CHAPTER 5

FUZZY LOGIC CONTROL

5.1 Introduction

Fuzzy logic is a soft computing tool for embedding structured human knowledge into workable algorithms. The idea of fuzzy logic was introduced by Dr. Lofti Zadeh of UC/Berkeley in 1960’s as a means of model of uncertainty of natural languages (Zadeh 1965). Fuzzy logic is considered as a logical system that provides a model for human reasoning modes that are approximate rather than exact. The fuzzy logic system can be used to design intelligent systems on the basis of knowledge expressed in human language. There is practically no area of human activity left untouched by intelligent systems as these systems permits the processing of both symbolic and numerical information. The systems designed and developed based on fuzzy logic methods have been proved to be more efficient than those based on conventional approaches. Fuzzy logic has been recently applied in process control, modeling, estimation, identification, stock market prediction, diagnostics, military science, agriculture and so on. One of the pioneering applications of fuzzy logic is in control systems.

Fuzzy logic based control is used in various applications. During the past several years, fuzzy logic based control has emerged as one of the most active and fruitful areas for research in the application of fuzzy logic theory, especially in wide range of industrial process which lacks quantitative data regarding the input-output relations. Some important systems for which fuzzy logic based controllers have been extensively used are water quality
control system, elevator control system, automatic train operation system, automatic transmission control, nuclear reactor control system, washing machine etc.

5.2 Basics of Fuzzy logic Theory

Fuzzy logic is another form of artificial intelligence, but its history and applications are more recent than artificial intelligence based expert systems. Fuzzy logic is based on the fact that human thinking does not always follow crispy “YES” – “NO” logic, but it is often vague, uncertain, and indecisive (Dote and Ovaska 2001, Driankov et al. 1996, Kecman 2001, Tao et al. 2010, Wang and Lin 1999, Yagiz and Hacioglu 2005, Yu and Kaynak 2009). Fuzzy logic deals with problems that have vagueness, uncertainty, imprecision, approximations or partial truth or qualitative mess. The fuzzy logic is based on fuzzy set theory in which a particular object or variable has a degree of membership in a given set which may be anywhere in the range of 0 to 1. This is different from conventional set theory based on Boolean logic in which a particular object or variable is either a member (logic 1) of a given set or it is not (logic 0). The basic set operations like union (OR), intersection (AND) and complement (NOT) of Boolean logic are also valid for fuzzy logic (Kecman 2001, Ross 2005).

5.2.1 Fuzzy Sets

A fuzzy set is a set without a crisp, clearly defined boundary. It can contain elements with a partial degree of membership. In other words, a fuzzy set is a set containing element with varying degree of membership in
the set. Fuzzy set is different from the classical crisp set because members of
crisp set will not be members unless their membership is full in that set.

Let $X$ be a space of points or objects (Universe of discourse), with
$x$ as a generic element of $X$, then a fuzzy set $A$ in $X$ is characterized by a
membership function $\mu_A(x)$ which associates a real number in the closed
interval $[0,1]$ with each point $x$ in $X$, the value of $\mu_A(x)$ at $x$ representing the
membership degree of $x$ in $A$.

### 5.2.2 Basic Fuzzy Set Operations

Out of many fuzzy set operations, the three basic and most
common operations are union, intersection and complement operations. Though, there are many operators to model the union and intersection
operations, ‘max’ and ‘min’ operators are commonly used in engineering
applications (Kecman 2001, Ross 2005).

Fuzzy Union of two fuzzy sets $A$ and $B$ in the universe of
discourse $X$, characterized by membership functions $\mu_A(x)$ and $\mu_B(x)$
respectively is a fuzzy set $A \cup B$ in the universe of discourse $X$ characterized
by a membership function $\mu_{A\cup B}(x)$ such that

$$
\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x))
$$

(5.1)

Fuzzy intersection of two fuzzy sets $A$ and $B$ in the universe of
discourse $X$, characterized by membership functions $\mu_A(x)$ and $\mu_B(x)$
respectively is a fuzzy set $A \cap B$ in the universe of discourse $X$ characterized
by a membership function $\mu_{A\cap B}(x)$ such that

$$
\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))
$$

(5.2)
Fuzzy complement of a fuzzy set $A$ in the universe of discourse $X$, characterized by membership functions $\mu_A(x)$ is a fuzzy set $\overline{A}$ in the universe of discourse $X$ characterized by a membership function $\mu_{\overline{A}}(x)$ such that

$$\mu_{\overline{A}}(x) = 1 - \mu_A(x)$$

(5.3)

### 5.2.3 Fuzzy Relation

Consider two sets $A$ and $B$ in the universes $X$ and $Y$ respectively, then a fuzzy relation $R$ between $A$ and $B$ is a relational matrix in Cartesian product space $X \times Y$, characterized by membership function $\mu_R(x, y)$. The elements of the relational matrix are membership degrees $\mu_R(x, y)$ which represent the degrees at which a specific pair $(x, y)$ belongs to the relation $R$.

### 5.2.4 Fuzzy Composition

Fuzzy composition combines fuzzy relations in different Cartesian product spaces with each other. Similarly, fuzzy set can also be combined with a fuzzy relation. Out of many different versions of composition operators, the most commonly used operators are max-min operator, max-prod operator and max-average operator.

Let $R_1$ and $R_2$ are two fuzzy relations in Cartesian product spaces $X \times Y$ and $Y \times Z$ respectively, then composition $R_1 \circ R_2$ is a fuzzy relation in $X \times Z$ characterized by the membership function $\mu_{R_1 \circ R_2}(x, z)$. The max-min composition is defined as

$$\mu_{R_1 \circ R_2}(x, z) = \max_y \{\min\{\mu_{R_1}(x, y), \mu_{R_2}(y, z)\}\}$$

(5.4)
The max-prod composition is
\[ \mu_{R_1 \circ R_2}(x, z) = \max_y \{\mu_{R_1}(x, y) \cdot \mu_{R_2}(y, z)\} \] (5.5)

The max-average composition is
\[ \mu_{R_1 \circ R_2}(x, z) = \frac{1}{z} \max_y \{\mu_{R_1}(x, y) + \mu_{R_2}(y, z)\} \] (5.6)

5.3 Basics of Fuzzy Logic Control

Fuzzy logic provides a non-analytic alternative to the classical analytical control theory. Hence, fuzzy logic control is a powerful control tool for systems or processes which are complex and precise mathematical modeling are not possible. Fuzzy logic control is inherently robust and does not require precise, noise free inputs or the measurement or computation of change of parameters. Since the fuzzy logic controller uses defined rules governing the target systems, it can be tuned easily to improve system performance. The fuzzy logic controller is a suitable candidate for the control of multiple input-multiple output systems as it can accept many feedback inputs and generate many control outputs and it needs less storage of data in the form of membership functions and rules than the conventional look up table for non-linear controllers (Dote and Ovaska 2001, Kecman 2001, Raviraj and Sen 1997, Ross 2005, Tao et al. 2010, Wang and Lin 1999, Yagiz and Hacioglu 2005, Yu and Kaynak 2009). The block diagram of a closed loop fuzzy logic controller is shown in Figure 5.1.
In fuzzy logic control of a process or a system, there are two links between the fuzzy logic controller and the process or the system under control, namely the input and output links. The inputs of fuzzy logic
controller are linked to the process through sensors and the outputs of the fuzzy logic controller are linked to the process through actuators. The fuzzy logic controller comprises of fuzzification, inference mechanism and defuzzification which are executed using information stored in the knowledge base (Dote and Ovaska 2001, Kecman 2001, Raviraj and Sen 1997, Ross 2005, Tao et al. 2010, Yagiz and Hacioglu 2005, Yu and Kaynak 2009).

5.3.1 Knowledge Base

The knowledge base can be divided into two sub-blocks namely the Data Base and Rule Base. The data base consists of the information required for fuzzifying the crisp input and later defuzzifying the fuzzy outputs to a crisp output. It consists of the membership functions for various fuzzy variables or sets used in the controller design. The rule base consists of a set of rules, which are usually formulated from the expert knowledge of the system. The rules are typically of the form “If……………, then………………” rules. The antecedent part of the rule may be a simple statement or a compound statement using connectives like “and”, “or” etc. The consequent part may contain a fuzzy set (Mamdani type and Tsukamoto type) or a linear or a quadratic function of the crisp input variables (Sugeno type). The knowledge base is the heart of fuzzy logic based system it has to be designed with utmost care and requires a lot of expertise in the knowledge of the system into which fuzzy logic controller is being incorporated (Kecman 2001, Raviraj and Sen 1997, Tao et al. 2010, Yu and Kaynak 2009).
5.3.2 Fuzzification

As the inputs of fuzzy logic controller are from sensors and the data from sensors are crisp in nature, the fuzzy logic controller cannot use this data directly. Hence, there exists the need for converting this data to the form comprehensible to the fuzzy system. Fuzzification is the process of converting a real scalar crisp value into a fuzzy quantity. This is done by assigning appropriate membership values to each input. The data required to change the crisp value to the fuzzy quantity is stored in the knowledge base in the form of membership functions associated with various linguistic fuzzy variables.

A membership function is a function that defines how each point or object in the universe of discourse is assigned a degree of membership or membership value between 0 and 1. The membership function can be an arbitrary curve that is suitable in terms of simplicity, convenience, speed and efficiency (Kecman 2001, Ross 2005).

Though a membership function can be an arbitrary curve, there are eleven standard membership functions that are commonly used in engineering applications (Ross 2005). These membership functions can be built from several basic functions: piecewise linear functions, sigmoid curve, Gaussian distribution function, and quadratic and cubic polynomial curves. The simplest membership functions can be formed using straight lines. They may be triangular membership function which is a collection of three points forming a triangle or trapezoidal membership function which has a flat top and is just a truncated triangle curve. These membership functions built out of straight lines have the advantage of simplicity. The examples of triangular
and trapezoidal membership functions are given in Figure 5.2 and Figure 5.3 respectively.

![Figure 5.2 Triangular Membership function](image)

![Figure 5.3 Trapezoidal Membership function](image)

Two types of membership functions can be built using Gaussian distribution curve, a simple Gaussian curve shown in Figure 5.4 and a two-sided composite of two different Gaussian curves. The generalized bell membership function is specified by three parameters as shown in Figure 5.5.
Because of their smoothness and nonzero values at all points, Gaussian and bell membership functions are popular membership functions for specifying fuzzy sets. However, they are unable to specify asymmetric membership functions, which are important in many applications.

Figure 5.4 Gaussian Membership function

Figure 5.5 Generalized bell Membership function
Three types of membership functions are built using sigmoid curves—left open, right open and closed. Two types of closed sigmoid membership functions can be synthesized using two sigmoid functions. They are d-sigmoid membership function using difference between two sigmoid functions, p-sigmoid membership function using the product of two sigmoid functions. The membership functions based on sigmoid curves are asymmetric.

![Figure 5.6 Sigmoid Membership function](image)

![Figure 5.7 d-sigmoid Membership function](image)
Z-shaped membership functions, S-shaped membership functions and Pi-shaped membership functions are the asymmetrical membership functions which can be built using polynomial based curves. The Z-shaped membership function is the polynomial curve open to the left, S-shaped membership function is the polynomial curve open to the right, and Pi-shaped membership function has zero on both extremes with a rise in the middle.
There is a wide choice of membership function types when a membership function is to be selected. There is no hard and strict rule on the selection of membership functions. The membership functions can be selected to suit the applications in terms of simplicity, convenience, speed and efficiency (Kecman 2001, Ross 2005). The following principles should
be adopted for designing membership functions for fuzzy logic controllers (Ross 2005, Yu and Kaynak 2009).

- Each membership function overlaps only with the closest neighbouring membership functions.
- For any input data, the sum of its membership values in all the fuzzy sets (membership functions) should be 1.

5.3.3 Fuzzy Inference Engine

Fuzzy inference is the process of converting fuzzy input to fuzzy output according to fuzzy rules in the knowledge base. Three types of fuzzy inference mechanisms are commonly used.

5.3.3.1 Mamdani-Type Inference

In Mamdani-type systems, the antecedent part of each rule may be a simple statement or a compound statement using connectives like “and”, “or” etc. and the consequent part is represented by a fuzzy set. If the antecedent of a rule has only one part (input), membership value of the input in the corresponding fuzzy set gives the truth value of the rule. If the antecedent of a rule has more than one part (more than one input), the fuzzy operator (“min” operator if the connective is “and”, “max” operator if the connective is “or”) is applied for the membership values to obtain the truth value of the rule. This truth value is then applied to the output fuzzy set. A rule base containing rules of the form “$R_i: \text{If } x \text{ is } A_i \text{ and } y \text{ is } B_i, \text{ then } z \text{ is } C_i$” is implemented as shown in Figure 5.12. The fuzzy outputs corresponding to different rules are then aggregated.
Mamdani-type systems are intuitive and are widely used (Ross 2005).

Figure 5.12 Mamdani-type Inference (Ross 2005)

5.3.3.2 Sugeno-type Inference

A Sugeno-type system differs from a Mamdani-type system in the nature of inference rules. Unlike Mamdani-type system, this method uses fuzzy sets only for input variables, but not for output variables and the consequent part of each rule of Sugeno-type system contains a linear or a quadratic function of the crisp input variables. If the antecedent of a rule has only one part (input), membership value of the input in the corresponding...
fuzzy set gives the truth value of the rule. If the antecedent of a rule has more than one part (more than one input), the fuzzy operator (“min” operator if the connective is “and”, “max” operator if the connective is “or”) is applied for the membership values to obtain truth value of the rule. This truth value is then applied to the output function. A rule base containing rules of the form “$R_i: If \; x \; is \; A_i \; and \; y \; is \; B_i, \; then \; z \; is \; \mu_{A_i}(x) \; \mu_{B_i}(y) + r_i$” is implemented as shown in Figure 5.13. The crisp values computed corresponding to different rules are combined to form the final output value by computing the weighted average of the crisp values, the weights being the truth degrees of their respective rule conditions. Hence, Sugeno-type inference directly leads to real crisp values without defuzzification (Ross 2005).

\[ z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2} \]

Figure 5.13 Sugeno-type Inference (Ross 2005)
Sugeno-type systems are computationally efficient, work well with linear techniques like PID controllers, optimization and adaptive techniques. They have continuous output surface (Ross 2005).

5.3.3.3 Tsukamoto-Type Inference

In Tsukamoto-type system, the consequent part of each fuzzy rule is a fuzzy set with a monotonic membership function (Ross 2005). If the antecedent of a rule has only one part (input), membership value of the input in the corresponding fuzzy set gives the truth value of the rule. If the antecedent of a rule has more than one part (more than one input), the fuzzy operator ("min" operator if the connective is "and", "max" operator if the connective is "or") is applied for the membership values to obtain the truth value of the rule. This truth value is then applied to the output fuzzy set and the crisp value of the output, having the membership value equal to the truth value of the rule is determined. The crisp values computed corresponding to different rules are combined to form the final output value by computing the weighted average of the crisp values, the weights being the truth degrees of their respective rule conditions (Ross 2005). An example of Tsukamoto-type system having the rules of the form “$R_i: If \ x \ is \ A_i \ and \ y \ is \ B_i, \ then \ z \ is \ C_i$” is given in Figure 5.14.
Figure 5.14 Tsukamoto-type Inference (Ross 2005)

As Tsukamoto method is restricted to monotonic output membership functions, it is not a general approach and must be used in specific situations (Ross 2005).

5.3.4 Defuzzification

The outputs from the inference engine (Mamdani type) are fuzzy and they need to be converted to crisp outputs before sending them to actuators to control the process. The conversion of a fuzzy quantity to a crisp value is called defuzzification. Some of the commonly used defuzzification methods are discussed here. Centroid method, Center of largest area method,
Height method, First of maxima method, Last of maxima method, Mean of maxima method are based on aggregated fuzzy output, i.e., all fuzzy outputs corresponding to different rules are aggregated using a union operator (max operator) to an aggregated fuzzy output before defuzzification. The Weighted average method and Center of sums method are based on individual output fuzzy sets (Ross 2005).

Centroid method or center of gravity method is the most commonly used defuzzification method as this method is very accurate and gives smooth output. In centroid defuzzification method, the defuzzified value is calculated as

\[ z^* = \frac{\int \mu_c(z) z \, dz}{\int \mu_c(z) \, dz} \]  

(5.7)

where \( z^* \) is the defuzzified value, \( z \) is the output variable, \( \int \mu_c(z) \) is the membership function of the aggregated fuzzy output. This method is computationally complex.

The center of largest area defuzzification can be used if the aggregate fuzzy set has at least two convex sub-regions (convex region is the region in which the membership values are strictly monotonically increasing or strictly monotonically decreasing or strictly monotonically increasing and then strictly monotonically decreasing with increasing values of points in the universe). Here, the centroid of the convex sub-region with the largest area is determined to obtain the defuzzified value.

\[ z^* = \frac{\int \mu_{cm}(z) z \, dz}{\int \mu_{cm}(z) \, dz} \]  

(5.8)

where \( cm \) represents the convex sub-region that has the largest area.
Max-membership method or height method is restricted to peaked aggregate membership function. This scheme is given by the mathematical expression

$$\mu_c(z^*) \geq \mu_c(z) \text{ for all } z \in Z$$

(5.9)

where $z^*$ is the defuzzified output.

First of maxima method determines the smallest value of the domain of the aggregate membership function having maximum membership degree in it. Similarly, last of maxima method determines the largest value of the domain of the aggregate membership function having maximum membership degree in it. Mean of maxima method determines the mean of smallest and largest values of the domain of the aggregate membership function having maximum membership degree in it.

Weighted average defuzzification scheme is the most commonly used one in fuzzy logic applications because of its computational efficiency. However, it has the disadvantage that it is limited to symmetrical membership functions. It is given by the mathematical expression.

$$z^* = \frac{\sum \mu_c(\bar{z}) \cdot z}{\sum \mu_c(\bar{z})}$$

(5.10)

where $\bar{z}$ is the centroid of each membership function corresponding to different rules and $\mu_c(\bar{z})$ is the membership value of $\bar{z}$ in that membership function.

Center of sums method is a computationally efficient method applicable to symmetrical or unsymmetrical membership functions. This scheme is given by
\[ Z^* = \frac{\int z \sum_{k=1}^{n} \mu_{C_k}(z) \, dz}{\int \sum_{k=1}^{n} \mu_{C_k}(z) \, dz} \]  \hspace{1cm} (5.11)

Center of sums method is similar to weighted average method as both methods are weighted methods. In center of sums method, weights are the areas of the membership functions, whereas the weights are membership values in weighted average method.

The performance of the defuzzification methods are measured using five criteria (Hellendoorn and Thomas 1993, Ross 2005). The first one is continuity which means that a small change in the input should not produce a large change in the output. The second criterion is disambiguity which means that the defuzzification method should always result in a unique defuzzified value. The center of largest area method does not satisfy this criterion as there is ambiguity in selecting a defuzzified value when the aggregate membership function has two or more convex sub-regions with the largest area. The third criterion is plausibility which means that the defuzzified value should lie approximately in the middle of the support region and should have high membership degree. The centroid method does not satisfy this criterion as the defuzzified value determined using centroid method may not have high membership degree in the aggregate membership function though the value lies in the middle of the support region. The fourth one is computational simplicity. The first of maxima method, last of maxima method, mean of maxima method and height method are computationally simpler than centroid method and weighted methods. The fifth criterion is weighting method. The weighted average method is computationally efficient than center of sums method (Hellendoorn and Thomas 1993, Ross 2005).
5.4 Summary

The basic principles and the components of Fuzzy logic control are discussed. Fuzzy logic can embed human knowledge into workable algorithms. Fuzzy logic controllers are suitable for the control of systems or processes whose exact mathematical models are not known. Fuzzy logic controllers can be designed based on approximate reasoning based on human experience.