### Chapter-3: LINEAR QUADRATIC POWER SYSTEM STABILIZERS FOR SMIB

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3.1 Introduction

Several researchers are of the opinion that the stabilizers of the Linear Quadratic Regulator (LQR) power systems command higher control capabilities in contrast to the stabilizers of the customary lead-lag power systems. Besides the state variables of the power system being mostly not quantifiable, the non-availability of the measurement of the generator’s rotor angle in particular in a majority of power plants, renders it impractical for consideration during the design of LQR. This chapter proposes the application of modified Heffron Phillip’s model derived by referencing the step-up transformer’s secondary bus voltage in place of the infinite bus, and its application in the LQR controller’s design for the removal of the short comings of the design in vogue.

3.2. Conventional Power System Stabilizer

The excitation systems of high performance maintain steady state as well as the transient stability of the current synchronous generators in addition to facilitating efficient terminal voltage control. Damping through power system stabilizers that are the additional controllers of the excitation system constitute a cost-effective and viable solution to the issues of oscillatory instability. The principal objective of the design of the power system stabilizers (PSS) is to furnish supplementary damping torque at critical frequencies of oscillations without adversely affecting the synchronizing torque.
3.2.1. Control Signals

The rotor velocity deviation is the largely employed control signal as an input to the PSS. Accelerating power, bus frequency and electrical power are the other signals employed. There is a local availability of all the considered control signals such as electrical power, rotor speed and frequency.

3.2.2. Structure and tuning of PSS

Some major components of the PSS are washout circuit, dynamic compensator, limiter and torsional filter. The standard PSS is shown in block diagram in figure 3.1. PSS is provided to maximise the transfer of power in the network. Even during the instances of the system’s exposure to major disturbances, the PSS must ensure proper functioning.

![Figure 3-1: Generic Block Diagram of PSS](image)

**Washout Circuit**

The provision of washout circuit removes the steady-state bias present in the PSS output. The PSS should respond only to the transient differences and not to the dc offsets present in the input signal. The washout circuit functions as a high pass filter and allows all frequencies of interest. The time constant $T_w$ can be selected in a range of 1-2, in case of the interest being only in the local
modes. Conversely, the choice of $T_w$ in the range of 10-20 is required to damp the inter-area modes also.

**Dynamic Compensator**

The following transfer function with two lead-lag stages is normally used as a dynamic compensator

$$T(s) = \frac{K_s(1 + sT_1)(1 + sT_3)}{(1 + sT_2)(1 + sT_4)}$$

The gain of the PSS, $K_s$ and the time constants $T_1$ to $T_4$ are selected for providing the input signal a phase lead in the frequency range between 0.1 and 3.0 Hz of the input signal. One lead-lag stage is sufficient for static exciters. The following transfer function in general can be considered for choosing the dynamic compensator.

The effects of the torsional filter and the washout circuit can be ignored for the purposes of designing.

Two design criteria exist here [10].

1. In order to accomplish damping torque, the constants of time in the equation $T_1$ to $T_4$ need to be preferred from the phase compensation requirements.

2. Selection of PSS gain provides the required damping to all critical modes in diverse conditions of operation. The most critical aspect to be considered is the tuning of PSS at specific condition of operation (conditions of full load either with weak or strong AC system). Even if the PSS is tuned to provide the
needed damping in the specified condition, its performance may not be optimal under different conditions.

In ideal situations, in order to give pure damping torque at all frequencies, the PSS should maintain a right balancing of its phase characteristics with those of the transfer function of the power system called GEP(s) at all frequencies. For single machine infinite bus system the GEP(s) transfer function can be obtained from the Heffron Phillip’s model as shown in Figure 3-2.

The following criteria are preferred in the design of PSS phase compensation [10], [12] as this transfer function cannot be compensated in practice for diverse range of operations.

(a) The compensated phase lag (phase of P(s)= GEP(s) PSS(s) must pass through $90^0$ at a frequency around 3.5 Hz (for frequency input signal which can be reduced to 2.0 Hz).

(b) The compensated phase lag at local mode frequency must be under 450, in all possibility near 20$^0$.

(c) The compensator’s gain in conditions of high frequencies (this is proportional to $T_1T_3/T_2T_4$) must be decreased.
The first criterion is vital to prevent the intra-plant frequency destabilization. Lagging of the compensated phase at inter-area modes is also preferable to enable the PSS furnish some synchronizing torque at the considered frequencies. The washout circuit’s constant of time can influence the compensated phase lag. The requirement of the third criterion is basically for reduction in noise amplification through PSS. The following sections in chapter 2 derive a novel design approach for the linear quadratic regulator power system stabilizer adopting the modified Heffron Phillip’s model.

3.3. Proposed Linear Quadratic Power System Stabilizer

The linearized state equation of the modified Heffron Phillip’s model of SMIB is given by

\[
\dot{X} = AX + BV_{PSS} + B_i \left[ \Delta \theta_s \right] \left[ \Delta V_s \right]
\]  

(3.1)

where
The state variables are

\[
X = [\Delta \delta_S; \Delta S_m; \Delta E'_q; \Delta E_{fL}] = [\delta_S - \delta_{S0}, S_m - S_{m0}, E'_q - E'_{q0}, E_{fL} - E_{fL0}]
\]

\(\delta_S\) and \(E'_q\) can be obtained by

\[
\delta_S = \tan^{-1}\left(\frac{P_S (X_t + X_q) - Q_S (R_a + R_t)}{P_S (R_a + R_t) + Q_S (X_t + X_q) + V_S^2}\right)
\]

(3.3)

Where \(P_S = V_S I_a \cos \theta_p\) and \(Q_S = V_S I_a \sin \theta_p\)

\[
E'_q = \frac{(X_t + X_d)}{X_t} \sqrt{V_t^2 - \left(\frac{X_q}{(X_t + X_q) V_S \sin \delta_S}\right)^2 - \frac{X_d}{X_t} V_S \cos \delta_S}
\]

(3.4)

Power system control applications applying equations (3.3) and (3.4) have been patented [52]. \(V_{PSS}\) constitutes the input design for damping the oscillations of the system. A linear quadratic regulator is designed taking \(B_1\) as the term for disturbance to minimize the objective function

\[
J = \lim_{\tau \to \infty} \frac{1}{\tau} E \left[ \int_0^\tau \left( X^T Q X + V_{PSS}^T R V_{PSS} \right) dt \right]
\]

(3.5)

The controller of the state feedback is achieved as

\[
V_{PSS} = -K_{lqr} X
\]

(3.6)

where
and the matrix $P$ represents the solution to the Algebraic Riccati Equation (ARE).

$$PA + A^T P - PBR^I {B^T}P + Q = 0$$ \hfill (3.8)

There is no requirement of any external system information for the considered controller design as the local measurements can obtain all the needed state variables. The suggested PSS attempts to control the computed rotor angle pertaining to the local bus instead of the measured angle $\delta$ pertaining to the remote bus.

### 3.4. Simulation Results

Fig 3.3 illustrates the wide ranging simulations held on SMIB test system in order to evaluate the performance of the proposed LQR stabilizer over vastly varying conditions of operation. The results are measured against those of a conventional lead-lag PSS (CPSS). Table 3.1 depicts the considered conditions of operation under weak, strong and nominal scenarios of the system for the purpose of evaluation. Appendix C furnishes the data of both the system and the PSS for conventional design as well as for the proposed method. The reactance of the transformer $X_t$ is 0.1p.u, and the reactance of the line $X_L$ is 0.3p.u. The overall impedance between the infinite bus and the generator bus is denoted by $X_e$. $Q = \text{Diag}([1.0, 1.0, 0.000001, 0.000001])$ and $R=0.8$ are selected for the proposed LQR design. For the design purpose of the proposed LQR, $Q = \text{Diag}([1.0, 1.0, 0.000001, 0.000001])$ and $R=0.8$ are chosen.
Table 3.1 Operating Conditions Tested for SMIB

<table>
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<th>$X_e$</th>
<th>$P_t$</th>
<th>$Q_t$</th>
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<td>Nominal : 0.4-0.6</td>
<td>0.8&amp;1.0</td>
<td>0.2to0.2</td>
</tr>
<tr>
<td>Strong : 0.2-0.3</td>
<td>0.8&amp;1.0</td>
<td>0.41to0.37</td>
</tr>
<tr>
<td>Weak : 0.6-0.8</td>
<td>0.8&amp;1.0</td>
<td>0.5to0.2</td>
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Figure 3-4 illustrates the response of the system in terms of variation in slip $S_m$ following a 5% step change at $V_{ref}$ input of the generator during the nominal conditions of operation ($S = P + jQ = 1 + j0.2 \text{pu}$, $X_e = 0.4 \text{pu}$). The system becomes volatile in this condition of operation in the absence of a PSS. The conventional as well as the proposed LQR are capable of effectively damping the oscillations of the system.
Figure 3-4: $S_m$ for 5% step change in $V_{ref}$. $S = 1 + j0.2\, p.u., X_e = 0.4\, p.u.$

Figure 3-5 depicts the responses of the system during similar conditions of operation as above, for a 3\(\phi\) fault of 55ms duration at the transformer bus cleared by tripping one of the parallel lines. After clearance of the defect the system becomes feeble with $X_e = 0.7\, p.u.$ The system maintains stability and stays well-damped both with the LQR and with the CPSS. Figure 3.6 explains the slip speed $S_m$ response of a strong system ($S = P + jQ = 1 + j0.15\, p.u., X_e = 0.3\, p.u.$) for a 5% step change in $V_{ref}$. The response of the system with the considered LQR shows lesser overshoot and undershoot in comparison with those obtained with CPSS. Figure 3.7 depicts the slip speed $S_m$ response of a strong system and the condition of leading power factor.
\((S = P + jQ = 1 + j0.05pu., X_e = 0.3pu.)\). The response of the system for a step change of 5\% in generator’s input \(T_m\) is furnished. The response of the system with the proposed LQR is well-damped as against the response acquired with CPSS.

Figure 3-8 depicts a feeble system’s slip speed \(S_m\) response \((S = P + jQ = 1 + j0.5pu., X_e = 0.8pu.)\) for a step change of 5\% in generator’s input \(V_{ref}\). Even here the response of the system with the proposed LQR is well-damped in contrast to the response obtained with CPSS.

![Graph](image)

Figure 3-5: Response, 3-Phase fault cleared by line tripping, Nominal system
Figure 3-6: $S_m$ for 5% step change in $V_{ref}$, $s = 1 + j0.15\,p.u.$, $X_e = 0.3\,p.u.$

Figure 3-7: $S_m$ for 5% step change in $T_m$, $s = 1 + j0.05\,p.u.$, $X_e = 0.3\,p.u.$
Figure 3-8: $S_m$ for 5% step change in $V_{ref}$, $S = 1 + j0.5 \text{ p.u.}$, $X_e = 0.8 \text{ p.u.}$

Figure 3-9: $S_m$ for 5% step change in $V_{ref}$, $S = 0.8 + j0.2 \text{ p.u.}$, $X_e = 0.4 \text{ p.u.}$
Figure 3.10: $S_m$ for 5% step change in $V_{\text{ref}}$, $S = 0.8 - j0.2 \, \text{p.u.}, \, X_e = 0.4 \, \text{p.u.}$

Figure 3.10 shows the slip speed $S_m$ response of a nominal system ($S = P + jQ = 0.8 - j0.2 \, \text{p.u.}, \, X_e = 0.4 \, \text{p.u.}$) for a 5% step change in $V_{\text{ref}}$ input of the generator. Figure 3.11 shows the rotor angle $\delta$ response of a nominal system ($S = P + jQ = 0.8 - j0.2 \, \text{p.u.}, \, X_e = 0.4 \, \text{p.u.}$) for a 5% step change in $V_{\text{ref}}$ input of the generator.
Figure 3.11: Rotor angle $\delta$ for 5% step change in $V_{\text{ref}}$, $S = 0.8 - j0.2 \text{p.u.}$, $X_e = 0.4 \text{p.u.}$

Figure 3.12 shows the slip speed $S_m$ response of a strong system \((S = P + jQ = 1 - j0.55 \text{p.u.}, \ X_e = 0.2 \text{p.u.})\) for a 5% step change in $V_{\text{ref}}$ input of the generator.

Figure 3.13 depicts the slip speed $S_m$ response of a strong system \((S = P + jQ = 0.8 + j0.2 \text{p.u.}, \ X_e = 0.2 \text{p.u.})\). The response of the system for a step change of 10% in the generator's input $T_m$, is provided.
Figure 3-12: $S_m$ for 5% step change in $V_{ref}$, $S = 1 - j0.55 \text{ p.u.}, X_e = 0.2\text{ p.u.}$

Figure 3-13: $S_m$ for 10% step change in $T_m$, $s = 0.8 + j0.2 \text{ p.u.}, X_e = 0.2\text{ p.u.}$

Figure 3-14: Response, 3-Phase fault cleared by line tripping, Nominal System
Figure 3.15 Response of Rotor angle $\delta$ with 3-Phase fault cleared by line tripping, Nominal System.

Figures 3.14 and 3.15 illustrate the responses of the system in the similar conditions of operation as above, cleared by tripping one of the parallel lines for a $3\phi$ fault of 55ms time frame at the transformer bus. The system turns nominal with $X_e = 0.5\, p.u.$ when the fault is cleared. Figure 3-16 illustrates the slip speed $S_m$ response of a strong system.
\( S = P + jQ = 0.8 - j0.2 \text{ pu}, \ X_e = 0.2 \text{ pu} \) for a step change of 5% in \( V_{\text{ref}} \) the generator’s input. Figure 3-17 shows the Rotor angle \( \delta \) response of a strong system \( (S = P + jQ = 0.8 - j0.2 \text{ pu}, \ X_e = 0.2 \text{ pu}) \) for a 5% step change in \( V_{\text{ref}} \) input of the generator.

Figure 3-16: \( S_m \) for 5% step change in \( V_{\text{ref}} \), \( S = 0.8 - j0.2 \text{ pu}, \ X_e = 0.2 \text{ pu} \).

![Figure 3-17: Rotor angle \( \delta \) for 5% step change in \( V_{\text{ref}} \), \( S = 0.8 - j0.2 \text{ pu}, \ X_e = 0.2 \text{ pu} \)](image)

The applied simulation results clearly demonstrate the superior performance of the proposed LQR to that of the conventional PSS under conditions of feeble, strong and leading power factors and almost identical to the performance of the CPSS in nominal conditions of the system. The design
and implementation of the controller needs local information and hence it can be extended directly to multi-machine systems. Further details are explained in the next chapter.

3.5. Conclusions

A new perspective for the PSS design based on full state feedback linear quadratic regulator is put forward in this chapter. A modified Heffron Phillip’s model is evolved in this chapter, by considering the step-up transformer’s secondary voltage as reference in place of the infinite bus. The proposed LQRPSS performs better under conditions of feeble, strong and leading power factors while the conventional PSS fails in similar conditions and performs almost identically as the conventional PSS under nominal conditions as observed. As the local measurements can realize the proposed controller, an easy extension of this approach to multi-machine systems is possible.