Chapter 4

Characterization of source parameters and some empirical relations between them for Kutch region.

‘An earthquake of consequence is never an isolated event’.

-Charles Richter.

4.1 Introduction

Seismic hazard analysis probabilistic and deterministic both requires an assessment of the future earthquake potential in a given region. Specifically, it is often necessary to estimate the size of the largest earthquakes that might be generated by a particular fault or earthquake source. Thus, the future earthquake potential of a fault commonly is evaluated from estimates of fault rupture parameters which are directly related to earthquake magnitude. It has been known for some time that earthquake magnitude may be correlated with rupture parameters such as length and displacement (e.g., Tocher, 1958; Iida, 1959; Chinnery, 1969). Accordingly, paleoseismic and geologic studies of active faults focus on estimating these source characteristics. For example, data from geomorphic and geologic investigation of faults may be used to assess the timing of past earthquakes, the amount of displacement per event and the segmentation of the fault zone (e.g., Schwartz and Coppersmith, 1986; Schwartz, 1988; Coppersmith, 1991). To translate these source characteristics into estimation of earthquake size, relationships between rupture parameters and the measures of earthquake size, typically, in terms of moment and magnitude, are required. Thus the knowledge of source parameters and relations amongst one another for a given region together provide a strong base to assess the seismic hazards for a given region.

As we know, earthquakes are caused when the brittle part of the Earth’s crust is subjected to stress that exceeds its breaking strength. Sudden rupture mostly occurs along pre-existing faults or sometimes along newly formed faults. For very large earthquakes, the length of the ruptured zone may be as much as thousand kilometers and the slip along the
fault may reach several meters. Source parameters of small and large earthquakes are important for understanding the differences and similarities between dynamic ruptures of small and large earthquakes and clarifying the scaling relations. However, it is often difficult to accurately determine source parameters of small earthquakes because the relatively high-frequency seismic waves excited by small earthquakes are easily scattered and attenuated along the path.

Numerous published literatures show empirical relationships between magnitude and various faults rupture parameters. Typically, magnitude is related to surface rupture length as a function of slip type. Additional relationships that have been investigated include displacement versus rupture length, magnitude versus maximum surface displacement, magnitude versus total fault length, and magnitude versus surface displacement times surface rupture length (Tocher, 1958; Iida, 1959; Albee and Smith, 1966; Chinnery, 1969; Ohnaka, 1978; Slemmons, 1977, 1982; Acharya, 1979; Bonilla and Buchanon, 1970; Bonilla et al., 1984; Slemmons et al., 1989). Other studies relate magnitude and seismic moment to rupture radius and rupture area, dimensions of the aftershock zone or earthquake source time functions (Utsu and Seki, 1954; Utsu, 1969; Kanamori and Anderson, 1975; Wyss, 1979; Singh et al., 1980; Purcaru and Berckhemer, 1982; Scholz, 1982; Wesnousky, 1986 and Darragh and Bolt, 1987).

The purpose of this study is to present new and revised empirical relationships between various source parameters to describe the empirical data base used to develop these relationships and to draw first-order conclusions regarding the trends in the relationships for Kutch region. Specifically, this study refines the data sets and extends previous studies by including data from recent earthquakes and from new investigations of older earthquakes for Kutch region. The new data provide a much larger and more comprehensive data base than was previously available. In the subsequent topics, the observational data base is described and then the statistical relationships developed among different source parameters particularly earthquake size and rupture parameters and then used to evaluate the relationships in terms of their statistical significance, relative stability and overall usefulness. I was more interesting in static stress drop and radiated energy,
explicitly as these two parameters are good representatives of dynamic behavior of rupture processes and they are less discussed rather rarely discussed for Kutch region. I focused on static stress drops and radiated seismic energies for earthquakes of moderate kind and investigated how these two parameters are related to the values for smaller and larger earthquakes.

In this study, the independent source parameters discussed most often are seismic moment-\( M_0 \), moment magnitude-\( M_w \), stress drop-\( \Delta \sigma \), apparent stress-\( \sigma_{app} \), rupture radius-\( R \), Rupture area-\( A \), radiated seismic energy-\( E_R \) and scaled energy-\( \hat{\varepsilon} \). Assumptions of similarity and scaling and spectral source theories are used to interrelate some of these parameters.

4.2 Data and Methodology

4.2.1 Data

For the present study, total 195 aftershocks of magnitude more than 4.0 (only some events are of \( 3.5 < M_w < 4.0 \) are included) for aftershock sequence of Jan 26, 2001 Bhuj earthquake of Kutch region have been used. The data include shallow-focus (hypo central depth less than 40 km), continental intraplate earthquakes from Jan 26, 2001 to Dec 31, 2010 recorded by different seismological observatories of India Meteorological Department. The epicentral locations of all earthquakes under the study are shown in Figure 4.1.
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Figure 4.1 Epicentral locations of seismic events analyzed for the present study.

For each earthquake in the data used, earthquake source parameters were estimated including seismic moment, moment magnitude, rupture radius and rupture area, stress drop, apparent stress and radiated energy, which describe the source dynamics in the best possible way. The most important objective of this study is to identify the most accurate value for each parameter or the average value where the accuracy of individual values could not be determined. Data are categorized by type of measurement and the most accurate value is selected for further analysis.

To develop empirical relationships among various source parameters for Kutch region, some of the aftershocks from the larger database, those are evaluated but excluded from further study because of poor-quality of data.
4.2.2 Methodology

There are two approaches to describe source parameters. The simplified approach describes the seismic source by limited number of parameters such as the origin time and location i.e. initial rupture, magnitude, intensity or acceleration of measured ground shaking and sometimes fault plane solution. These are easily obtainable parameters and provide quick information to the public and concern authorities. They are fundamental for other research but are not sufficient to describe the true nature and geometry or the energy release by the source.

The second approach is the detailed analysis of a given event i.e., analyzing near and far-field waveforms and spectra of various kinds of seismic waves. The spectra should be in a broad frequency range. It provides detailed important information on energy distribution in the frequency domain about the source of the seismic event. The second approach is adopted, i.e. the spectral technique for source parameter estimation, where S-wave displacement spectrum is considered. The uncorrected displacement spectra recorded at a given station were first corrected to base line by instrument response. The onset of S-wave arrival time was estimated from these spectra. Then, the corrected spectra were band pass filtered between 0.1 and 20.0 Hz. From the filtered waveform, a time window of 10 second duration was selected for all the analysis undertaken in this study. Subsequently, Fast Fourier Transform (FFT) is applied to compute displacement spectrum, approximated by two straight lines. The first line represents a constant amplitude level $\Omega_0$ at lower frequencies and the second line is the approximation of decay spectrum at higher frequencies. The intersection of these two lines is the point at which stress releases and is known as corner frequency $f_c$. The observed waveforms are corrected for instrument response, geometrical spreading and surface effects. These corrections are made for the corresponding wave parameters of each station. An example of the three-component seismogram recorded at Bhuj for moderate earthquake of magnitude $M_w \approx 5.5$ is shown in Figure 4.2.
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Figure 4.2 Three-component seismogram recorded at Bhuj for the event Jan 28, 2001($M_w$ 5.5).

The Brune’s (1970) source model is followed for the estimation and computed S-wave displacement spectra applying required corrections. While deriving spectral analysis using Brune (1970) model, a single trace mode is selected considering the geometrical spreading and inelastic attenuation. Figure 4.3 shows displacement spectra for some of seismic events recorded at Bhuj station. As it is difficult to display spectra of all 195 aftershocks, it is restricted to present only thirty displacement spectra of different aftershocks in this thesis.
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(Figure 4.3 continued)
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(Figure 4.3 continued)
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(Figure 4.3 continued)
Theoretical displacement spectrum \( d(f) \) (Brune, 1970) can be given,

\[
d (f) = \frac{G (r, h) \ast D (f) \ast M_0 \ast R_{\phi}}{\left( 1 + \left( \frac{f}{f_c} \right) \ast 2 \right) \ast 4 \pi \rho V_{P,s}^3}
\]  
(4.1)
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Where,

\[ G(r,h) = \text{Geometrical Spreading} \]
\[ D(f) = \text{Inelastic attenuation.} \]
\[ M_0 = \text{Static Seismic Moment (N.m)} \]
\[ R_{\text{psf}} = \text{Radiation pattern function.} \]

= 0.55 (average value, 0.63 for strike slip fault, and 0.51 for normal fault)

\[ f = \text{Frequency (Hz)} \]
\[ f_c = \text{Corner frequency (Hz)} \]
\[ \rho = \text{Density at the source (kg/m}^3\text{)} \]
\[ = 2700 \text{ kg/m}^3 \]
\[ V_{\text{p,s}} = \text{Seismic-wave velocity of the corresponding phase (km/sec)} \]
\[ = 6.2 \text{ km/sec for p-wave} \]
\[ = 3.6 \text{ km/sec for s-wave} \]

In the equation (4.1), the term \( G(r,h) \) represents geometrical spreading, for S-wave, it has been considered to depend on distance and depth. For distance \( r \) and depth \( h \), geometrical spreading \( G(r,h) = 1/r \) (for \( r < 100 \text{ km} \)) and \( G(r,h) = 1/(100 \times r)^{1/2} \) (Herrmann and Kijko, 1983). In the present study, all earthquakes are of shallow depth i.e. depth less than 40 km, thus for this study \( G(r,h) = 1/r \) is considered. According to the Brune’s (1970) source model, \( D(f) \) is the diminution function due to inelastic attenuation and can be given as follow,

\[ D(f) = \exp(-\pi kf) \times \exp((-\pi ft) / Q(f)) \]  

(4.2)

Where, \( k \) accounts for kappa, the near-surface high frequency attenuation (Singh et al., 1982) with the constant kappa having a value of 0.025 for Kutch region, \( t \) is taken as travel time and \( Q(f) \) is frequency dependent inelastic attenuation. Furthermore, taking the attenuation factor into account and following the Andrews (1986), the amplitude spectrum in frequency domain is a product of a source and site spectral function and a propagation path term. Thus the amplitude spectrum of the \( i^{th} \) event recorded at \( j^{th} \) station at a distance of \( r \) and for frequency \( f \) can be written as (Lermo and Chavez-Garcia 1993; Nath et al. 2002a, 2002b),
\[ A(r_{ij}, f) = SO_i(f) \times SI_j(f) \times P(r_{ij}, f) \] (4.3)

Where, \( SO_i \) and \( SI_j \) are source and site spectral function respectively and \( P(r_{ij}, f) \) is the propagation path term which can be expressed as,

\[ P(r_{ij}, f) = G(r_{ij}, h) \times \exp((-\pi fR)/Q(f) \times V_s) \] (4.4)

The term \( G(r_{ij}, h) \) is geometrical spreading and similar to \( G(r, h) \) of equation (4.1) as discussed earlier and \( V_s \) is assumed to be 3.6 km/sec based on a velocity model of the Kutch region (Mandal and Pujol, 2006; Mandal, 2007). Here is considered a known \( Q(f) \) relation based on the result of the coda-\( Q_c \) study of the Kutch region by Mandal et al. (2004b) and expressed by Equation (4.5),

\[ Q(f) = 102 f^{0.98} \] (4.5)

In the coda \( Q_c \) study, Mandal et al. (2004b) have used Aki and Chouet’s (1975) technique, where the coda waves were assumed to be composed of the backscattered S-waves. Thus, the \( Q_c \) estimates obtained using Aki and Chouet’s (1975) technique can be considered to be very close to the estimates of \( Q(f) \) (Mandal et al., 2005). Mandal et al. (2008) also experimented the effect of \( Q \) on the site response estimates considering three available frequency dependent relations, i.e., \( Q = 102 f^{0.98} \) (from local coda waves by Mandal et al., 2004b), \( Q = 508 f^{0.48} \) (from regional \( Lg \) waves by Singh et al., 2004) and \( Q = 790 f^{0.22} \) (obtained by Bodin and Horton, 2004 from the study of ground-motion prediction) and found no significant difference in the shape and amplitude of the site response estimates using the aforementioned three \( Q \) values.

In a homogeneous half-space, \( M_0 \) can be determined from the spectra of seismic waves observed at the Earth’s surface by using the relationship:

\[ M_0 = \frac{4\pi \rho \times R \times V_s^3 \times \Omega_0}{R_{\varphi \phi}} \] (Brune, 1970) (4.6)

Where,

\[ R = \text{Epicentral Distance (km)} \]
\[ \Omega_0 = \text{Long period amplitude level. (m-sec)} \]
From the spectral parameters, other parameters like moment magnitude, source radius, stress drop, apparent stress and radiated energy can be calculated as follows:

Moment Magnitude,
\[
M_w = \frac{2}{3} \left( \log M_o - 9.1 \right) \quad \text{(Hanks and Kanamori, 1979)} \quad (4.7)
\]

Rupture radius and area,
\[
r = \frac{2.34 V}{2\pi f_c} \quad \text{(Ben-Menahem, 1965)} \quad (4.8)
\]

And \( A = \pi r^2 \)

Where, \( r \) = Rupture radius (km)
\( f_c \) = Corner frequency (Hz)
\( A \) = Rupture Area (sq km)

Stress Drop,
\[
\Delta \sigma = \frac{7M_o}{16r^3} \quad \text{(Kellis-Borok, 1959)} \quad (4.9)
\]

And apparent stress drop,
\[
\sigma_{app} = \mu \frac{E_s}{M_o} \quad \text{(Wyss and Brunei, 1968)} \quad (4.10)
\]

Where, \( \mu \) = Shear Modulus
\( = 3 \times 10^{10} \text{ N/m}^2 \)
\( E_s \) = Radiated Seismic Energy

Radiated Seismic Energy,
\[
E_R = \frac{21.1 \rho R\Omega_0 f_c^3 M_o}{\mu} \quad \text{(Ritcher, 1958)} \quad (4.11)
\]

Scaled Energy is defined as,
\[
\tilde{E} = \frac{E_s}{M_o} \quad (4.12)
\]
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4.3 Results

On the basis of methodology described above displacement spectra of 195 aftershocks are analyzed and estimated source parameters. The result can be summarized in the Table 4.1.

### Table 4.1 Estimated Source Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower bound value</th>
<th>Upper bound value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment Magnitude-$M_w$</td>
<td>3.5</td>
<td>5.7</td>
</tr>
<tr>
<td>Seismic Moment-$M_0$(N.m)</td>
<td>$2.0 \times 10^{14}$</td>
<td>$6.3 \times 10^{17}$</td>
</tr>
<tr>
<td>Corner Frequency-$f_c$(Hz)</td>
<td>0.624</td>
<td>8.187</td>
</tr>
<tr>
<td>Stress Drop-$\Delta \sigma$(bars)</td>
<td>68.4</td>
<td>299.8</td>
</tr>
<tr>
<td>Rupture Radius-$r$ (km)</td>
<td>0.168</td>
<td>2.100</td>
</tr>
<tr>
<td>Rupture Area-$A$(sq km)</td>
<td>0.088</td>
<td>13.847</td>
</tr>
<tr>
<td>Radiated Seismic Energy-$E_R$ (J)</td>
<td>$2.0 \times 10^{10}$</td>
<td>$1.8 \times 10^{13}$</td>
</tr>
</tbody>
</table>

The estimated value of different source parameters like seismic moment, corner frequency, rupture radius and rupture area, stress drop and radiated seismic energy are ranging from $2.0 \times 10^{14}$ N.m to $6.3 \times 10^{17}$ N.m, 0.624Hz to 8.187 Hz, 0.168 km to 2.100km, 0.088 sq km to 13.847sq km, 68.4 bars to 299.8 bars and $2.0 \times 10^{10}$J to $1.8 \times 10^{13}$ J respectively. Errors associated in estimation of source parameters can be represented by standard deviation and mean values. The mean and standard deviation values in estimation of seismic moment can be given below (Archuleta et al., 1982),

Mean of seismic moment,

$$\bar{M}_0 = \text{anti log} \left[ \frac{1}{NS} \sum_{i=1}^{NS} \log M_{0i} \right]$$  \hspace{1cm} (4.13)
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Standard deviation of seismic moment,

\[ sd(\log \bar{M}_o) = \left[ \frac{1}{NS - 1} \sum \left( \log M_{oi} - \log \bar{M}_{ob} \right) \right]^{1/2} \] (4.14)

The same procedure as mentioned above has been followed to calculate mean and standard deviation in corner frequency, rupture radius and rupture area. Stress drop values are highly deviate from the mean hence percentage error has been derived for stress drop which can be defined as,

\[ \% \text{error of } \Delta \sigma = \frac{\text{Mean } \Delta \sigma - \text{actual } \Delta \sigma}{\text{actual } \Delta \sigma} \times 100\% \] (4.15)

Mean values of seismic moment, corner frequency, rupture radius, rupture area and stress drop are 2.3x10^{16} N.m, 2.803 Hz, 0.617 km, 1.605 sq km and 197.5 bars respectively. While standard deviation in estimation of seismic moment, corner frequency and rupture radius and rupture area are 0.664 N.m, 1.299 Hz and 0.361 km and 2.204 sq km respectively. An average % error is calculated in the estimation of stress drop as stress drop values are discrete and they found to be 0.2%.

In the process, empirical relation between the estimated source parameters are further analyzed which are described individually in following subsequence. The empirical relations between source parameters are grouped in three different categories:

(a) Relation between corner frequency with other source parameters.

(b) Relation between moment magnitude, seismic moment and rupture parameters.

(c) Relation between Radiated seismic energy with seismic moment and moment magnitude.

In the following subsections, empirical relations between source parameters derived in the present study are presented.
4.4 Discussion and empirical relations between source parameters

4.4.1 Moment magnitude

The magnitude is the first and an immediate quantitative measure of the strength of an earthquake. It is still the most widely used earthquake parameter even though other parameters can measure the earthquake as accurately as the magnitude.

The concept of magnitude was introduced by Richter (1935) to provide an objective instrumental measure of the size of earthquakes. Magnitudes are derived from ground motion amplitudes and signal duration measured from instrumental records and adjusted for epicentral distance and source depth. Standardized instrument characteristics were originally used to avoid instrumental effects on the magnitude estimates. The original Richter magnitude $M_L$, was based on maximum amplitudes measured in Displacement-proportional records from the standardized short-period Wood-Anderson (WA) seismometer network. Later many geoscientists have introduced different magnitude scales based on internationally recommended standards and now-a-days commonly and widely used for surface waves and different types of body waves. These magnitude scales have one or another kind of limitation and useful for a particular application. Kanamori (1977) proposed a moment magnitude $M_w$ which is directly determined from seismic moment and would not get saturated at larger earthquakes. Moment magnitude is also crucial for the quantitative classification and statistical treatment of seismic events aimed at assessing seismic activity and hazard, studying variations of seismic energy release in space and time and so forth. Consequently, it is also significant in earthquake prediction research.

The seismic moment $M_o$ is related to the final static displacement after an earthquake and hence the moment magnitude $M_w$ is more closely related to the tectonic effects of an earthquake. It is also the most difficult parameter to relate theoretically to other important source characteristics such as moment, stress drop, strain energy release, rupture dimensions and radiated seismic energy. These other parameters can be related to one to another. Their relationship to magnitude requires a spectral description of the seismic source. On the basis of digital broadband records at modern observatories and network
centers, it has become routine to determine moment magnitude $M_w$. The most advantage of it is that it equally applies to the scale smaller, moderate or larger events at local, regional and teleseismic range.

In the process, the relationship between moment magnitude and seismic moment, rupture parameters and corner frequency is shown. Its relation with stress drop and radiated energy will be discussed in subsequent section of respective source parameters.

### 4.4.1.1 Relation between moment magnitude and seismic moment

![Figure 4.4 Relation between Moment magnitude $M_w$ and Seismic moment $M_0$](image)

According to Gutenberg and Richter (1956), relation between released seismic strain energy $E_s$ and magnitude $M_s$ is,

$$\log E_s = 4.8 + 1.5M_s \quad (4.16)$$

Extending above equation and replacing $M_s$ with $M_w$ have,

$$M_w = \frac{2}{3} (\log M_0 - 9.1) \quad (4.17)$$

i.e. $M_w = 0.66 \log M_0 - 6.1 \quad (4.18)$
For the present study, it is found from the graph, for events analyzed,

\[ M_w = 0.59 \log M_0 - 4.9 \tag{4.19} \]

The computed \( M_0 \) depends on details of the individual inversion methodologies and thus related \( M_w \) may vary.

### 4.4.1.2 Moment magnitude and corner frequency

As shown in above graph, moment magnitude is higher for lower corner frequencies. Here, moment magnitude in the range of 3.5 to 5.7 and corresponding corner frequency ranges from 0.60 Hz to 8.0Hz are studied and showing the power co-relation. Graph shows corner frequency as high as 8Hz for magnitude of 3.5.

### 4.4.2 Seismic moment

In view of limitations to traditional magnitude scales, by both the frequency response of the Earth and the response of the recording seismograph, seismic moment \( M_0 \) is a
physically meaningful link between earthquake size and fault rupture parameters. It is directly related to rupture and static shear of the source and the best represents the characteristics of the source. According to Hanks and Kanamori (1979) seismic moment is related to rupture area and average displacement after the rupture can be given as follow.

\[ M_0 = \mu \bar{D} A \]  \hspace{1cm} (4.20)

Where, \( \mu \) = Rigidity or shear modulus of the medium
\( \bar{D} \) = average final displacement after the rupture
\( A \) = the surface area of the rupture.

\( M_0 \) is a measure of the irreversible inelastic deformation in the rupture area.

In above equation, the product \( \bar{D}A \) is inelastic strain. Seismic moment \( M_0 \) can be computed from the source spectra of body and surface waves or it is derived from a moment tensor solution (Hanks et al., 1975; Kanamori and Anderson, 1975; Dziewonski et al., 1981). \( M_0 \) is considered a more reliable measure of the energy released during an earthquake (Hanks and Kanamori, 1979).

As mentioned earlier, in a homogeneous half-space, \( M_0 \) can be determined from the spectra of seismic waves observed at the Earth’s surface which are shown in Figure 4.3. The relation between seismic moment and moment magnitude is discussed in section 4.4.1.1 and its relation with corner frequency and rupture parameters are discussed in 4.4.2.1 and 4.4.2.2 respectively.
4.4.2.1 Relation between seismic moment and corner frequency

Relation between seismic moment $M_0$ and corner frequency $f_c$ is shown in Figure 4.6.

![Figure 4.6 Relation between seismic moment $M_0$ and corner frequency $f_c$.](image)

It can be observed that the corner frequency is decreasing with increasing $M_0$ in this study. The analysis suggests that

$$M_0 \propto f_c^{-2.98} \quad (4.21)$$

Izutani (2005) derived the relation between moment and corner frequency for the 2004 $M_w 6.7$ Niigata, Japan earthquake and found the relation,

$$M_0 \propto f_c^{-3.3} \quad (4.22)$$

Whereas Mayeda et al (2012) found $M_0 \propto f_c^{-3.0}$ for the $M_w 5.65$, 2011 Virginia earthquake.

Kanamori and Rivera (2004) have proposed that the scaling between moment and corner frequency could take on the form,

$$M_0 \sim f_c^{(-3+\varepsilon)} \quad (4.23)$$

Where, $\varepsilon$ represents the deviation from self-similarity and is usually thought to be a small positive number. For example, Walter et al. (1995) found $\varepsilon$ to be close to 0.5 for the Hector Mine main shock and its aftershocks using independent spectral methods.
4.4.2.2. Relation between seismic moment and rupture parameters

Relation between seismic moment and rupture parameters are shown in Figure 4.7 (a) and Figure 4.7 (b).

\[ M_0 = 7.0 \times 10^{15} A^{1.4966} \]

This is close to Kanamori et al (1975). According to Abercrombie (1995) the general trend follows the \( M_0 \propto A^{3/2} \) scaling with stress drop ranging from 1 to 1000bars. The average relationship suggested that \( M_0 = 1.33 \times 10^{15} A^{3/2} \). This is identical with the relation by Purcaru and Berckhemer (1982), \( \log M_0 = (1.5 \pm 0.02) \log A + (15.25 \pm 0.05) \).

4.4.2.3 Relation between Seismic moment and epicentral distance and hypocentral depth

In addition, it is also represented the distribution of \( M_0 \) with epicentral distance and hypocentral depth over the region. These relationships represented in Figure 4.8 and Figure 4.9 and no any systematic dependency observed.
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Figure 4.8 Relation between seismic moment $M_0$ and epicentral distance

Figure 4.9 Relation between seismic moment $M_0$ and hypocentral depth
4.4.3 Radiated seismic energy

To reduce the hazard from earthquakes, it is imperative that we understand the physics of the processes that occur during earthquakes. As we know, an earthquake happens when the accumulated strain energy in the earth is released suddenly. The energy that releases during an earthquake can be categorized as fracture energy, frictional energy and wave energy. The fracture energy is used in mechanical processes other than frictional heating on the fault zone as the rupture propagates; while frictional energy is dissipated as heat on the fault surface and finally wave energy, that is responsible for particles motion on the fault, generating seismic waves. The wave energy is only one that can be felt by people and recorded by instruments all over the world. Thus the only part of the energy released in an earthquake that we have direct access to be the wave energy and referred to as radiated seismic energy. According to Haskell (1964), radiated energy can be defined as the wave energy that would be transmitted to infinity if an earthquake occurred in an infinite, lossless medium.

Radiated seismic energy is an important parameter that represents the dynamic characteristic of the earthquake rupture. It has been observed that each earthquake has unique characteristics even if they are of same category and of same region, if we study all interplate earthquakes or all intraplate earthquakes or all deep earthquakes. For example, tsunami earthquakes propagate with small rupture velocities and produce mild shaking but are followed by destructive tsunamis (Kanamori, 1972; Polet and Kanamori, 2000). The 1994 deep Bolivian earthquake is the largest deep earthquake instrumentally recorded and seems to have propagated very slowly and a large amount of the total energy available for faulting was probably dissipated on the fault zone causing frictional melting on the fault (Kanamori et al., 1998). These striking differences observed between different earthquakes are probably due to the different physical mechanisms that control the rupture of these earthquakes. Thus, to understand the differences between these earthquakes, it is important that we understand the physics that controls their rupture mechanics.
Traditionally, radiated energy has been used as a measure of the size of earthquakes. Early attempts to quantify the size of earthquakes were based on estimating the intensity of damage in earthquakes (Wood and Neumann, 1931). With the advent of instrumental seismology, the size of an earthquake was measured as a function of the amplitude of different seismic waves (Richter, 1935). The idea of using a physical and fundamental quantity to measure the size of the earthquakes led Gutenberg to use radiated energy as a measure of earthquake size (Gutenberg, 1942, 1956; Gutenberg and Richter, 1956a, 1956b). Based on the then available seismic data, Gutenberg introduced an empirical scale (the Gutenberg-Richter energy-magnitude relation) that related radiated energy, (in ergs), to surface wave magnitude, and is given as $\log E = 1.5M_s + 11.8$. However, due to poor seismic data quality and limited computing facilities, estimating radiated energy was difficult.

An earthquake generates seismic waves that travel through the earth and are recorded by seismometers on the surface of the earth. When the seismic waves travel through the earth, the earth structure acts like a filter and modifies the waves. Thus, to calculate radiated energy, we have to correct the recorded seismic waves for propagation path effects like dissipation of energy due to attenuation, site effects and geometric spreading and the corrections for a source effect known as directivity.

In an attempt to understand the physics of the rupture process, many recent studies have focused on the determination of spatial and temporal variations of slip, estimation of critical slip and fracture energy from detailed inversion of seismic wave-forms (Boatwright and Cocco, 1996; Guatteri and Spudich, 2000; Ide and Takeo, 1997; Ji et al., 2002b; Pulido and Irikura, 2000; Wald and Heaton, 1994). An alternative approach would be to estimate the radiated energy, because radiated energy reflects the overall frictional conditions during rupture (Kanamori and Heaton, 2000). Thus, instead of trying to understand the small-scale complex details of the rupture process, we use the 2-macroscopic parameters–radiated energy, seismic moment, rupture area and rupture velocity to better understand the dynamics of earthquake rupture. Subsequently, with the introduction of seismic moment (Aki, 1966) the moment magnitude scale came into vogue (Kanamori, 1977).
Kanamori (1977) determined radiated energy from seismic moment. Later, on the availability of broadband seismometers data, we can now directly integrate seismic velocity data to determine radiated energy (Boatwright and Choy, 1986).

Thus, we have two independent measures of earthquake size, seismic moment and radiated energy and the relationship between these two can be used to better understand the earthquake source. Energy gives a different measure of earthquake size than moment. Energy is derived from the velocity power spectra, while moment is derived from the low-frequency asymptote of the displacement spectra. Thus, energy is a better measure of the severity of shaking and thus of the seismic potential for damage, while moment being related to the final static displacement is more related to the long-term tectonic effects of the earthquake process. The radiated energy is calculated as defined by Richter (1958),

\[
E_R = \frac{21.1\rho R \Omega_0 f c^3 M_0}{\mu} \quad \text{(Richter, 1958)} \quad (4.24)
\]

Where, \( \mu = \text{Shear Modulus (N/m}^2\) \)

For this analysis, radiated energy ranges from \(2.2 \times 10^{10}\)J to \(2.0 \times 10^{13}\)J.

Figure-4.10(a) and (b) show interrelation of radiated energy to seismic moment and moment magnitude.
4.4.3.1 Relation between radiated seismic energy and moment magnitude

![Figure 4.10(a) Relation between Radiated seismic energy and moment magnitude.](image)

It is found that radiated seismic energy ranges from $2.2 \times 10^{10}$ J to $2.0 \times 10^{13}$ J for the magnitude range from $3.5 < M_w < 5.7$. For magnitude it is found, $\log E_R = 1.2 M_w + 6.4$ which is similar to Gutenberg-Richter energy relation $\log E_R = 1.5 M_w + 4.8$ and Choy and Boatwright (1995), $\log E_R = 1.5 M_w + 4.4$.

4.4.3.2 Relation between radiated seismic energy and seismic moment

In the present study, empirical relations are derived between radiated energy to seismic moment and moment magnitude.
As shown in Figure 4.10(a) and Figure 4.10(b), radiated energy exhibits linear relation with $M_w$ and $M_0$. Results from analysis of 394 shallow-focus earthquakes over the globe show the following relation between radiated energy and moment (Choy and Boatwright, 2012); $E_R = 1.6 \times 10^{-5} M_0$. While for this study it is found $E_R = 2.0 \times 10^{-5} M_0 + 3 \times 10^{11}$. These results are close to global values of radiated energy for intraplate earthquakes.

Thus, radiated energy is not only a measure of the size of the earthquake, but also a macroscopic parameter that can be used to obtain insights into the rupture mechanisms of earthquakes. Systematic variations in the release of energy as a function of faulting type and tectonic setting is now identified which were previously undetectable because of lack of reliable energy estimates.

**4.4.4 Scaled energy**

Now to understand the physical processes of earthquake source, we have two the most important and fundamental macroscopic parameters i.e. seismic moment $M_0$ and radiated seismic energy $E_R$. The seismic energy reflects the dynamic characteristics of earthquake source while the seismic moment represents static one. And the ratio of seismic energy to seismic moment $E_R/M_0$ can be interpreted as the radiated energy per unit area and per unit
slip on the fault plane. The seismic-wave energy calculated from near-field data is generally larger than that estimated from far-field ones (Smith et al. 1991; Singh and Ordaz 1994; Hwang et al. 2001). This might be due to existence of more high-frequency signals in near-field seismograms than in far-field ones. Additionally, scaled energy - the ratio of the radiated energy to seismic moment, $E_R/M_0$, for small earthquakes are observed to be significantly smaller than those for large earthquakes (Abercrombie, 1995). This leads to conclude that energy release mechanisms for small and large earthquakes are drastically different (Kanamori and Heaton, 2000). However, the uncertainties in the currently available estimates of radiated energy are large and the differences in the $E_R/M_0$ ratio between small and large earthquakes may be due to errors in the radiated energy estimates; more tightly constrained estimates are required to understand these differences and to validate the proposed mechanisms.

As stated above, scaled energy is defined as,

$$\bar{\varepsilon} = \frac{E_R}{M_0} \quad (4.25)$$

For the present study, it is found that scaled energy ranges from $8.3 \times 10^{-6}$ to $2.0 \times 10^{-4}$ for the magnitude range from $3.5 < M_w < 5.7$. Ide and Beroza (2001) examined previous studies of ratios of radiated seismic energy to seismic moment. They found that scaled energies were almost constant in the range of $10^{-6}$ to $10^{-3}$ for earthquakes with $4 < M_w < 8$. Their results emphasized the importance of the high frequency energy content that is sometimes not sufficiently recorded in the seismograms. Prieto et al. (2004) studied source parameters by analyzing spectra of small earthquakes ($M_l = 0.5–3.4$) in southern California. They found that scaled energies were about $10^{-5}$ to $10^{-3}$ and nearly constant over the magnitude range from $M_l = 1.8$ to $3.4$. Izutani (2005) analyzed radiated seismic energies for the 2004 Niigata, Japan, earthquake and its aftershocks and found that the scaled energies were from $10^{-6}$ to $10^{-3}$. Mayeda et al. (2005) estimated radiated seismic energies of five large earthquakes and their aftershocks ($3.7 \leq M_w \leq 7.4$). They concluded that the scaled energies were in the range between $10^{-7}$ to $10^{-4}$. Venkataraman et al. (2004) analyzed spectral similarities of micro earthquakes in western Nagano, Japan and estimated their radiated seismic energies. They found that scaled energies were from $10^{-6}$ to $10^{-4}$. It indicates that the scaled energies were between $10^{-7}$ to $10^{-3}$ for earthquakes with seismic
moment ranging from $10^9$ N.m to $10^{22}$ N.m. When examining the relations; it shows that scaled energy varies randomly with seismic moment and moment magnitude as displayed in Figure 4.11(a) and Figure 4.11(b).

![Figure 4.11 Relation of Scaled energy with $M_0$ and $M_w$.](image)

### 4.4.5 Stress drop

Stress drop change on the fault is one of the significant indicators specifying dynamic behavior of earthquake ruptures (Brune 1970). It can be defined as the change in the average state of stress on a fault before and after rupture. The static stress drop ($\Delta \sigma$), the seismic moment ($M_0$), and the radius of a circular rupture ($r$) are related, in the context of a circular dislocation, through equation given by Kellis-Borok (1959).

**Static stress drop**, 

$$\Delta \sigma = \frac{7M_0}{16r^3} \quad \text{(Kellis-Borok, 1959)} \quad (4.26)$$

We can write for apparent stress, 

$$\sigma_{app} = \mu \frac{E_s}{M_0} \quad \text{(Wyss and Brune, 1968)} \quad (4.27)$$

Where, $\mu$ = Shear Modulus 

= $3 \times 10^{10}$ N/m$^2$ 

$E_s$ = Radiated Seismic Energy
Several earlier studies noted that intraplate earthquakes had systematically larger static stress drops as compared to interplate earthquakes (e.g., Molnar and Wyss, 1972; Kanamori and Anderson, 1975; Scholz et al., 1986). Kanamori and Allen (1986) also studied the repeat times of earthquakes and concluded that faults with longer repeat times had higher stress drops. They observed that interplate earthquakes on mature faults occur more frequently than intraplate earthquakes and thus intraplate faults have larger stress drop. Large earthquake populations reveal strong variations in stress drop but little in the way of systematic behavior or dependence on seismic moment (Aki, 1972; Hanks, 1977; Allmann and Shearer, 2009). Because static stress drop measurements depend on the corner frequency cubed thus small uncertainties in corner frequency map into large uncertainties in the stress drop and it’s often unclear how much of this variability is due to measurement error rather than variability in source properties (Sonley and Abercrombie, 2006; Prieto et al., 2007).

While stress drop values were estimated for aftershock sequence of Jan 26, 2001 Bhuj earthquake, it is found that stress drop values for Kutch region ranges from 68.4 to 299.8 bar, which is in accordance with global values of other intraplate earthquake. No systematic relationship of stress drop and apparent stress could found with seismic moment and other parameters. However, several anomalous events with very high or very low stress drops than average values are found. The energetic events could have high stress drop or high rupture velocity which suggests that there might be a population of earthquakes that have particularly intense strong ground motion for their size. Also Earthquakes with higher stress drops will have more intense ground motions. Stress drop distribution for Kutch region is shown in location map and by contours in Figure 4.12 and Figure 4.13 respectively.
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Figure 4.12 Distribution of stress drop over Kutch region.

Figure 4.13 Contours for stress drop distributed over Kutch region.
Stress drop distribution for Kutch region is less discussed so far. Important result from our study can be summarized that high stress drop values (> 200 bars) are observed away from the fault in the most of cases. While low (50-100 bars) and moderate stress drop values (100-200 bars) are observed for events at the fault or near the fault. Not a single event located at the fault which exhibits high stress drop value except one at North Wagad fault in this study. Events with high stress drop are scattered and as stated above they are far away from the fault at least 25 km away from the nearest fault. Fault wise stress drop distribution can be describe as follow:

- Events located near Gora Dungar fault shows lower stress drop values compared to other faults of Kutch region.
- Gedi fault having moderate stress drop events.
- Western part of widely known North Wagad fault exhibits moderate stress drop values while Eastern part exhibits that of higher values.
- NW of South Wagad fault can be characterized by higher stress drop values. Only few events with high stress drop values are observed at Kutch Mainland fault-KMF and that is again at extreme Eastern end of KMF. While North of KMF and South of KMF are confined with high stress drop events.

This difference in stress drop from one fault to another can be explained with fault geometry, frictional strength of faults and crustal brittleness. The inferred high heat flow for Kutch region can be attributed to the presence of mafic instructive at lower crustal depths in the region (Mandal et al., 2004), which lead to higher deep crustal temperatures. This cold mafic crust provides enough brittleness to generate large earthquake with high stress drops in the lower crust.

In turn, apparent stress also calculated for the region and apparent stress values are found considerably high ranging from 2.2 bars to 115.4 bars for this analysis. In addition to stress drop distribution discussed above, relations of stress drop to other source parameter were also studied but due to its distinctive pattern over the region, it is difficult to establish the relation of stress drop to other source parameters.
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(Figure 4.14 continued)
The relationship between stress drop, apparent stress and earthquake moment remains a controversial topic due to the difficulty in correction for propagation effects over the broad frequency range required to measure the radiated energy (Ide et al., 2003). Some studies find no dependence of apparent stress on moment, while others observe a systematic increase in apparent stress with moment (Walter et al., 2006). Studies of stress drop and apparent stress indicate that they two vary together (Abercrombie, 1995; Ide et al., 2003). Uchida et al. (2010) studied the smaller earthquakes in the sequence of the Kamaishi earthquake of 2008. They found to have stress drops between 30 bars and 110 bars with a stress drop of 270 bars for the 2008 main event. For global earthquakes from $M_w$ 5.2 to 8.3, Allmann and Shearer (2009) estimated stress drops ranging from 3 bars to 500 bars with the median value of 40 bars independent of moment.

**4.4.6 Zuniga parameter for stress release mechanism**

In 1993, Ramon Zuniga introduced a parameter denoted by $\varepsilon$ and proposed a stress model other than Brune’s final stress model i.e. partial-stress-drop model. Later he described rupture processes by frictional overshoot mechanism.
Defining Zuniga parameter-\( \varepsilon \),

\[
\varepsilon = \frac{\Delta \sigma}{\sigma_a + \frac{\Delta \sigma}{2}} \quad (4.28)
\]

Now, \( \varepsilon \) represents two stress models, i.e., for frictional overshoot model \( \varepsilon > 1 \) and for partial-stress-drop model \( \varepsilon < 1 \).

On the basis of Brune’s stress drop model, Kikuchi and Fukao (1988), Kanamori (1994) and Smith et al (1991) described rupture processes using static stress drop, apparent stress and their ratio and explained the importance of dynamic stress and final stress during earthquake rupturing processes. A stress model specified with frictional overshoot as the final stress level being lower that the dynamic one (Kikuchi and Fukao, 1988). While Ramon Zuniga considered partial-stress-drop model and interpreted earlier observations. He concluded that frictional overshoot model and partial-stress-drop models have distinction due to differences in seismic wave energy from teleseismic data and the frictional overshoot model is more appropriate than the partial stress drop model. Hwang et al (2001) studied 1999 Chi-Chi, Japan earthquake and obtained the same results and came to the same conclusion.

When Zuniga parameter is studied for Kutch region, it is found to range from 0.62 to 1.95. When Zuniga parameter \( \varepsilon \) scaled as a function of time, it helped to understand the stress release mechanism during long aftershock sequence. Figure 4.15 (a) and (b) show Zuniga parameter \( \varepsilon \) for first 100 days and for entire aftershock series respectively. It describes how Zuniga parameter \( \varepsilon \) varies with time.
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It can be observed that value of Zuniga parameter is comparatively higher. It is found that $\varepsilon < 1$ during initial days only, which are the cases of final stress greater than the frictional stress and can be explained with partial-stress-model. Most of aftershocks having higher values of Zuniga parameter i.e. $\varepsilon > 1$ which satisfies the conditions of final stress lower than the frictional stress and can be understood with frictional overshoot model. It is worth notice point that we do not have any event having $\varepsilon = 1$, the case of final stress equals to the frictional stress. This kind of case when $\varepsilon = 1$ can be explained with the help of faulting model of Orowan (1960).

In addition, variation in Zuniga parameter with magnitude is checked and shown in Figure 4.16.
It is found from this study that Zuniga parameter does not show any dependancy on magnitude for Kutch region. In the case of Kutch region, for most of the events $\varepsilon$ found more than 1. For very few events $\varepsilon < 1$ is found and again it is for $M_w \leq 4.5$ while for $M_w \geq 5.0$; $\varepsilon > 1.5$ are observed and for $4.5 \leq M_w \leq 5.0$ values of $\varepsilon$ were between $1 < \varepsilon < 1.5$. An important observation of this study is that higher values of $\varepsilon$ is observed for higher magnitude.