CHAPTER 5

FETAL ECG EXTRACTION USING SURELET, SPLINELET AND ANFIS TRANSFORM

In this chapter, a new approach to signal de-noising, based on the image-domain minimization of an estimate of the mean squared error—Stein's unbiased risk estimate (SURELET) and Splinelet is presented. The attractive characteristics of SURELET and Splinelet is used to extract fetal ECG accurately with minimum error.

5.1 INTRODUCTION ABOUT SURELET AND SPLINELET

5.1.1 SURELET Transform

Unlike most existing de-noising algorithms, using the SURE makes it needless to hypothesize a statistical model for the noiseless image. A key point of the approach is that, although the (nonlinear) processing is performed in a transformed domain—typically, an undecimated discrete wavelet transform, but nonorthonormal transforms—the minimization performed in the image domain is also addressed. Indeed, when the transform is a “tight” frame (an undecimated wavelet transform using orthonormal filters), separate subband minimization yields substantially worse results. In order for this approach to be viable, another principle which can be added here is that the denoising process can be expressed as a linear combination of elementary denoising processes—linear expansion of thresholds (LET). Armed with the SURE and LET principles, a denoising algorithm merely amounts to solving
a linear system of equations which is obviously fast and efficient is showed. Quite remarkably, the very competitive results obtained by performing a simple threshold on the undecimated Haar wavelet coefficients show that the SURE-LET principle has a huge potential.

SURE is an unbiased statistical estimate of the mean squared error (MSE) between an original unknown signal and a processed version of its noisy observation. This estimate depends only on the saved data and does not require any prior assumption on the noise-free signal. The only statistical assumption is made on the noise: additive and Gaussian. Denoting by \( \hat{v} \), an estimate of the noise-free video \( v \), the global MSE is defined as

\[
MSE = \frac{1}{NT} \sum_{t=1}^{T} \sum_{n=1}^{N} e_t^T (\hat{v}_n - v_n)(\hat{v}_n - v_n)^T e_t
\]

where \( e_t^T x_n = \theta_t^j (y_n, p_n) \) is the nth pixel of the jth wavelet sub-band of the denoised frame \( t \). It is obtained by thresholding the \( n^{th} \) pixel of the \( j^{th} \) wavelet subband of the noisy frame \( t \), taking into account (some of) its neighboring frames. From now on, neglect the subband superscript “\( j \)” and the time frame indication “\( t \)” for the sake of clarity, when no ambiguities arise. Considering this multiframe processing \( \theta: \mathbb{R}^T \times \mathbb{R}^T \rightarrow \mathbb{R} \), the MSE of any wavelet subband \( j \) of any frame \( t \) can be estimated without bias by

\[
e = \frac{1}{N} \sum_{n=1}^{N} \left[ (\theta(y_n, p_n) - e_i^T y_n)^2 + 2e_i^T R \hat{\nu}_i \theta(y_n, p_n) - Ne_i^T R e_i \right]
\]

(5.2)
Here, $p^n$ denotes any random variables statistically independent of $y_n$. $\&_1$ stands for the gradient operator relatively to the first variable of the function i.e., $y_n$.

The thresholding function is specified by a linear combination of basic thresholding functions, a strategy that is coined LET, as

$$
\theta(y_n, p_n) = \left[ \begin{array}{c}
\theta_1(y_n, p_n) \\
\theta_2(y_n, p_n) \\
\vdots \\
\theta_K(y_n, p_n)
\end{array} \right]
$$

where $a$ and $\theta$ are both $K \times 1$ vectors. Each $\theta_k : \mathbb{R}^T \times \mathbb{R}^T \rightarrow \mathbb{R}^T$ is an arbitrary vector-valued thresholding.

The steps for the SURELET denoising are as follows:

Step 1: Obtain the noisy abdominal signal.

Step 2: Set the number of iterations.

Step 3: Compute the standard deviation of the noise.

Step 4: Compute the Surelet transform of the signal.

Step 5: Denoise the signal using the calculated parameters.

Step 6: Reconstruct to obtain the denoised signal.

5.1.2 SPLINES AND WAVELETS

Spline wavelets are extremely regular and usually symmetric or anti-symmetric. They can be designed to have compact support and to achieve optimal time-frequency localization. Fractional spline wavelets are the
shortest and most regular scaling functions of order L. Splines are the best for approximating smooth functions.

The construction of spline wavelets starts with the specification of the multiresolution function spaces (polynomial splines). Thus, spline wavelets can be characterized explicitly; this is in contrast with most other constructions where the scaling function is specified indirectly via a two-scale relation. The main advantage of an explicit construction is that one does not have to worry about the delicate issues of the convergence of the iterated filterbank. It also makes the study of filterbank much more transparent.

5.1.2.1 Polynomial Splines

A polynomial spline of degree n is made up of polynomial segments of degree n that are connected in a way that guarantees the continuity of the function and of its derivative up to order n-1. The joining points between the polynomial segments are called knots. In the context of the wavelet transform, the knots are equally spaced and typically positioned at the integers. A hierarchy of spline subspaces of degree n is defined as \( \{ V_i^n \}_{i \in \mathbb{Z}} \) where \( V_i^n \) is the subspace of \( L^2_{2^i} \)-functions that are (n-1) times continuously differentiable and are polynomials of degree n in each interval \([2^k, 2^{k+1})\), \( k \in \mathbb{Z} \). The spacing between the knot points \( 2^i \) is controlled by the scale index i. Clearly, a function \( f(x) \in V_{i_0}^n \) that is piecewise polynomial on each segment \([2^{i_0} k, 2^{i_0} (k+1)]\) is also included in any of the finer subspaces \( V_i^n \) with \( i \leq i_0 \). Thus, the inclusion property is

\[
L_2 \supseteq V_{-1}^n \supseteq V_0^n \supseteq V_1^n \supseteq \{0\} \tag{5.4}
\]

Any \( L_2 \)–function can be approximated by a spline as closely as one wishes by letting the knot spacing (or scale) go to zero \( (i \to -\infty) \). This means
that the above sequence of nested subspaces is dense in $L_2$ and therefore meets all the requirements for a multiresolution analysis of $L_2$.

Schoenberg’s representation of splines in terms of the B-spline basis functions is used to proceed. In order to satisfy the multiresolution inclusion property of any degree $n$, causal B-splines can be constructed from the $(n+1)$-fold convolution of the indicator function in the unit interval (causal B-spline of degree 0).

$$\phi^n(x) = \phi^0 * ... * \phi^0 (x) \quad (n+1)\text{times}$$

(5.5)

where

$$\phi^0 (x) = \begin{cases} 1, & 0 \leq x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(5.6)

The B-spline of degree $n$ satisfies the two-scale relation

$$\phi^n(x / 2) = \sqrt{2} \sum_{k \in \mathbb{Z}} h^n(k) \phi^n(x - k)$$

(5.7)

where $h^n (k)$ is the binomial filter of order $n+1$ whose transfer function is

$$h^n(k) \rightarrow H^n(z) = \sqrt{2} \left( \frac{1 + z^{-1}}{2} \right)^{n+1}$$

(5.8)

In 1946, Schoenberg proved that any polynomial spline of degree $n$ with knots at the integers could be represented as a linear combination of shifted B-splines. Thus, the basic spline space $V_o^n$ can also be specified as

$$V_o^n = \left\{ s_0(x) = \sum_{k \in \mathbb{Z}} c(k) \phi^n(x - k) \big| c \in l_2 \right\}$$

(5.9)
where the weights $c(k)$ are the so-called B-spline coefficients of the spline function $s_0(x)$. In addition, the B-splines $\{\varphi^n(x-k)\}_{k \in \mathbb{Z}}$ constitute a Riesz basis of $V_0^n$ in the sense that there exist two constants $A_n > 0$ and $B_n < +\infty$ such that

$$\forall c \in L_2, A_n \|c\|^2 \leq \left\| \sum_{k \in \mathbb{Z}} c(k) \varphi^n(x-k) \right\|_{L_2}^2 \leq B_n \|c\|^2$$

(5.10)

The lower inequality implies that the B-splines are linearly independent (i.e., $s_0(x) = 0 \Rightarrow c(k) = 0$). The upper inequality guarantees that $V_0^n \subset L_2$. Hence, any polynomial spline has a unique representation in terms of its B-spline coefficients $c(k)$. Schoenberg also proved that the B-splines are the shortest possible spline functions.

### 5.1.2.2 Spline Wavelets

Spline wavelet transform is produced whenever the synthesis functions ($\Psi(x)$ and $\Phi(x)$) are polynomial splines of degree $n$. This means that the synthesis wavelet can also be represented by its B-spline expansion.

$$\psi^n(x/2) = \sum_{k \in \mathbb{Z}} w(k) \varphi^n(x-k)$$

(5.11)

It is important to observe that the scaling function $\Phi(x) \in V_0^n$ is not necessarily the B-spline of degree $n$- unless $h(k)$ is precisely the binomial filter. This function is usually specified indirectly as the solution of the two-scale relation

$$\varphi(x/2) = \sqrt{2} \sum_{k \in \mathbb{Z}} h(k) \varphi(x-k)$$

(5.12)
where $h(k)$ is the corresponding (low pass) reconstruction filter. However in the spline case, there will always exist a sequence $p(k)$ such that

$$\varphi(x) = \sum_{k \in \mathbb{Z}} p(k) \varphi'(x-k)$$  \hspace{1cm} (5.13)

Such specific B-spline characterizations for various kinds of spline scaling functions (orthogonal, dual, or interpolating) can be found. Note that the sequence $p(k)$ defines an invertible convolution operator from $l_2$ into $l_2$ which performs the change from one coordinate system to the other (i.e., $\Phi$ to $\Phi^n$). The basic requirement for $\{\varphi(x-k)\}_{k \in \mathbb{Z}}$ to form a Riesz basis of $V^n_0$ is that there exist two constants $A_p > 0$ and $B_p < +\infty$ such that $A_p \leq |P(e^{i\omega})|^2 \leq B_p$ almost everywhere, where $P(e^{i\omega})$ denotes the Fourier transform of $p$.

The B-spline coefficients of the wavelet $\psi(x)$ is obtained as

$$w(k) = (p * g)(k) \quad \text{with} \quad W(z) = P(z)G(z)$$  \hspace{1cm} (5.14)

These spline wavelets are quite attractive because they are extremely regular. In fact, any spline wavelet of degree $n$ is $(n+1)$ times differentiable almost everywhere. It has Sobolev regularity index $S_{\text{max}} = n + 1/2$ meaning that all its functional derivatives up to $S_{\text{max}}$ are well defined in the $L_2$ sense.

A very important wavelet parameter is the order of the representation determined from the zero-properties of the refinement filter $h$. By definition, the order $L$ is the largest integer such that the transfer function can be factorized, where $Q(z)$ is the $z$-transform of a stable filter. Thus, splines have an order of approximation $L = n+1$ which is one more than the degree. This order property has some remarkable consequences such as the vanishing moments of the analysis wavelet, the ability of the scaling function
to reproduce polynomials of degree \( n=L-1 \), and the special eigen-structure of the two scale transition operator.

The steps for the Splinelet denoising are as follows:

Step 1: Obtain the noisy abdominal signal

Step 2: Decompose the signal into low frequency and high frequency bands using splinelet transform.

Step 3: Analyze the bands to identify the high frequency noise.

Step 4: Remove the noise by replacing those bands with null bands.

Step 5: Reconstruct to get the noiseless signal.

5.2 SIMULATION RESULTS

5.2.1 SURELET Denoising

The simulated abdominal signal is denoised using SURELET transform technique and the denoised signal is fed into ANFIS for FECG extraction. Similarly, SURELET post-processing is done by denoising the FECG extracted using the ANFIS. Upon comparison, FECG signal extraction using SURELET post-processing resulted in a more accurate estimation in the FECG signal extraction. The results are provided in Figure 5.1 and 5.2.
Figure 5.1 FECG signal extraction using SURELET pre-processing

Figure 5.2 FECG signal extraction using SURELET post-processing
5.2.2 Fractional Spline Denoising

Similar to the SURELET denoising technique, the fractional spline transform is used to remove the random noise in the abdominal signal by decomposing the signal into low frequency and high frequency bands. Now, the bands are analyzed to identify the high frequency noise and the noise will be removed by replacing those bands by null bands. Now, the signal is reconstructed to produce the denoised FECG signal. The results are provided in Figure 5.3 and 5.4.

![Figure 5.3 FECG signal extraction using Splinelet pre-processing](image)

Figure 5.3 FECG signal extraction using Splinelet pre-processing
The values of MSE and PSNR for SURELET transform pre-processing and post-processing and SPLINELET transform pre-processing and post-processing are compared and it is tabulated as shown in table 5.1.

### Table 5.1 MSE and PSNR comparison of SURELET and SPLINELET

<table>
<thead>
<tr>
<th>Method</th>
<th>MSE</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SURELET pre-processing</td>
<td>2.2627e-04</td>
<td>84.5846</td>
</tr>
<tr>
<td>SURELET post-processing</td>
<td>2.1542e-04</td>
<td>84.7979</td>
</tr>
<tr>
<td>Fractional spline pre-processing</td>
<td>3.1663e-004</td>
<td>83.1252</td>
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<tr>
<td>Fractional spline post-processing</td>
<td>2.1679e-004</td>
<td>84.7703</td>
</tr>
</tbody>
</table>
5.4 CONCLUSION

In this chapter, implementation of SURELET and Fractional spline wavelet along with ANFIS is presented and have been analyzed for the quality of extracted FECG. The parameters used to assess the quality of the FECG are peak signal to noise ratio (PSNR) and mean square error (MSE). The results of performance evaluation for SURELET pre-processing and post-processing, Splinelet pre-processing and post-processing are compared. Upon comparison, de-noising using SURELET post-processing achieved the best results and can be concluded as the better of all methods discussed so far.