-g-open in \( X \). Let \( F \) be any sg-closed set in \( Y \). Then \( F^C \) is sg-open in \( Y \). By assumption, \( f^{-1}(F^C) \) is \( \tau^* \)-g-open in \( X \). But \( f^{-1}(F^C) = X - f^{-1}(F) \). Thus \( X - f^{-1}(F) \) is \( \tau^* \)-g-open in \( X \) and so \( f^{-1}(F) \) is \( \tau^* \)-g-closed in \( X \). Therefore \( f \) is \( \tau^* \)-sg-continuous. Hence (b) \( \implies \) (c). Thus (a) and (b) are equivalent.

(ii) Assume that \( f \) is \( \tau^* \)-sg-continuous. Suppose \( A \) is a subset of \( X \). Then \( \text{cl}(f(A)) \) is sg-closed in \( Y \). By assumption, \( f^{-1}(\text{cl}(f(A))) \) is \( \tau^* \)-g-closed in \( X \) and it contains \( A \). But \( \text{cl}_\tau^*(A) \) is the intersection of all \( \tau^* \)-g-closed set containing \( A \). Therefore \( \text{cl}_\tau^*(A) \subseteq f^{-1}(\text{cl}(f(A))) \) and so \( f(\text{cl}_\tau^*(A)) \subseteq \text{cl}(f(A)) \).

**Theorem 6.2.3:** If \( f : X \to Y \) is continuous then it is \( \tau^* \)-sg-continuous provided \( Y \) is both \( T_{b*} \)-space and \( T_b \)-space.
**Proof:** Let \( f : X \to Y \) be continuous. Suppose \( V \) is a sg-closed set in \( Y \). Since \( Y \) is both \( T_{gs} \)-space and \( T_b \)-space, \( V \) is closed in \( Y \). By the assumption, \( f^{-1}(V) \) is closed in \( X \). Also by Theorem 3.2.10, \( f^{-1}(V) \) is \( \tau^* \)-g-closed. Thus, \( f \) is \( \tau^* \)-sg-continuous.

**Remark 6.2.4:** The above theorem need not be true as seen from the following example if \( Y \) is not \( T_{gs} \)-space and \( T_b \)-space.

**Example 6.2.5:** Let \( X=Y= \{a, \ b, \ c\} \), \( \tau = \{X, \phi, \{b\}, \{c\}, \{b, \ c\}, \{a, \ c\}\} \) and \( \sigma = \{Y, \phi, \{b\}, \{a, \ c\}\} \). Let \( f : X \to Y \) be an identity function. Here \( Y \) is not a \( T_b \)-space. Then \( f \) is continuous. On the other hand, it is not \( \tau^* \)-sg-continuous, since for the sg-closed set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not \( \tau^* \)-g-closed in \( X \).

**Example 6.2.6:** Let \( X=Y= \{a, \ b, \ c\} \), \( \tau = \{X, \phi, \{b\}, \{c\}, \{b, \ c\}\} \) and \( \sigma = \{Y, \phi, \{c\}\} \). Let \( f : X \to Y \) be an identity function. Here \( Y \) is not a \( T_{gs} \)-space. Then \( f \) is continuous. On the other hand, it is not \( \tau^* \)-sg-continuous, since for the sg-closed set \( \{b\} \) in \( Y \), the inverse image of \( \{b\} \) is not \( \tau^* \)-g-closed in \( X \).

**Theorem 6.2.7:** If \( f : X \to Y \) is g-continuous then it is \( \tau^* \)-sg-continuous provided \( Y \) is both \( T_{gs} \)-space and \( T_b \)-space.

**Proof:** Let \( f : X \to Y \) be g-continuous. Suppose \( V \) is a sg-closed set in \( Y \). Since \( Y \) is both \( T_{gs} \)-space and \( T_b \)-space, \( V \) is closed in \( Y \). By the assumption, \( f^{-1}(V) \) is g-closed in \( X \). Also by Theorem 3.2.13, \( f^{-1}(V) \) is \( \tau^* \)-g-closed. Thus, \( f \) is \( \tau^* \)-sg-continuous.

**Remark 6.2.8:** The above theorem need not be true as seen from the following example if \( Y \) is not \( T_{gs} \)-space and \( T_b \)-space.
Example 6.2.9: Let $X=Y=\{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{a, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Here $Y$ is not a $T_0$-space. Clearly $f$ is $g$-continuous. But it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{a, b\}$ in $Y$, the inverse image is not $\tau^*$-g-closed in $X$.

Example 6.2.10: Let $X=Y=\{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Here $Y$ is not a $T_{gs}$-space. Clearly $f$ is $g$-continuous. But it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-g-closed in $X$.

Theorem 6.2.11: If $f : X \rightarrow Y$ is strongly sg-continuous then it is $\tau^*$-sg-continuous.

Proof: Let $f : X \rightarrow Y$ be strongly sg-continuous. Suppose $F$ is a sg-closed set in $Y$. Then $F^C$ is sg-open in $Y$. By the assumption, $f^{-1}(F^C)$ is open in $X$. But $f^{-1}(F^C) = X - f^{-1}(F)$. Therefore $f^{-1}(F)$ is closed in $X$. By Theorem 3.2.10, $f^{-1}(F)$ is $\tau^*$-g-closed in $X$. Thus, $f$ is $\tau^*$-sg-continuous.

Remark 6.2.12: Converse of the above theorem need not be true as seen from the following example.

Example 6.2.13: Let $X=Y=\{a, b, c\}$, $\tau = \{X, \phi, \{c\}, \sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. However, it is not strongly sg-continuous, since for the sg-open set $\{a, b\}$ in $Y$, the inverse image of $\{a, b\}$ is not open in $X$.

Theorem 6.2.14: If $f : X \rightarrow Y$ is strongly gs-continuous then it is $\tau^*$-sg-continuous.
**Proof:** Let \( f : X \to Y \) be strongly gs-continuous. Suppose \( V \) is a sg-closed set in \( Y \). Since every sg-closed set is gs-closed, \( V \) is gs-closed and so \( V^C \) is gs-open. By the assumption of \( f \), \( f^{-1}(V^C) \) is open in \( X \). But \( f^{-1}(V^C) = X - f^{-1}(V) \). Therefore \( f^{-1}(V) \) is closed in \( X \). By Theorem 3.2.10, \( f^{-1}(V) \) is \( \tau^* \)-g-closed in \( X \). Thus, \( f \) is \( \tau^* \)-sg-continuous.

**Remark 6.2.15:** Converse of the above theorem need not be true as seen from the following example.

**Example 6.2.16:** Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}, \{b, c\}\} \) and \( \sigma = \{Y, \phi, \{a\}, \{a, b\}, \{a, c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \tau^* \)-sg-continuous. On the other hand, it is not strongly gs-continuous, since for the gs-open set \( \{a, b\} \) in \( Y \), the inverse image of \( \{a, b\} \) is not open in \( X \).

**Remark 6.2.17:** Following examples show that \( \tau^* \)-sg-continuous function is independent from the semi-continuous function.

**Example 6.2.18:** Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}, \{a, c\}\} \) and \( \sigma = \{Y, \phi, \{a\}, \{a, b\}, \{a, c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is semi-continuous. On the contrary, it is not \( \tau^* \)-sg-continuous, since for the sg-closed set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not \( \tau^* \)-g-closed in \( X \).

**Example 6.2.19:** Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{c\}\} \) and \( \sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \tau^* \)-sg-continuous. On the other hand, it is not semi-continuous, since for the closed set \( \{a, c\} \) in \( Y \), the inverse image of \( \{a, c\} \) is not semi-closed in \( X \).

**Remark 6.2.20:** Following examples show that \( \tau^* \)-sg-continuous function is independent from the sg-continuous function.
Example 6.2.21: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\text{sg}$-continuous. On the other hand, it is not $\tau^*$-$\text{sg}$-continuous, since for the $\text{sg}$-closed set $\{a, b\}$ in $Y$, the inverse image $\{a, b\}$ is not $\tau^*$-$\text{g}$-closed in $X$.

Example 6.2.22: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-$\text{sg}$-continuous. However, it is not $\text{sg}$-continuous, since for the closed set $\{a, c\}$ in $Y$, the inverse image of $\{a, c\}$ is not $\text{sg}$-closed in $X$.

Remark 6.2.23: Following examples show that $\tau^*$-$\text{sg}$-continuous function is independent from the $\text{gs}$-continuous function.

Example 6.2.24: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\text{gs}$-continuous. But it is not $\tau^*$-$\text{sg}$-continuous, since for the $\text{gs}$-closed set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-$\text{g}$-closed in $X$.

Example 6.2.25: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-$\text{sg}$-continuous. On the other hand, it is not $\text{gs}$-continuous, since for the closed set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\text{gs}$-closed in $X$.

Remark 6.2.26: Following examples show that $\tau^*$-$\text{sg}$-continuous function is independent from the $\text{gsp}$-continuous function.

Example 6.2.27: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\text{gsp}$-continuous. However, it is not $\tau^*$-$\text{sg}$-continuous, since for the $\text{sg}$-closed set $\{c\}$ in $Y$, the inverse image of $\{c\}$ is not $\tau^*$-$\text{g}$-closed in $X$. 
**Example 6.2.28:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. But it is not gsp-continuous, since for the closed set $\{c\}$ in $Y$, the inverse image of $\{c\}$ is not gsp-closed in $X$.

**Remark 6.2.29:** Following examples show that $\tau^*$-sg-continuous function is independent from the $\alpha g$-continuous function.

**Example 6.2.30:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{a, b\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\alpha g$-continuous. However, it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-g-closed in $X$.

**Example 6.2.31:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. But it is not $\alpha g$-continuous, since for the closed set $\{b\}$ in $Y$, the inverse image of $\{b\}$ is not $\alpha g$-closed in $X$.

**Remark 6.2.32:** Following examples show that $\tau^*$-sg-continuous function is independent from the pre-continuous function.

**Example 6.2.33:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is pre-continuous. On the other hand, it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-g-closed in $X$.

**Example 6.2.34:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$, $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. However, it is not pre-continuous, since for the open set $\{b, c\}$ in $Y$, the inverse image of $\{b, c\}$ is not pre-open in $X$. 
Remark 6.2.35: Following examples show that $\tau^*$-sg-continuous function is independent from the $\alpha$-continuous function.

Example 6.2.36: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{c\}, \{b, c\}\}$ and $\sigma=\{Y, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\alpha$-continuous. On the contrary, it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{b\}$ in $Y$, the inverse image of $\{b\}$ is not $\tau^*$-g-closed in $X$.

Example 6.2.37: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\sigma=\{Y, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. But it is not $\alpha$-continuous, since for the open set $\{a, c\}$ in $Y$, the inverse image of $\{a, c\}$ is not $\alpha$-open in $X$.

Remark 6.2.38: Following examples show that $\tau^*$-sg-continuous function is independent from the sp-continuous function.

Example 6.2.39: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{a, c\}\}$ and $\sigma=\{Y, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is sp-continuous. On the other hand, it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{c\}$ in $Y$, the inverse image of $\{c\}$ is not $\tau^*$-g-closed in $X$.

Example 6.2.40: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{c\}\}$, $\sigma=\{Y, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. However, it is not sp-continuous, since for the open set $\{b\}$ in $Y$, the inverse image of $\{b\}$ is not sp-open in $X$.

Remark 6.2.41: Following examples show that $\tau^*$-sg-continuous function independent from the weakly sg-continuous function.
Example 6.2.42: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is weakly sg-continuous. On the other hand, it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{b\}$ in $Y$, the inverse image of $\{b\}$ is not $\tau^*$-g-closed in $X$.

Example 6.2.43: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. But it is not weakly sg-continuous, since for the sg-open set $\{a, c\}$ in $Y$, the inverse image of $\{a, c\}$ is not semi-open in $X$.

Remark 6.2.44: Following examples show that $\tau^*$-sg-continuous function is independent from the weakly gs-continuous function.

Example 6.2.45: Let $X = Y = \{a, b, c\}$ $\tau = \{X, \phi, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is weakly gs-continuous. On the contrary, it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-g-closed in $X$.

Example 6.2.46: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. However, it is not weakly gs-continuous, since for the gs-open set $\{c\}$ in $Y$, the inverse image of $\{c\}$ is not semi-open in $X$.

Remark 6.2.47: Following examples show that $\tau^*$-sg-continuous function is independent from the sg*-continuous function.

Example 6.2.48: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is sg*-continuous. On the other hand, it is not $\tau^*$-sg-continuous, since for the sg-closed set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-g-closed in $X$. 
Example 6.2.49: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. On the contrary, it is not $\sg^*$-continuous, since for the semi-open set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\sg^*$-open in $X$.

Remark 6.2.50: Following examples show that $\tau^*$-sg-continuous function is independent from the $\sg^*$-continuous function.

Example 6.2.51: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\sg^*$-continuous. On the other hand, it is not $\tau^*$-sg-continuous, since for the $\sg$-closed set $\{a, c\}$ in $Y$, the inverse image of $\{a, c\}$ is not $\tau^*$-g-closed in $X$.

Example 6.2.52: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. However, it is not $\sg^*$-continuous, since for the semi-open set $\{a, c\}$ in $Y$, the inverse image of $\{a, c\}$ is not $\sg^*$-open in $X$.

Remark 6.2.53: Following examples show that $\tau^*$-sg-continuous function is independent from the $\tau^*$-g-continuous function.

Example 6.2.54: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a\}, \{c\}, \{a, c\}\}$. Let $f : X \to Y$ be a function defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is $\tau^*$-g-continuous. But it is not $\tau^*$-sg-continuous, since for the $\sg$-closed set $\{a\}$ in $Y$, the inverse image of $\{b\}$ is not $\tau^*$-g-closed in $X$.

Example 6.2.55: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{a, b\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. On the other hand, it is not $\tau^*$-g-continuous, since for the g-closed set $\{b, c\}$ in $Y$, the inverse image of $\{b, c\}$ is not $\tau^*$-g-closed in $X$. 
Remark 6.2.56: From the above discussion, we get the following Figure 6.1.

\[ \begin{array}{cccc}
\alpha\text{-continuous} & \tau^*\text{-g-continuous} & \text{gsp-continuous} & \text{sp-continuous} \\
\alpha g\text{-continuous} & & & \text{gs-continuous} \\
\text{weakly sg-continuous} & \tau^*\text{-sg-continuous} & \text{weakly gs-continuous} & \\
\text{sg}-\text{continuous} & & & \text{gs}-\text{continuous} \\
\text{semi-continuous} & \text{pre-continuous} & \text{sg-continuous} & \\
\end{array} \]

Figure 6.1 Remoteness of \( \tau^* \)-sg-continuous function

6.3 \( \tau^* \)-GENERALIZED SEMI CONTINUOUS FUNCTION IN TOPOLOGICAL SPACES

In this section, a new class of function namely \( \tau^* \)-generalized semi continuous function in topological spaces is introduced and some of its properties and relationship with some existing functions are studied.

Definition 6.3.1: A function \( f : X \rightarrow Y \) from a topological space \( X \) into a topological space \( Y \) is called \( \tau^* \)-generalized semi continuous function (briefly \( \tau^* \)-gs-continuous) if the inverse image of every gs-open set in \( Y \) is \( \tau^* \)-g-open in \( X \).

Theorem 6.3.2: Let \( f : X \rightarrow Y \) be a function from a topological space \( (X, \tau^*) \) into a topological space \( (Y, \sigma^*) \).

(i) The following statements are equivalent:

(a) \( f \) is \( \tau^* \)-gs-continuous.
(b) The inverse image of each gs-open set in $\mathcal{Y}$ is $\tau^*-g$-open in $\mathcal{X}$.

(ii) If $f: \mathcal{X} \rightarrow \mathcal{Y}$ is $\tau^*-gs$-continuous, then $f(\text{cl}_{\tau^*}(A)) \subseteq \text{cl}(f(A))$ for every subset $A$ of $\mathcal{X}$.

**Proof:**

(i) Assume that $f: \mathcal{X} \rightarrow \mathcal{Y}$ is $\tau^*-gs$-continuous. Suppose $G$ is a gs-open set in $\mathcal{Y}$. Then $G^c$ is gs-closed in $\mathcal{Y}$. By the assumption, $f^{-1}(G^c)$ is $\tau^*-g$-closed in $\mathcal{X}$. But $f^{-1}(G^c) = \mathcal{X} - f^{-1}(G)$. Thus $\mathcal{X} - f^{-1}(G)$ is $\tau^*-g$-closed in $\mathcal{X}$ and so $f^{-1}(G)$ is $\tau^*-g$-open in $\mathcal{X}$. Therefore (a) $\Rightarrow$ (b).

Conversely assume that the inverse image of each gs-open set in $\mathcal{Y}$ is $\tau^*-g$-open in $\mathcal{X}$. Suppose $F$ is a gs-closed set in $\mathcal{Y}$. Then $F^c$ is gs-open in $\mathcal{Y}$. By the assumption, $f^{-1}(F^c)$ is $\tau^*-g$-open in $\mathcal{X}$. But $f^{-1}(F^c) = \mathcal{X} - f^{-1}(F)$. Thus $\mathcal{X} - f^{-1}(F)$ is $\tau^*-g$-open in $\mathcal{X}$ and so $f^{-1}(F)$ is $\tau^*-g$-closed in $\mathcal{X}$. Therefore $f$ is $\tau^*-gs$-continuous. Hence (b) $\Rightarrow$ (c). Thus (a) and (b) are equivalent.

(ii) Assume that $f$ is $\tau^*-gs$-continuous. Suppose $A$ is a subset of $\mathcal{X}$. Then $\text{cl}(f(A))$ is gs-closed in $\mathcal{Y}$. By the assumption of $f$, $f^{-1}(\text{cl}(f(A)))$ is $\tau^*-g$-closed in $\mathcal{X}$ and it contains $A$. But $\text{cl}_{\tau^*}(A)$ is the intersection of all $\tau^*-g$-closed set containing $A$. Therefore $\text{cl}_{\tau^*}(A) \subseteq f^{-1}(\text{cl}(f(A)))$ and so $f(\text{cl}_{\tau^*}(A)) \subseteq \text{cl}(f(A))$.

**Theorem 6.3.3:** If $f: \mathcal{X} \rightarrow \mathcal{Y}$ is continuous then it is $\tau^*-gs$-continuous provided $\mathcal{Y}$ is a $\text{T}_{1\beta}$-space.
Proof: Let \( f : X \to Y \) be continuous. Suppose \( V \) is a gs-open set in \( Y \). Then \( V^c \) is gs-closed in \( Y \). Since \( Y \) is a \( T_b \)-space, \( V^c \) is closed in \( Y \). Since \( f \) is continuous, \( f^{-1}(V^c) \) is closed in \( X \). By Theorem 3.2.10, \( f^{-1}(V^c) \) is \( \tau^* \)-closed in \( X \). But \( f^{-1}(V^c) = X - f^{-1}(V) \). Thus \( X - f^{-1}(V) \) is \( \tau^* \)-closed in \( X \). Therefore \( f^{-1}(V) \) is \( \tau^* \)-open in \( X \). Hence \( f \) is \( \tau^* \)-gs-continuous.

Remark 6.3.4: The above theorem need not be true as seen from the following example if \( Y \) is not a \( T_b \)-space.

Example 6.3.5: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\} \) and \( \sigma = \{Y, \phi, \{a\}, \{b, c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is continuous. But it is not \( \tau^* \)-gs-continuous, since for the gs-open set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not \( \tau^* \)-gs-open in \( X \).

Theorem 6.3.6: If a function \( f : X \to Y \) from a topological space \( X \) into a topological space \( Y \) is g-continuous, then it is \( \tau^* \)-gs-continuous provided \( Y \) is a \( T_b \)-space.

Proof: Let \( f : X \to Y \) be g-continuous. Suppose \( V \) is a gs-open set in \( Y \). Then \( V^c \) is gs-closed in \( Y \). Since \( Y \) is a \( T_b \)-space, \( V^c \) is closed in \( Y \). By the assumption, \( f^{-1}(V^c) \) is g-closed in \( X \). Also by Theorem 3.2.13, \( f^{-1}(V^c) \) is \( \tau^* \)-g-closed. But \( f^{-1}(V^c) = X - f^{-1}(V) \). Consequently \( X - f^{-1}(V) \) is \( \tau^* \)-g-closed in \( X \). Thus \( f^{-1}(V) \) is \( \tau^* \)-g-open in \( X \). Therefore \( f \) is \( \tau^* \)-gs-continuous.

Remark 6.3.7: The above theorem need not be true as seen from the following example if \( Y \) is not a \( T_b \)-space.

Example 6.3.8: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\} \) and \( \sigma = \{Y, \phi, \{a, b\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is
g-continuous. However, it is not $\tau^*$-gs-continuous, since for the gs-open set \{a\} in Y, the inverse image of \{a\} is not $\tau^*$-g-open in X.

**Theorem 6.3.9:** If a function $f : X \to Y$ from a topological space X into a topological space Y is strongly sg-continuous, then it is $\tau^*$-gs-continuous.

**Proof:** Let $f : X \to Y$ be a strongly sg-continuous function. Suppose F is a sg-open set in Y. Then $F^c$ is sg-closed in Y. Since every sg-closed set is gs-closed, $F^c$ is gs-closed in Y. By the assumption, $f^{-1}(F^c)$ is closed in X. Also by Theorem 3.2.10, $f^{-1}(F^c)$ is $\tau^*$-g-closed in X. But $f^{-1}(F) = X - f^{-1}(V^c)$, As a result, $f^{-1}(V)$ is $\tau^*$-g-closed in X. Thus $f^{-1}(V)$ is $\tau^*$-g-open in X. Therefore f is $\tau^*$-gs-continuous.

**Remark 6.3.10:** The converse of the above theorem need not be true as seen from the following.

**Example 6.3.11:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let $f : X \to Y$ be an identity function. Then f is strongly sg-continuous. On the other hand, it is not $\tau^*$-gs-continuous, since for the gs-open set \{b\} in Y, the inverse image of \{b\} is not $\tau^*$-g-open in X.

**Theorem 6.3.12:** If a function $f : X \to Y$ from a topological space X into a topological space Y is strongly gs-continuous, then it is $\tau^*$-gs-continuous.

**Proof:** Let $f : X \to Y$ be a strongly gs-continuous function. Suppose V is a gs-closed set in Y. Then $V^c$ is gs-open in Y. Then by assumption, $f^{-1}(V^c)$ is open in X. By Theorem 3.2.10, $f^{-1}(V^c)$ is $\tau^*$-g-open in X. Thus, f is $\tau^*$-gs-continuous.
Remark 6.3.13: The converse of the above theorem need not be true as seen from the following example.

Example 6.3.14: Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$, $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-gs-continuous. On the contrary, it is not strongly gs-continuous, since for the gs-open set $\{c\}$ in $Y$, the inverse image of $\{c\}$ is not open in $X$.

Theorem 6.3.15: If a function $f : X \to Y$ from a topological space $X$ into a topological space $Y$ is $\tau^*$-sg-continuous, then it is $\tau^*$-gs-continuous, provided $Y$ is a T$_{gs}$-space.

Proof: Let $f : X \to Y$ be a $\tau^*$-sg-continuous function. Suppose $V$ is a gs-open set in $Y$. Since $Y$ is a T$_{gs}$-space, $V$ is sg-open in $Y$. Then $V^c$ is sg-closed in $Y$. By assumption, $f^{-1}(V^c)$ is $\tau^*$-g-closed in $X$. Also, $f^{-1}(V^c) = X - f^{-1}(V)$. Thus $X - f^{-1}(V)$ is $\tau^*$-g-closed in $X$. As a result, $f^{-1}(V)$ is $\tau^*$-g-open in $X$. Therefore $f$ is $\tau^*$-gs-continuous.

Remark 6.3.16: The above theorem need not be true as seen from the following example if $Y$ is not a T$_{gs}$-space.

Example 6.3.17: Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-sg-continuous. On the other hand, it is not $\tau^*$-gs-continuous, since for the gs-open set $\{c\}$ in $Y$, the inverse image of $\{c\}$ is not $\tau^*$-g-open in $X$.

Remark 6.3.18: Following examples show that $\tau^*$-gs-continuous function is independent from the semi-continuous function.
Example 6.3.19: Let $X = Y = \{ a, b, c \}$ and let $\tau = \{ X, \phi, \{ b \}, \{ a, b \} \}$ and $\sigma = \{ Y, \phi, \{ b \}, \{ a, b \}, \{ b, c \} \}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is semi-continuous. However, it is not $\tau^*$-gs-continuous, since for the gs-open set $\{ b, c \}$ in $Y$, the inverse image of $\{ b, c \}$ is not $\tau^*$-g-open in $X$.

Example 6.3.20: Let $X = Y = \{ a, b, c \}$ and let $\tau = \{ X, \phi, \{ a \} \}$ and $\sigma = \{ Y, \phi, \{ a \}, \{ c \}, \{ a, c \}, \{ b, c \} \}$. Define $f : X \rightarrow Y$ by $f(a) = b, f(b) = c$ and $f(c) = a$. Then $f$ is $\tau^*$-gs-continuous. On the contrary, it is not semi-continuous, since for the closed set $\{ b \}$ in $Y$, the inverse image of $\{ b \}$ is not semi-closed in $X$.

Remark 6.3.21: Following examples show that $\tau^*$-gs-continuous function is independent from the sg-continuous function.

Example 6.3.22: Let $X = Y = \{ a, b, c \}$ and let $\tau = \{ X, \phi, \{ b \}, \{ a, b \}, \{ b, c \} \}$ and $\sigma = \{ Y, \phi, \{ a, b \} \}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is sg-continuous. On the other hand, it is not $\tau^*$-gs-continuous, since for the gs-open set $\{ a \}$ in $Y$, the inverse image of $\{ a \}$ is not $\tau^*$-g-open in $X$.

Example 6.3.23: Let $X = Y = \{ a, b, c \}$ and let $\tau = \{ X, \phi, \{ b \} \}$ and $\sigma = \{ Y, \phi, \{ a \}, \{ a, b \}, \{ a, c \} \}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is $\tau^*$-gs-continuous. However, it is not sg-continuous, since for the closed set $\{ b \}$ in $Y$, the inverse image of $\{ b \}$ is not sg-closed in $X$.

Remark 6.3.24: Following example shows that $\tau^*$-gs-continuous function is independent from the gsp-continuous function.

Example 6.3.25: Let $X = Y = \{ a, b, c \}$ and let $\tau = \{ X, \phi, \{ a \}, \{ c \}, \{ a, c \}, \{ a, b \} \}$ and $\sigma = \{ Y, \phi, \{ a, b \} \}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is gsp-continuous. On the contrary, it is not $\tau^*$-gs-continuous, since for the gs-open set $\{ b \}$ in $Y$, the inverse image of $\{ b \}$ is not $\tau^*$-g-open in $X$. 
Example 6.3.26: Let \( X = Y = \{a, b, c\} \) and let \( \tau = \{X, \phi, \{c\}\} \), \( \sigma = \{Y, \phi, \{b\}, \{a, b\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \tau^* \)-gs-continuous. But it is not gsp-continuous, since for the closed set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not gsp-closed in \( X \).

Remark 6.3.27: Following examples show that \( \tau^* \)-gs-continuous function is independent from the \( \alpha g \)-continuous function.

Example 6.3.28: Let \( X = Y = \{a, b, c\} \) and let \( \tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\} \), \( \sigma = \{Y, \phi, \{a, c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \alpha g \)-continuous. On the other hand, it is not \( \tau^* \)-gs-continuous, since for the gs-open set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not \( \tau^* \)-g-open in \( X \).

Example 6.3.29: Let \( X = Y = \{a, b, c\} \) and let \( \tau = \{X, \phi, \{b\}\}, \sigma = \{Y, \phi, \{a\}, \{a, b\}, \{a, c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \tau^* \)-gs-continuous. However, it is not \( \alpha g \)–continuous, since for the closed set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not \( \alpha g \)-closed in \( X \).

Remark 6.3.30: Following examples show that \( \tau^* \)-gs-continuous function is independent from the \( g\alpha \)-continuous function.

Example 6.3.31: Let \( X = Y = \{a, b, c\} \) and let \( \tau = \{X, \phi, \{b\}, \{b, c\}\} \), \( \sigma = \{Y, \phi, \{c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( g\alpha \)-continuous. But it is not \( \tau^* \)-gs-continuous, since for the gs-open set \( \{a\} \) in \( Y \), the inverse image of \( \{a\} \) is not \( \tau^* \)-g-open in \( X \).

Example 6.3.32: Let \( X = Y = \{a, b, c\} \) and let \( \tau = \{X, \phi, \{b\}\} \) and \( \sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is
\( \tau^* \)-gs-continuous. On the other hand, it is not \( g\alpha \)-continuous, since for the closed set \( \{b, c\} \) in \( Y \), the inverse image of \( \{b, c\} \) is not \( g\alpha \)-closed in \( X \).

**Remark 6.3.33**: Following examples show that \( \tau^* \)-gs-continuous function is independent from the pre-continuous function.

**Example 6.3.34**: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{c\}, \{b, c\}\} \) and \( \sigma = \{Y, \phi, \{b\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is pre-continuous. However, it is not \( \tau^* \)-gs-continuous, since for the gs-open set \( \{a\} \) in \( Y \), the inverse image of \( \{a\} \) is not \( \tau^* \)-g-open in \( X \).

**Example 6.3.35**: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{b\}\} \) and \( \sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \tau^* \)-gs-continuous. But it is not pre-continuous, since for the open set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not pre-open in \( X \).

**Remark 6.3.36**: Following examples show that \( \tau^* \)-gs-continuous function is independent from the \( \alpha \)-continuous function.

**Example 6.3.37**: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\} \) and \( \sigma = \{Y, \phi, \{b\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \alpha \)-continuous. On the other hand, it is not \( \tau^* \)-gs-continuous, since for the gs-open set \( \{b\} \) in \( Y \), the inverse image of \( \{b\} \) is not \( \tau^* \)-g-open in \( X \).

**Example 6.3.38**: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}\} \), \( \sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\} \). Let \( f : X \to Y \) be an identity function. Then \( f \) is \( \tau^* \)-gs-continuous. However, it is not \( \alpha \)-continuous, since for the open set \( \{c\} \) in \( Y \), the inverse image of \( \{c\} \) is not \( \alpha \)-open in \( X \).
**Remark 6.3.39:** Following examples show that $\tau^*$-gs-continuous function is independent from the sp-continuous function.

**Example 6.3.40:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is sp-continuous. On the contrary, it is not $\tau^*$-gs-continuous, since for the gs-open set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-open in $X$.

**Example 6.3.41:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}\}$, $\sigma = \{Y, \phi, \{a, b, a, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is $\tau^*$-gs-continuous. But it is not sp-continuous, since for the open set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not sp-open in $X$.

**Remark 6.3.42:** Following examples show that $\tau^*$-gs-continuous function is independent from the weakly sg-continuous function.

**Example 6.3.43:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is weakly sg-continuous. On the other hand, it is not $\tau^*$-gs-continuous, since for the gs-open set $\{b\}$ in $Y$, the inverse image of $\{b\}$ is not $\tau^*$-open in $X$.

**Example 6.3.44:** Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : X \rightarrow Y$ be an identity function. Then $f$ is $\tau^*$-gs-continuous. However, it is not weakly sg-continuous, since for the sg-open set $\{c\}$ in $Y$, the inverse image of $\{c\}$ is not semi-open in $X$.

**Remark 6.3.45:** Following examples show that $\tau^*$-gs-continuous function is independent from the weakly gs-continuous function.
Example 6.3.46: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{a, b\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is weakly $\text{gs}$-continuous. But it is not $\tau^*$-gs-continuous, since for the gs-open set $\{b, c\}$ in $Y$, the inverse image of $\{b, c\}$ is not $\tau^*$-g-open in $X$.

Example 6.3.47: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-gs-continuous. On the contrary, it is not weakly gs-continuous, since for the gs-open set $\{b\}$ in $Y$, the inverse image of $\{b\}$ is not semi-open in $X$.

Remark 6.3.48: Following examples show that $\tau^*$-gs-continuous function is independent from the weakly sg*-continuous function.

Example 6.3.49: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is sg*-continuous. But it is not $\tau^*$-gs-continuous, since for the gs-open set $\{a, c\}$ in $Y$, the inverse image of $\{a, c\}$ is not $\tau^*$-g-open in $X$.

Example 6.3.50: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is $\tau^*$-gs-continuous. On the other hand, it is not sg*-continuous, since for the semi-open set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not sg-open in $X$.

Remark 6.3.51: Following examples show that $\tau^*$-gs-continuous function is independent from the $\text{gs}$*-continuous function.

Example 6.3.52: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$, and $\sigma = \{Y, \phi, \{a, b\}\}$. Let $f : X \to Y$ be an identity function. Then $f$ is gs*-continuous. However, it is not $\tau^*$-gs-continuous, since for the gs-open set $\{a\}$ in $Y$, the inverse image of $\{a\}$ is not $\tau^*$-g-open in $X$. 
Example 6.3.53: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a, b\}\} \) and \( \sigma = \{Y, \phi, \{a, b\}\} \). Let \( f : X \rightarrow Y \) be an identity function. Then \( f \) is \( \tau^* \)-gs-continuous. On the contrary, it is not gs*-continuous, since for the semi-open set \( \{a, c\} \) in \( Y \), the inverse image of \( \{a, c\} \) is not gs-open in \( X \).

Remark 6.3.54: Following examples show that \( \tau^* \)-gs-continuous function is independent from the gs-continuous function.

Example 6.3.55: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{c\}\} \) and \( \sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\} \). Let \( f : X \rightarrow Y \) be an identity function. Then \( f \) is gs-continuous. But, it is not \( \tau^* \)-gs-continuous, since for the gs-closed set \( \{a\} \) in \( Y \), the inverse image of \( \{a\} \) is not \( \tau^* \)-g-closed in \( X \).

Example 6.3.56: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}\} \) and \( \sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}\} \). Let \( f : X \rightarrow Y \) be an identity function. Then \( f \) is \( \tau^* \)-sg-continuous. On the other hand, it is not gs-continuous, since for the closed set \( \{a\} \) in \( Y \), the inverse image of \( \{a\} \) is not gs-closed in \( X \).

Remark 6.3.57: Following examples show that \( \tau^* \)-gs-continuous function is independent from the \( \tau^* \)-g-continuous function.

Example 6.3.58: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a, b\}\} \) and \( \sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\} \). Let \( f : X \rightarrow Y \) be defined by \( f(a) = c \), \( f(b) = a \) and \( f(c) = b \). Then \( f \) is \( \tau^* \)-g-continuous. However, it is not \( \tau^* \)-sg-continuous, since for the sg-closed set \( \{a\} \) in \( Y \), the inverse image of \( \{b\} \) is not \( \tau^* \)-g-closed in \( X \).

Example 6.3.59: Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}\} \) and \( \sigma = \{Y, \phi, \{b\}, \{a, b\}\} \). Let \( f : X \rightarrow Y \) be an identity function. Then \( f \) is \( \tau^* \)-sg-continuous. But it is not \( \tau^* \)-g-continuous, since for the g-closed set \( \{b, c\} \) in \( Y \), the inverse image of \( \{b, c\} \) is not \( \tau^* \)-g-closed in \( X \).
Remark 6.3.60: From the above discussion, we obtain the following Figure 6.2.

\[\begin{array}{cccc}
\alpha\text{-continuous} & \text{gsp-continuous} & \text{sp-continuous} \\
\alpha g\text{- continuous} & \tau^*\text{-gs-continuous} & g\alpha\text{- continuous} \\
\text{weakly sg-continuous} & \tau^*\text{-gs-continuous} & \text{weakly gs-continuous} \\
sg^*_\text{-continuous} & \text{pre-continuous} & gs^*_\text{-continuous} \\
\text{semi-continuous} & \text{pre-continuous} & \text{sg- continuous}
\end{array}\]

Figure 6.2 Solitude of $\tau^*$-gs-continuous function

6.4 CONCLUSION

In this chapter, $\tau^*$-semi generalized continuous function and $\tau^*$-generalized semi continuous function in topological spaces have been defined and their basic properties are investigated. These two continuous functions have been compared with several other functions and relationship among the functions has been obtained.