CHAPTER 4

SOLVING INTUITIONISTIC FUZZY LINEAR PROGRAMMING BY USING RATIO RANKING METHOD

4.1 INTRODUCTION

This chapter focuses on solving intuitionistic fuzzy linear programming by ratio ranking method. Fundamental theorem of IFLP, necessary and sufficient conditions of optimality are proved. Using the concept of value and ambiguity, IFLP problem is solved, where the data are triangular intuitionistic fuzzy number.

Intuitionistic fuzzy set is applied in many fields such as medical diagnosis, decision making and logic programming. Vagueness is usually expressed by linguistic terms such as fuzzy numbers or intuitionistic fuzzy numbers. In practice, it is observed that human expressions like perception, knowledge and behavior are represented by intuitionistic fuzzy sets rather than fuzzy sets.

4.2 LITERATURE REVIEW

Intuitionistic fuzzy optimization is introduced by Hassan Mishmast Nehi (2005). Arithmetic operations of triangular intuitionistic fuzzy number were introduced by Deng Feng Li (2010, 2011) and he proposed a ratio ranking method to rank the triangular intuitionistic fuzzy numbers. Mohammad Modaress & Sohail Sadi-Nezhad (2001) defined preference...
function in which fuzzy numbers are measured point by point and at each point, the most preferred number is identified. The average ranking index is introduced by Mijanur Rahaman Seikh (2012) to find out order relations between two generalized TIFNs. Dipti Dubey & Aparna Mehra (2011) solved the linear programming with triangular intuitionistic fuzzy numbers in which triangular intuitionistic fuzzy numbers are converted to crisp set and solved linearly. Deng Feng Li & Wang (2008) have approached multi attribute decision making under intuitionistic fuzzy environment.

In this chapter, IFLPP is solved by using the concept of Deng Feng Li (2011) ratio ranking method. In which IFLPP solved directly without converting to crisp LPP and by taking intuitionistic fuzzy numbers as variables.

4.3 RATIO RANKING METHOD

Definition 4.3.1

A triangular intuitionistic fuzzy number \( \tilde{a}^t = (\mu_{\tilde{a}}, \tilde{\mu}_{\tilde{a}}) \) is a special IFS on a real number set \( R \), whose membership function and non-membership function are defined as below.

\[
\varrho_{\tilde{a}}(x) = \begin{cases} 
(x-a_1)\mu_{\tilde{a}} / (a-a_1) & \text{if } a_1 \leq x < a \\
\mu_{\tilde{a}} & \text{if } x = a \\
(a_2-x)\mu_{\tilde{a}} / (a_2-a) & \text{if } a < x \leq a_2 \\
0 & \text{if } x \leq a_1 \text{ or } x > a_3 
\end{cases}
\]

and

\[
\nu_{\tilde{a}}(x) = \begin{cases} 
\left[ a - x + \mu_{\tilde{a}} \left( x-a_1 \right) \right] / (a-a_1) & \text{if } a_1 \leq x < a \\
\mu_{\tilde{a}} & \text{if } x = a \\
\left[ x - a + \mu_{\tilde{a}} \left( a-x \right) \right] / a_2 - a & \text{if } a < x \leq a_2 \\
1 & \text{if } x \leq a_1 \text{ or } x > a_3 
\end{cases}
\]

(4.1)
The values $\omega_\tilde{a}$ and $\mu_\tilde{a}$ represent the maximum degree of membership and the minimum degree of membership respectively and satisfy the condition

$$0 \leq \omega_\tilde{a} \leq 1, \quad 0 \leq \mu_\tilde{a} \leq 1 \text{ and } 0 \leq \omega_\tilde{a} + \mu_\tilde{a} \leq 1.$$  

Let $\pi_\tilde{a}(x) = 1 - \omega_\tilde{a}(x)$, $\mu_\tilde{a}(x)$ is called hesitancy of $x$ in $\tilde{a}$.

**Definition 4.3.2**

Let $\tilde{a}^i = \langle a_1, a_2 \rangle$ and $\tilde{b}^i = \langle b_1, b_2 \rangle$ be two TIFNs and $\lambda$ be a real number. The arithmetical operations are defined below.

\[
\begin{align*}
\tilde{a}^i + \tilde{b}^i &= \langle a_1 + b_1, a + b, a_2 + b_2 \rangle \\\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) \\
\tilde{a}^i - \tilde{b}^i &= \langle a_1 - b_1, a - b, a_2 - b_1 \rangle \\\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) \\
\tilde{a}^i \tilde{b}^i &= \begin{cases}
\langle (a_1 b_2, ab, a_2 b_1) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 > 0 \text{ and } b_2 > 0 \\
\langle (a_2 b_1, ab, a_1 b_2) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 > 0 \text{ and } b_2 < 0 \\
\langle (a_2 b_1, ab, a_1 b_2) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 < 0 \text{ and } b_2 > 0 \\
\langle (a_1 b_2, ab, a_2 b_1) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 < 0 \text{ and } b_2 < 0
\end{cases}
\end{align*}
\]

\[
\tilde{a}^i / \tilde{b}^i = \begin{cases}
\langle (a_2 b_1, ab, a_1 b_2) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 > 0 \text{ and } b_2 > 0 \\
\langle (a_2 b_1, ab, a_1 b_2) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 > 0 \text{ and } b_2 < 0 \\
\langle (a_2 b_1, ab, a_1 b_2) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 < 0 \text{ and } b_2 > 0 \\
\langle (a_2 b_1, ab, a_1 b_2) \rangle \\text{min}(\omega_\tilde{a}, \sigma_{\tilde{b}}) \\text{max}(\mu_\tilde{a}, \mu_{\tilde{b}}) 
& \text{if } a_2 < 0 \text{ and } b_2 < 0
\end{cases}
\]

\[
(\tilde{a}^i)^{-1} = \langle (1 / a_2, 1 / a, 1 / a_1) \rangle \\sigma_{\tilde{a}} / \omega_{\tilde{a}}.
\]

**Definition 4.3.3**

Let $\tilde{a}^i = \langle (0, 0, 1) \rangle$ is said to be zero vectors for TrIFN.

For example, let $\tilde{a}^i = \langle (0, 0, 1) \rangle$ and $\tilde{b}^i = \langle (1, 2, 3) \rangle$ and $\tilde{x}^i = \langle (1, 0) \rangle$. Then
Definition 4.3.4

Let $\bar{a}^I = \langle (0,1,0); 1,0 \rangle$ is said to be unit TIFN.

For example, let $\bar{a}^I = \langle (0,1,0); 1,0 \rangle$ and $\bar{b}^I = \langle (5,6,7); 0,4,0,5 \rangle$. Then

$$\bar{a}^I * \bar{b}^I = \langle (0,6,0); 0,4,0,5 \rangle$$

$$\bar{a}^I / \bar{b}^I = \langle (0,1/6,0); 0,4,0,5 \rangle$$

$$\bar{a}^I + \bar{b}^I = \langle (5,7,7); 0,4,0,5 \rangle$$

$$\bar{a}^I - \bar{b}^I = \langle -7, -5, -5; 0,4,0,5 \rangle$$

Definition 4.3.5

Let $\bar{a}^I = \langle (a_1, a_2); \omega_a, \mu_a \rangle$ be TrIFN. The value and ambiguity of $\bar{a}^I$ are defined below.

The value of the membership function of $\bar{a}^I$ is $v_{\mu}(\bar{a}^I) = (a_1 + 4a + a_2) \omega_a / 6$ and the value of non-membership of $\bar{a}^I$ is $v_{\nu}(\bar{a}^I) = (a_1 + 4a + a_2)(1 - \mu_a) / 6$. 
The ambiguity of the membership function of $\tilde{a}'$ is

$$A_\mu(\tilde{a}') = \frac{(a_2 - a_1)}{3} \omega_a$$

The ambiguity of the non-membership function of $\tilde{a}'$ is

$$A_\mu(\tilde{a}') = \frac{(a_2 - a_1)}{3} (1 - \mu_a) .$$

It is noted that $0 \leq \omega_a + \mu_a \leq 1$. Obviously $A_\mu(\tilde{a}') \leq A_\mu(\tilde{a}')$.

**Definition 4.3.6**

Let $\tilde{a}' = (a_1, a, a_2, \omega_a, \mu_a)$ be a TIFN.

The value index is defined as $V(\tilde{a}', \lambda) = V_\mu(\tilde{a}') - \lambda (V_\gamma(\tilde{a}') - V_\mu(\tilde{a}'))$ and the ambiguity index is defined as $A(\tilde{a}', \lambda) = A_\mu(\tilde{a}') - \lambda (A_\gamma(\tilde{a}') - A_\mu(\tilde{a}'))$

where $\lambda \in [0, 1]$ is a weight which represents the decision maker’s choice.

The decision maker prefers negative feeling or uncertainty when $\lambda \in [0, 1/2]$ and prefers positive feeling or certainty when $\lambda \in [1/2, 1]$. In this, $\lambda = 1/2$ shows that the decision maker is indifferent between positive feeling and negative feeling, so $\lambda = 1/2$ is taken for the entire study.

**Definition 4.3.7**

A ratio of the value index to the ambiguity index for a TIFN is defined below:

$$R(\tilde{a}', \lambda) = \frac{V(\tilde{a}', \lambda)}{1 + A(\tilde{a}', \lambda)}$$  \hspace{1cm} (4.3)
Theorem 4.3.8 (Fundamental theorem of IFLP)

If the feasible region of an IFLPP is a convex polyhedron, then there exists an intuitionistic fuzzy optimal to the IFLPP and at least one intuitionistic fuzzy basic feasible solution must be optimal.

Proof

Consider the IFLPP

\[
\text{Maximize } \bar{z}^l = \sum_{j=1}^{n} \bar{c}_j^l \bar{x}_j^l
\]

Subject to

\[
\sum_{j=1}^{n} \bar{a}_{ij}^l \bar{x}_j^l = \bar{b}_i^l, \quad \bar{x}_j^l \geq 0
\]

where

\[
\bar{c}_j^l = \left\{ c_{ij}, c_j, c_{2j}; \omega_{c_j}, \mu_{c_j} \right\}
\]

\[
\bar{b}_j^l = \left\{ b_{1j}, b_j, b_{2j}; \omega_{b_j}, \mu_{b_j} \right\}
\]

\[
\bar{a}_{ij}^l = \left\{ a_{ij}, a_i, a_{2j}; \omega_{a_j}, \mu_{a_j} \right\}
\]

The feasible region S of the IFLPP is given by

\[
s = \left\{ \bar{c}_j^l / \bar{a}_{ij}^l / \bar{x}_j^l = \bar{b}_i^l, \quad \bar{x}_j^l \geq 0 \right\}
\]

Since S is a convex set, it is non-empty closed and bounded and the objective function is continuous on S.

Therefore z attains its maximum on S and proves that the optimal solution exists.
To prove that at least one intuitionistic Fuzzy basic feasible solution must be intuitionistic fuzzy optimal.

Since \( S \) is a convex set, let 
\[
\{y_i, x_i, z_i; \omega_z, \mu_z\}, \ldots, \{y_k, x_k, z_k; \omega_z, \mu_z\}
\]
be finite number of extreme points.

Therefore, any \( \{y_i, x_i, z_i; \omega_z, \mu_z\} \in S \) can be expressed as a convex combination of the extreme points, say 
\[
\{y, x, z; \omega_z, \mu_z\} = \sum_{i=1}^{k} \alpha_i \{y_i, x_i, z_i; \omega_z, \mu_z\}; \alpha_i \geq 0, \sum \alpha_i = 1
\]

Let 
\[
\tilde{Z}_0^i = \max \{ \tilde{c} \{y_i, x_i, z_i; \omega_z, \mu_z\}, i = 1, 2, \ldots, k \}\]

Then for any \( \{y_j, x_j, z_j; \omega_z, \mu_z\} \in s \).

\[
\tilde{Z}^i = \tilde{c} \{y_j, x_j, z_j; \omega_z, \mu_z\} = \tilde{c} \sum_{i=1}^{k} \alpha_i \{y_i, x_i, z_i; \omega_z, \mu_z\}
\]

\[
= \sum \alpha_i \tilde{c} \{y_i, x_i, z_i; \omega_z, \mu_z\} \leq \sum \alpha_i z_0 = \tilde{Z}_0^i \cdot \alpha_i = 1
\]

Therefore \( \tilde{z}^i \leq \tilde{z}_0^i \). Thus the maximum value of \( z \) is obtained only at one extreme point of \( s \).
That is at least one extreme point of ‘S’ yields an intuitionistic fuzzy optimization solution.

**Theorem 4.3.9 (Intuitionistic Fuzzy Improved Basic Feasible solution)**

Let $\tilde{x}^l_B$ be an intuitionistic fuzzy basic feasible solution to the IFLPP:

\[
\text{Max } \tilde{z}^l = \tilde{c}^l \tilde{x}^l \quad \text{Subject to } \tilde{A}^l \tilde{x}^l = \tilde{b}^l \quad \tilde{x}^l \geq 0
\]

Let $\tilde{x}^l_B$ be another intuitionistic fuzzy basic feasible solution obtained by admitting a non-basic column vector $\tilde{a}^l_j$ on the basis for which the net evaluaton $\tilde{z}^l_j - \tilde{c}^l_j$ is negative. Then $\tilde{x}^l_B$ is an intuitionistic fuzzy improved basic feasible solution, that is $\tilde{c}^l_B \tilde{x}^l_B > \tilde{c}^l_B \tilde{x}^l_B$.

**Proof**

Let the IFLPP be

\[
\text{Max } \tilde{z}^l = \tilde{c}^l \tilde{x}^l \quad \text{where } \tilde{c}^l, \begin{bmatrix} \tilde{x}^l \\ \tilde{y}^l \end{bmatrix} \in \mathbb{R}^n
\]

Subject to $\tilde{A}^l \tilde{x}^l = \tilde{b}^l \quad \tilde{x}^l \geq 0$, $\begin{bmatrix} \tilde{b}^l \\ \tilde{y}^l \end{bmatrix} \in \mathbb{R}^n$, where $\tilde{A}^l$ is an $m \times n$ real matrix.

Let $\tilde{z}^l_j = \tilde{c}^l_B \tilde{x}^l_B$ and $\tilde{a}^l_j$ be the column vector introduced on the basis of $\tilde{z}^l_j - \tilde{c}^l_j < 0$.

Let $\tilde{b}^l_i$ be the vector removed from the basis, then

\[
\tilde{x}^l_B = \tilde{x}^l_B - \tilde{x}^l_B \frac{\tilde{y}^l_j}{\tilde{y}^l_j} \quad \text{and} \quad \tilde{x}^l_B = \tilde{x}^l_B \frac{\tilde{y}^l_i}{\tilde{y}^l_i}
\]
Hence, the new value of the objective function is

\[ \hat{z}^I = \sum_{i=1}^{m} \frac{\hat{c}^I_{Bi} \hat{x}^I_{Bi}}{y^I_{j}} \]

\[ = \sum_{i=1}^{m} \hat{c}^I_{Bi} \left( \frac{\hat{x}^I_{Bi} - \hat{x}^I_{Br} y^I_{j}}{y^I_{j}} \right) \]

\[ = \sum_{i=1}^{m} \hat{c}^I_{Bi} \left( \frac{\hat{x}^I_{Bi} - \hat{x}^I_{Br} y^I_{j}}{y^I_{j}} \right) + \hat{c}^I_{Br} \frac{\hat{x}^I_{Br}}{y^I_{j}} \]

\[ = \sum_{i=1}^{m} \hat{c}^I_{Bi} \left( \frac{\hat{x}^I_{Bi} - \hat{x}^I_{Br} y^I_{j}}{y^I_{j}} \right) + \hat{c}^I_{Br} \frac{\hat{x}^I_{Br}}{y^I_{j}} \quad (\because \hat{c}^I_{Br} = \hat{c}^I_{j}) \]

\[ = \tilde{z}^I - (\tilde{c}^I_{j} - \hat{c}^I_{j}) \frac{\hat{x}^I_{Br}}{y^I_{j}} \]

\[ > \tilde{z}^I_0 \quad (\because \frac{\hat{x}^I_{Br}}{y^I_{j}} > 0) \]

Hence, the new intuitionistic fuzzy basic feasible solution \( \hat{x}^I_{b} \), gives an improved value of the objective solution.

**Theorem 4.3.10 (Intuitionistic Fuzzy Unbounded Solution)**

Let there be a basic feasible solution to a given IFLPP. If for at least one \( j \), for which \( \tilde{y}^I_{ij} \leq 0 (i = 1, 2, ..., m) \) and \( \tilde{z}^I_{j} - \hat{c}^I_{j} \) is negative, then there does not exist any intuitionistic fuzzy optimum solution to this IFLPP.

**Proof**

Let an Intuitionistic fuzzy basic feasible solution to the IFLPP be \( \hat{x}^I_{b} \), so that \( \hat{B}^I \hat{x}^I_{b} = \hat{b}^I \) and \( \hat{x}^I_{b} \geq 0 \) with the value of the objective function
\[ z^I = z^I x^I_B \]

\[ = \sum_{i=1}^{m} c^I x^I_i \]

It can be written as

\[ \tilde{b}^I = \tilde{B}^I \tilde{x}^I + \xi \tilde{a}^I_j - \xi \tilde{a}^I_j \quad \text{where} \quad \tilde{a}^I_j \in \tilde{A}^I \quad \xi \quad \text{is a scalar}. \]

\[ = \sum_{i=1}^{m} \tilde{x}^I_i \tilde{b}^I_i + \xi \tilde{a}^I_j - \xi \sum_{j=1}^{n} \tilde{y}^I_j \tilde{b}^I_j \]

\[ = \sum_{i=1}^{m} (\tilde{x}^I_i - \xi \tilde{y}^I_j) \tilde{b}^I_i + \xi \tilde{a}^I_j \]

The value of the objective function is given by

\[ \tilde{z}^I = \sum_{i=1}^{m} (\tilde{x}^I_i - \xi \tilde{y}^I_j) \tilde{c}^I_i - \xi \tilde{c}^I_j \]

\[ = \sum_{i=1}^{m} \tilde{c}^I_i \tilde{x}^I_i - \xi \sum_{j=1}^{n} \tilde{c}^I_j \tilde{y}^I_j - \tilde{c}^I_j \]

\[ = \tilde{z}^I_0 - \xi (\tilde{z}^I_j - \tilde{c}^I_j). \]

\[ : \quad \tilde{z}^I \rightarrow +\infty \quad \text{as} \quad \xi \rightarrow +\infty \quad \text{(Since} \quad (\tilde{z}^I_j - \tilde{c}^I_j) < 0 \quad \text{and} \quad \xi > 0). \]

Hence there is no limit to the optimum value of \( \tilde{z}^I \) and hence there exists an intuitionistic fuzzy unbounded solution to the given IFLPP.

**Theorem 4.3.11 (Conditions of Intuitionistic Fuzzy Optimality)**

A sufficient condition for an intuitionistic fuzzy basic feasible solution to an intuitionistic fuzzy linear programming problem to be an optimum is that \( \tilde{z}^I_j - \tilde{c}^I_j \geq 0 \) for all \( j \) for which the column vector \( \tilde{a}^I_j \in \tilde{A} \) is not in the basis \( B \).
Proof

Let the IFLPP be

\[
\text{Max } \tilde{z}^t = \tilde{c}^t \tilde{x}^t ; \quad \text{where } \tilde{c}^t, [\tilde{x}^t]^t \in \mathbb{R}^n
\]

subject to \( \tilde{A}^t \tilde{x}^t = \tilde{b}^t \quad \tilde{x}^t \geq 0 \)

where \( \tilde{A}^t \) and \( \tilde{b}^t \) are \( m \times n \) and \( m \times 1 \) real matrices respectively.

Assume that there exists a basic feasible solution \( \tilde{x}_b^t \) to this IFLPP.

Let \( \tilde{c}_b^t \) be the cost vector corresponding to the basic variables. Then

\[
\tilde{B}^t \tilde{x}_b^t = \tilde{b}^t; \quad \tilde{x}_b^t \geq 0 \quad \text{and} \quad \tilde{z}^t = \tilde{c}_b^t \tilde{x}_b^t
\]

For all \( j \), for which \( \tilde{a}^t_j \notin \tilde{B}^t \), given that \( \tilde{z}^t_j - \tilde{c}^t_j \geq 0 \).

Let \( \tilde{a}_j^t = \tilde{b}_j^t \) for all such \( j \), for which \( \tilde{a}_j^t \)

Then, \( \tilde{y}_j^t = (\tilde{B}^t)^{-1} \tilde{b}^t = \tilde{c}_j^t \), unit vector and

\[
\tilde{z}^t_j - \tilde{c}^t_j = \tilde{c}_b^t \tilde{y}_j^t - \tilde{c}_j^t = \tilde{c}_b^t \tilde{e}_j^t - \tilde{c}_j^t
\]

\[
= \tilde{c}_b^t - \tilde{c}_j^t = 0
\]

Thus \( \tilde{z}^t_j - \tilde{c}^t_j \geq 0 \) for all \( j \) for which \( \tilde{a}_j^t \in A \).
Let $\tilde{x}^I$ be a feasible solution.

Then $\sum_{j=1}^n (\tilde{z}_j^I - \tilde{c}_j^I) \tilde{x}_j^I \geq 0$

$\implies \sum_{j=1}^n \tilde{z}_j^I \tilde{x}_j^I \geq \tilde{c}_j^I \tilde{x}_j^I$

$\implies \sum_{j=1}^n \tilde{c}_j^I \tilde{x}_j^I \geq \tilde{c}_j^I \tilde{x}_j^I$

$\implies \sum_{i=1}^m \tilde{c}_b^I \sum_{j=1}^n \tilde{y}_j^I \tilde{x}_j^I \geq \tilde{c}_j^I \tilde{x}_j^I$

for all $j$ for which $\tilde{a}_j^I \notin B$.

$\tilde{x}_b^I = (\tilde{B}^I)^{-1} \tilde{A}^I \tilde{x}^I = (\tilde{B}^I)^{-1} \tilde{A}^I \tilde{x}^I = \tilde{Y}^I \tilde{x}^I$

Since $\implies \tilde{x}_b^I = \sum_{j=1}^n \tilde{y}_j^I \tilde{x}_j^I$

Therefore, the above inequality can be written as

$\sum_{i=1}^m \tilde{c}_b^I \tilde{x}_b^I \geq \tilde{c}_j^I \tilde{x}_j^I$

$\implies \tilde{c}_b^I \tilde{x}_b^I \geq \tilde{c}_j^I \tilde{x}_j^I$

$\implies \tilde{z}_0^I \geq \tilde{z}_o^I$

where $\tilde{z}_o^I$ is the value of the objective function for the feasible solutions for which $\tilde{z}_j^I - \tilde{c}_j^I > 0$ for all $j$ such that $\tilde{a}_j^I \notin B$. 
Algorithm to Solve IFLPP by Ratio ranking method

Step 1

Take all the values as triangular intuitionistic fuzzy numbers. Apply simplex method procedure to obtain initial IFBFS.

Step 2

To find the most negative among $\tilde{Z}_j^l - \tilde{C}_j^l$, use ratio ranking method. Take the minimum value of $R(a, \lambda)$ which enters the basis $\tilde{y}_k$.

Step 3

Compute $\left\{ \frac{\tilde{x}_{bi}}{\tilde{y}_{ir}}, i = 1, 2, ..., m \right\}$ and choose minimum of them by using ratio ranking method. Then the vector $\tilde{y}_k$ would leave the basis. This element is called triangular intuitionistic fuzzy pivotal number.

Step 4

Convert pivotal element into unit triangular intuitionistic fuzzy number and all other elements in its column to zero triangular intuitionistic fuzzy number using the arithmetic operations given in equation (4.2).

Step 5

Repeat the procedure until an intuitionistic fuzzy optimum feasible solution is obtained.
Illustration 4.3.12

Max \( z = (6, 9, 12; 0.5, 0.4) \bar{x}_1^{I} + (2, 4, 8; 0.2, 0.7) \bar{x}_2^{I} \)

Subject to \((2.3,5,5;0.1,0.6)\bar{x}_1^{I} + (3.5,9; 0.2,0.7)\bar{x}_2^{I} \leq (4,8,11; 0.3,0.4)\)

\((6.9,14;0.2,0.5)\bar{x}_1^{I} +(7,8,11;0.4,0.5)\bar{x}_2^{I} \leq (9,12,15;0.7,0.2)\)

\(\bar{x}_1^{I}, \bar{x}_2^{I} \geq 0\)

By introducing intuitionistic fuzzy slack variables \(S_1\) and \(S_2\), the problem in the standard form becomes

Max \( z = (6, 9, 12; 0.5, 0.4) \bar{x}_1^{I} + (2, 4, 8; 0.2, 0.7) \bar{x}_2^{I} \)

\(+ (0,0,0;1,0) s_1 +(0,0,0;1,0) s_2\)

Subject to \((2.3,5,5;0.1,0.6)\bar{x}_1^{I} + (3.5,9; 0.2,0.7)\bar{x}_2^{I} -(0,1,0;1,0)S_1 = (4,8,11; 0.3,0.4)\)

\((6.9,14;0.2,0.5)\bar{x}_1^{I} +(7,8,11;0.4,0.5)\bar{x}_2^{I} -(0,1,0;1,0)S_2 = (9,12,15;0.7,0.2)\)

\(\bar{x}_1^{I}, \bar{x}_2^{I}, S_1 and S_2 \geq 0\)

The initial intuitionistic fuzzy basic feasible solution is given in Table 4.1.

### Table 4.1 Simplex table 1

<table>
<thead>
<tr>
<th>(c_B)</th>
<th>(y_B)</th>
<th>(x_B)</th>
<th>(Y_1)</th>
<th>(Y_2)</th>
<th>(Y_3)</th>
<th>(Y_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(S_1)</td>
<td>(4.8,11;0.3,0.4)</td>
<td>(2.3,5,5;0.1,0.6)</td>
<td>(3.5,9;0.2,0.7)</td>
<td>(0,1,0;1,0)</td>
<td>(0,0,0;1,0)</td>
</tr>
<tr>
<td>0</td>
<td>(S_2)</td>
<td>(9,12,15;0.7,0.2)</td>
<td>(6.9,14;0.2,0.5)</td>
<td>(7,8,11;0.4,0.5)</td>
<td>(0,0,0;1,0)</td>
<td>(0,1,0;1,0)</td>
</tr>
<tr>
<td>(\bar{Z}_j - \bar{C}_j)</td>
<td>-12.9,-6;0,0.4</td>
<td>-8.4,-2;0,0.9</td>
<td>(0,0,0,1,0,1,0)</td>
<td>(0,0,0,1,0)</td>
<td>(0,0,0,1,0,1,0)</td>
<td></td>
</tr>
</tbody>
</table>
Using ratio ranking method, the most negative among $\tilde{Z}_i^j - \tilde{C}_i^j$ is found.

$$\text{Min } \{R(\tilde{Z}_i^j - \tilde{C}_i^j) \} \text{ s.t. } Min (-1.35, -0.2) = -1.35$$

Corresponding $Y_i$ enters the basis. To find the leaving variable, the ratio ranking method is used.

$$\text{Min } = \left[ \frac{\tilde{x}_i^j}{\tilde{y}_i^j}, \frac{\tilde{x}_i^j}{\tilde{y}_i^j} \right] = \text{Min } \{0.8, 2.29, 5.5; 0.1, 0.6 \}, (1, 2.4, 5; 0.2, 0.7)$$

$$= \text{Min } R (0.46, 0.49) = 0.46.$$  

And the corresponding $S_1$ leaves the basis. $(2, 3.5, 5; 0.1, 0.6)$ is the pivotal element. The pivotal element is converted into unit element and the remaining element in the column into zero.

**Table 4.2 Simplex table**

<table>
<thead>
<tr>
<th>$c_B$</th>
<th>$y_B$</th>
<th>$x_B$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(6, 9.12; 0.5, 0.4)$</td>
<td>$y_1$</td>
<td>$(0.8, 2.3, 5.5; 0.1, 0.6)$</td>
<td>$(0.4, 1.2, 5; 0.1, 0.6)$</td>
<td>$(0.6, 2.6, 4.5; 0.1, 0.7)$</td>
<td>$(0.2, 0; 0.1, 0.6)$</td>
<td>$(0, 0, 0; 1, 0)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$S_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(-68, -6, 10.2; 0.1, 0.6)$</td>
<td>$(-29, 0, 11.6; 0.1, 0.6)$</td>
<td>$(-56, -15, 45; 0.1, 0.6)$</td>
<td>$(0, -8; 0.1, 0.6)$</td>
<td>$(0.1, 0; 1, 0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{Z}_i^j - \tilde{C}_i^j$</td>
<td>$(-9.6, 0.24; 0.1, 0.6)$</td>
<td>$(-4.4, 19, 45; 0.1, 0.7)$</td>
<td>$(0.18, 0; 0.1, 0.6)$</td>
<td>$(0, 0.0; 1, 0)$</td>
</tr>
</tbody>
</table>

In Table 4.2, all $\tilde{Z}_i^j - \tilde{C}_i^j \geq 0$ and $R(\tilde{Z}_i^j - \tilde{C}_i^j) \geq 0$, $i = 1, 2, 3, 4$ and so the optimal solution is obtained.

The intuitionistic fuzzy optimum feasible solution is

$$\text{Max } z = (4.8, 20.7, 66; 0.1, 0.6) \text{ at } x_1 = (0.8, 2.3, 5.5; 0.1, 0.6) \text{ and } x_2 = 0.$$
4.4 SUMMARY

Intuitionistic fuzzy optimum basic feasible solution is obtained by using ratio ranking method which is applied to multiattribute decision making problems in which the ratings of alternatives on attributes are expressed with TIFNs. This method is found to be very useful in the real world problems which are uncertain.