CHAPTER - 6

CHAPTER- 6

STRUCTURAL ERROR SYNTHESIS OF SIX BAR FUNCTION GENERATOR

INTRODUCTION

The optimal dimensions of six bar chains (Watt and Stephenson) for same function and range is obtained. Using optimal dimensions of link calculated and angles constants as parameters, structural error is calculated. From the numerical examples taken it is concluded that structural error for function generation is significantly less for Stephenson chain compared to Watt chain.

Literature Review

Unfortunately, four-bar linkages are quite often incapable of generating a prescribed motion within given design constraints, example are flap of airplane wing, the hood linkage of car, ...etc. The next more complex linkage with one degree of freedom is six-bar mechanisms. These mechanisms often provide more compact attractive solutions with less limitation on design constraints. However, six bar linkages cannot be decomposed into dyads, and therefore, rectified synthesis using algebraic methods cannot be implemented directly. Soni et al. [47] considered the synthesis of six-bar. They used the displacement matrix developed by such and Radcliffe [48] to generate a multitude of equations and unknowns, which were solved using Marquadt's numerical technique. This method could solve upto 21 precision points for path points for path generation with an eight-bar linkage, but it was a very complex method. McLaran [42] worked on Watt - II, Stephenson - II and Stephenson - III mechanisms for Function generation. He developed an algebraic expression, which could be solved using Newton - Raphson technique. However, the solution diverges when initial guess is far from solution Therefore, this method was restricted Patwardhan and Soni [49] provided graphical methods to design a Watt-I six-bar linkage Their work depended on synthesis of four bar linkage for a prescribed coupler and synthesis of another four-bar linkage [50] used the inversion method to synthesize a Watt-I linkage for motion generation with prescribed timing at five precision positions. In his method, Burmester theory was used along the procedure outlined by Patwardhan and Soni, but a moving pivot was selected on floating link to be tracer point of four-bar as suggested by Rao, Erdman and Sandor [51]. Erdman [52], Erdman and Sandor and Chase [53] used the complex - number method to synthesize different type of six-bar linkages. In the present work Gaussian-elimination method is adopted to overcome the difficulty experienced by McLaran [42] solution. Detailed procedure is given in the following paragraphs. Good convergence is obtained.

Nomenclature

a = Length of link

b = auxiliary variable which is combination of linkage parameters

F = dimensionless function of linkage parameter

n = number of precision points used in given problem

r = dimensionless ratio of linkage dimensions

α, λ = Angle (constant) used as linkage dimensions

β, γ, δ = angles (variable) used to designate positions of links other than driver & follower

θ = Angular position of input crank

φ = Angular position of output crank

θ_i = value of input angle at i^{th} precision point

ϕ_i = value of output angle at i^{th} precision point

Mechanism to Be Compared

Only two basic types of pin-connected Six-link Kinematic chains provide mechanisms, which give motion of an output crank different from that of a four-bar linkage. These are Watt and Stephenson Kinematic chains. The first mechanism studied is an inversion of Watt chain, which consists of two four bar linkage in series. It is shown in Fig 1. The second mechanism obtained is inversion of Stephenson chain, shown in Fig. 2. One important characteristic of these mechanisms is that the links of each form two independent complex position equations, which completely describe the relative positions of the links. The technique used here for synthesis of these linkages requires Combination of the two complex equations in such a way to eliminate the unwanted angles. The resulting single Scalar equation then relates only the link dimensions and input and output angles and such equations are called position equations of Synthesis as in [1].

$(b_3b_5-b_2b_6)^2 + (b_1b_6-b_3b_4)^2 - (b_1b_5-b_2b_4)^2 = 0$ where, (1A)

Where, for watt chain (1 B)

$b_1 = \cos\theta - r_1,$ and $r_1 = a_1 / a_2,$

$b_2 = \sin\theta,$ $r_2 = a_1 / a_4,$

$b_3 = r_3 - r_2 \cos\theta,$ $r_3 = (a_1^2 + a_2^2 - a_3^2 + a_4^2) / 2 a_2 a_4,$

$b_4 = \cos(\phi+\alpha) + r_4 \cos\alpha,$ $r_4 = a_8 / a_7,$

$b_5 = \sin(\phi+\alpha) + r_4 \cos\alpha,$ $r_5 = a_8 / a_5,$

$b_6 = r_6 + r_5 \cos\phi,$ $r_6 = (a_5^2 - a_6^2 + a_7^2 + a_8^2) / 2a_5 a_7,$

Similaraly for Stephenson chain

(1C)

$b_1 = r_1 - \cos\theta,$

$b_2 = - \sin\theta,$

$b_3 = r_3 - r_2 \cos\theta,$

$b_4 = r_4 \{r_6 [\cos(\phi-\alpha) + r_5 \cos (\lambda - \alpha)] - \cos (\theta-\alpha)\},$

$b_5 = r_4 \{r_6 [\sin(\phi-\alpha) + r_5 \sin (\lambda - \alpha)] - \sin (\theta-\alpha)\},$

$b_6 = r_7 - r_5 \cos(\theta-\lambda) - \cos (\theta-\phi) + r_5 r_6 \cos (\theta-\lambda),$

and

$r_1 = a_1 / a_2,$

$r_2 = a_1 / a_3,$

$r_3 = (a_1^2 + a_2^2 + a_3^2 - a_4^2) / 2 a_2 a_3,$

$r_4 = a_5 / a_7,$

$r_5 = a_8 / a_7,$

$r_6 = a_7 / a_2,$

$r_7 = [a_2^2 + a_5^2 - a_6^2 + a_7^2 + a_8^2] / 2 a_2 a_7,$

Procedure

In designing a mechanism to provide n - precision points, the values are assigned to all but n of its dimensions. The position equation, which is a single scalar equations relating only the link dimensions and input output angles is then written n times, once for each of the precision positions, with each set of precision points angles being used in one of the equations. This results in n non-linear algebraic equations in n unspecified linkage dimensions. Solution of (B) is accomplished numerically using the Gauss - elimination technique. Once the

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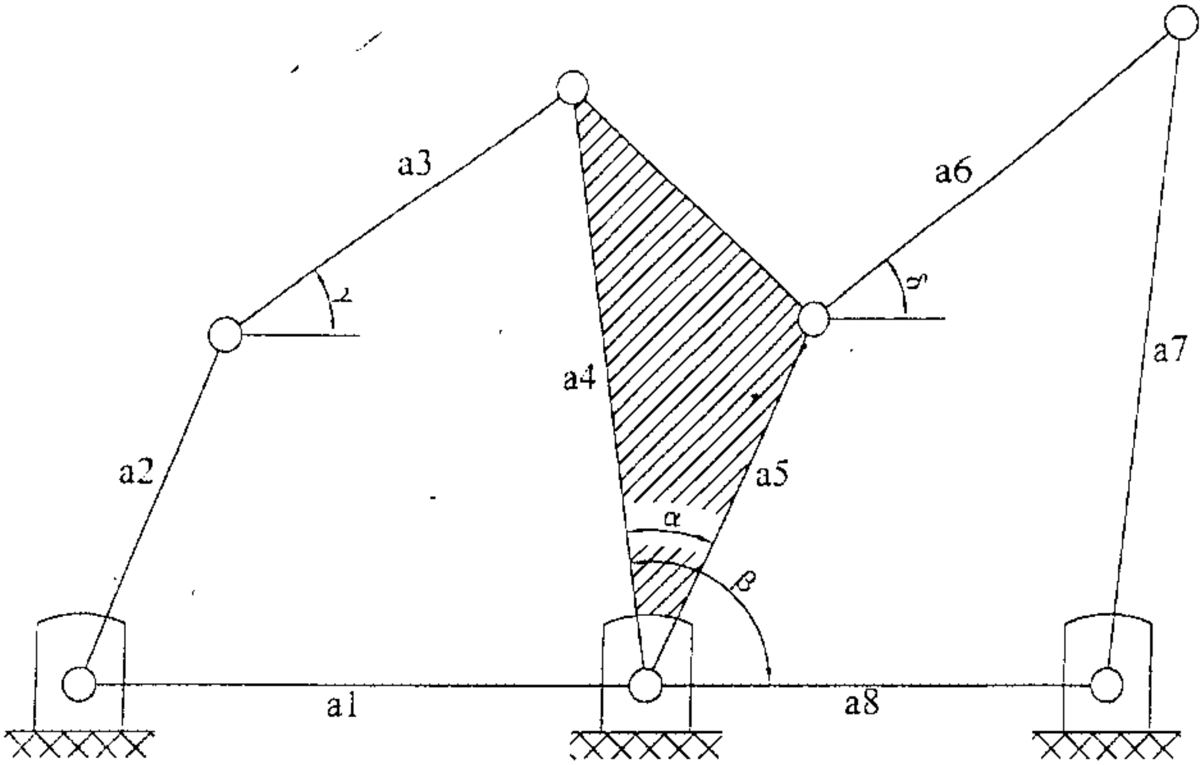
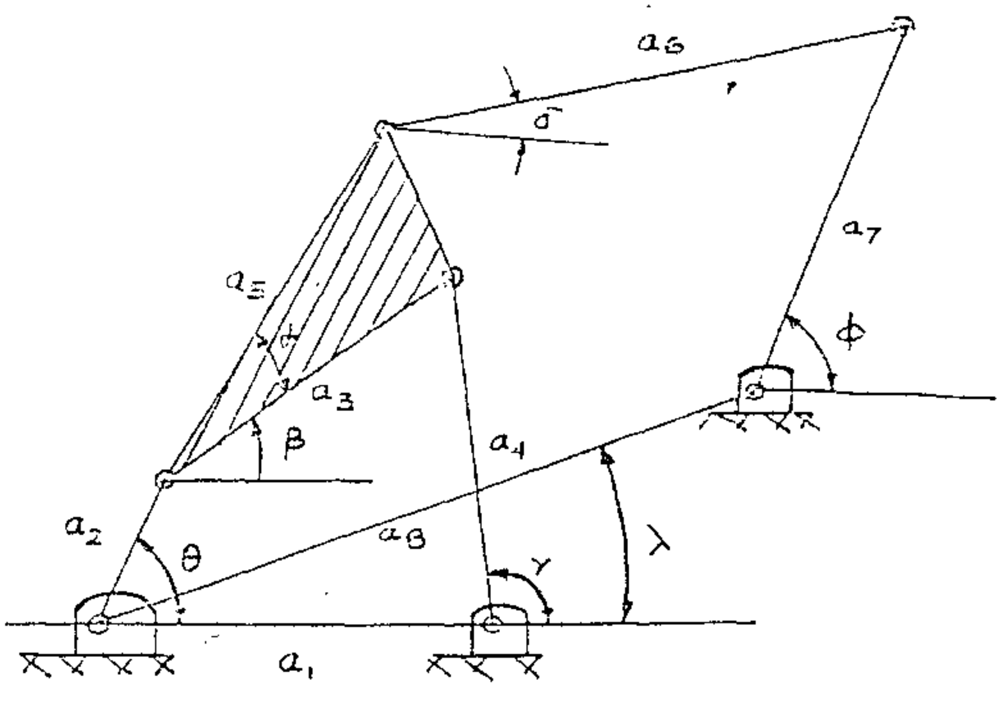


Fig-1

WATT LINKAGE



Stephenson linkage

Fig. 2

correction vector δ is known, the next approximation to unknown vector r is found from the recursion relation $r_{k+1} = r_k + \delta k / 5$. The process of forming partial derivatives solving B, and calculating new value of r is repeated until, for all values of r_i,

if $|r_i| \geq 10$ or,
$|\delta_i / r_i| \leq 10$

Hence optimized value of $r_1, \ldots\ldots, r_n$ is known, these are used to calculate the actual link through the relation, (1B). Using the calculated link dimensions, given alpha and θ, other than the precision point angle, ϕ is calculated using the relation (2C), which is solved using modified Newton-Raphson technique and roots are obtained. In precision point approach, the error is assumed to be zero at precision point but there is considerable amount of error in between the precision point. Cumulative error can be expressed as function of θ_s and ϕ_s. $E = f(\theta_s, \phi_s)$. Angle

ϕ corresponding to angle θ at intermediate points other than precision points can be obtained from position equation of Watt and Stephenson chain. The ϕ_s so obtained are substituted back with their corresponding θ's and optimized r in the position equation respectively and cumulative error is obtained.

Solution of n position equation

Following McLaran [42] position equations for the two mechanisms under consideration Fig (1,2) can be written in the same form. For simplicity in writing, this position equation will be written as, $F(r_1, r_2, r_3, \ldots\ldots, r_n) = 0$

The synthesis of mechanism to have n precision point is therefore, reduced to the problem of Solving n-simultaneous equations. As the equations are of highly non-linear character, a complete solution has not been possible. The procedure is as follows

Assume a set of n equations in the form,

$F_1\ (r_1, r_2, r_3, \ldots r_n) = 0$

$F_2\ (r_1, r_2, r_3, \ldots r_n) = 0$

$F_3\ (r_1, r_2, r_3, \ldots r_n) = 0$

$F_n\ (r_1, r_2, r_3, \ldots r_n) = 0$

Each of these equations can be expanded in the Taylor's series form as

$$F_i(r+\delta) = F_i(r) + \frac{\delta F_i}{\delta r_1}\delta_1 + \frac{\delta F_i}{\delta r_2}\delta_2 + \ldots\ldots + \frac{\delta F_n}{\delta r_n}\delta_n \ldots\ldots (A)$$

Where at a solution $F_i\ (r+\delta) \to 0$ $\quad i=1, 2, \ldots n$

Equation (A) can be arranged as a set of linear equation in matrix form.

$$\begin{bmatrix} \frac{\partial F_1}{\partial r_1} \cdots \frac{\partial F_1}{\partial r_2} \cdots \frac{\partial F_1}{\partial r_n} \cdots \\ \frac{\partial F_2}{\partial r_1} \cdots \frac{\partial F_2}{\partial r_2} \cdots \frac{\partial F_2}{\partial r_n} \\ . \\ . \\ . \\ . \\ . \\ \frac{\partial F_n}{\partial r_1} \cdots \frac{\partial F_n}{\partial r_2} \cdots \frac{\partial F_n}{\partial r_n} \end{bmatrix} \cdots \begin{bmatrix} \delta_1 \\ \delta_2 \\ . \\ . \\ . \\ . \\ \delta_n \end{bmatrix} = \cdots \begin{bmatrix} -F_1 \\ -F_2 \\ \\ \\ \\ \\ -F_n \end{bmatrix} \ldots\ldots (B)$$

The solution of (B) is accomplished numerically using the Gauss - elimination technique.

Results

Table-1 Dimensions and structural error of solutions to example problem: Generation of function $y = x^2$ for $0 \le x \le 1$ with Watt and Stephenson mechanisms for 90 degree rotation of input and output crank

Watt Mechanism

r (initial)	r (calculated)	θ (deg)	ϕ (precison point) (deg)	ϕ (calculated) (deg)
1.091675	1.009275	0.000000	0.000000	0.000000
0.177780	-0.815452	11.610000	15.480000	15.479998
1.099000	-0.993775	31.770000	38.670000	38.670000
1.277010	-1.011956	55.080000	61.160000	61.160000
1.185089	0.764725	75.240000	77.870000	77.870000
0.941000	-0.994362	86.930000	86.670000	86.670000

Final values of b for optimized value of r

-0.009275	0.000000	-0.178323	-0.011956	0.000000	-0.229637
-0.029735	0.201249	-0.195007	-0.048232	0.266902	-0.257378
-0.159106	0.526511	-0.300504	-0.231198	0.624834	-0.397297
-0.436843	0.819952	-0.526984	-0.529590	0.875970	-0.625485
-0.754504	0.967001	-0.786022	-0.801825	0.977673	-0.833670
-0.955719	0.998565	-0.950103	-0.953869	0.998312	-0.949942

Values of a

a_1	a_2	a_4	a_3	a_7	a_8	a_5	a_6
1.000000	0.990810	1.034692	-1.226314	1.307657	1.056709	-0.988186	1.000000

Structural error = 0.000005

Stephenson Mechanism

r (initial)	r (calculated)	θ (deg)	φ (precision point) (deg)	φ (calculated) (deg)
1.618983	0.300232	0.000000	1.617700	1.617876
0.917778	-0.344280	8.689000	11.820000	11.819833
0.431500	-1.022880	24.346900	30.748200	30.748143
2.074251	0.311224	43.871760	51.034000	51.033948
1.851501	0.402151	63.396522	68.577000	68.576932
2.218311	1.988960	79.050000	81.108000	81.107905
3.673220	1.137587	87.743511	87.574700	87.574579

Final values of b for optimized value of r

-0.699768	-0.000000	-0.678600	0.556478	0.017475	0.535378
-0.688291	-0.151071	-0.682552	0.547171	0.079780	0.524446
-0.610834	-0.412260	-0.709219	0.497384	0.188175	0.464855
-0.420661	-0.693047	-0.774691	0.413849	0.265601	0.358482
-0.147581	-0.894127	-0.868707	0.335661	0.297970	0.253733
0.110280	-0.981793	-0.957483	0.285501	0.306015	0.185479
0.260859	-0.999225	-1.009325	0.262877	0.307475	0.155605

Values of a

a_1	a_2	a_4	a_3	a_7	a_8	a_5	a_6
1.000000	3.330758	1.146713	4.154654	3.256255	1.000000	1.791071	1.674623

Structural error = .000001

Table-2 Dimensions and structural error of solutions to example problem. Generation of function y= log x for 1 < x < 2 with watt and Stephenson mechanisms for 90 degree rotation of input and output crank

Watt Mechanism

r (initial)	r (calculated)	θ (deg)	φ (precision point) (deg)	φ (calculated) (deg)
1.091675	0.872203	0.000000	0.000000	0.000000
0.177791	-0.370944	11.640000	1.970000	1.970000
1.099000	-0.519746	31.810000	12.750000	12.750000
1.277010	-0.959329	55.110000	36.880000	36.880000
1.185089	0.587063	75.280000	67.850000	67.850000
0.941000	-0.634414	86.920000	90.000000	90.000000

Final values of b for optimized value of r

0.127797	0.000000	-0.148801	0.040671	0.000000	-0.047351
0.107232	0.201762	-0.156430	0.040080	0.034376	-0.047698
-0.022402	0.527104	-0.204517	0.016014	0.220697	-0.061826
-0.300200	0.820252	-0.307565	-0.159435	0.600141	-0.164826
-0.618107	0.967179	-0.425491	-0.582296	0.926200	-0.413072
-0.818472	0.998555	-0.499815	-0.959329	1.000000	-0.634414

Values of a

a_1	a_2	a_3	a_5	a_8	a_6	a_4	a_7
1.000000	1 146523	2.523704	-2 695821	1.703395	1 653844	-1 042395	1.000000

Structural error = 0.000013

Stephenson Mechanism

r (initial)	r (calculated)	θ (deg)	φ (precision point) (deg)	φ (calculated) (deg)
1 608983	1.043483	0.000000	0.000000	0.000000
0 917778	0.994549	8.680000	1.083000	1.083000
0.431551	0.913385	24.340000	7.380000	7.379994
2.074025	0.875254	43.870000	23.060000	23.059990
1 851500	-0.032474	63.390000	47.430000	47.429986
2.218310	1.071992	79.050000	73.100000	73.099983
2.673240	0.941595	87.740000	89.980000	89.980005

Final values of b for optimized value of r

0.043483	-0.000000	-0.081164	0.032542	0.000000	-.060743
0.054936	-0.150916	-0.069773	0.042399	-0.114356	-0.052331
0.132367	-0.412151	0.007236	0.102566	-0.240217	-0.019850
0.322568	-0.693024	0.196401	0.201842	-0.239059	-0.001788
0.595567	-0.894076	0.467912	0.212219	-0.091557	-0.028863
0.853530	-0.981793	0.724468	0.076030	0.038427	-0.056969
1.004048	-0.999222	0.874166	-0.064657	0.063692	-0.056373

Values of a

a_1	a_2	a_3	a_5	a_8	a_6	a_4	a_7
1 000000	0.958329	0.953105	1.472253	1.049206	1 000000	0.000001	0.000000

Structural error = 0.00000

On comparing values of maximum structural error it is observed that it is significantly less for Stephenson chain compared to the Watt chain.

Discussion

The dimensional synthesis of Watt and Stephenson chain is attempted for the same function and range The procedure used is a modification of numerical technique used by Mc Laran [42] It is observed that structural error for function generation is significantly less for Stephenson chain compared to the Watt chain The convergence is obtained for large number of examples taken up by the author even if initial approximation is wilder. The iterations are no doubt large in number but they converge to optimum value, which is not the case with other methods reported

Conclusion

In the summary the thesis include synthesis of planar kinematic chains up to 13-link, 2-d.o.f. Using the information of generated chains, type of freedom in multi- degree of freedom chains, inversions, best of inversions for path and function generation and actuator pairs have been decided up to 12-link, 1-d.o.f chains. Rating of chains using symmetry as criterion is formulated and solving numerical example strengthens results.

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Appendix

(2 A)

Partial derivative co-efficient; (Watt chain) to be used in ----- (A).

$$\frac{\partial F_n}{\partial r_1} = [2\,(b_{1n}b_{6n} - b_{3n}\,b_{4n})\,(b_{6n}) - 2\,(b_{1n}b_{5n} - b_{2n}\,b_{4n})\,(b_{5n})] \times -1$$

$$\frac{\partial F_n}{\partial r_2} = [2\,(b_{3n}b_{5n} - b_{2n}\,b_{6n})\,(b_{5n}) - 2\,(b_{1n}b_{6n} - b_{3n}\,b_{4n})\,(-b_{4n})] - \cos\theta$$

$$\frac{\partial F_n}{\partial r_3} = [2\,(b_{3n}b_{5n} - b_{2n}\,b_{6n})\,(b_{5n}) + 2\,(b_{1n}b_{6n} - b_{3n}\,b_{4n})\,(-b_{4n})] \times 1$$

$$\frac{\partial F_n}{\partial r_4} = [2\,(b_{1n}b_{6n} - b_{3n}\,b_{4n})\,(-b_{3n}) - 2\,(b_{1n}b_{5n} - b_{2n}\,b_{4n})\,(-b_{2n})] - \cos\alpha + [\,2\,(b_{3n}b_{5n} - b_{2n}\,b_{6n})\,(b_{3n}) - 2\,(b_{1n}b_{5n} - b_{2n}\,b_{4n})\,(b_{1n})]\sin\alpha$$

$$\frac{\partial F_n}{\partial r_5} = [2\,(b_{3n}b_{5n} - b_{2n}\,b_{6n})\,(-b_{2n}) + 2\,(b_{1n}b_{6n} - b_{3n}\,b_{4n})\,(b_{1n})]\cos\phi$$

$$\frac{\partial F_n}{\partial r_6} = [2\,(b_{3n}b_{5n} - b_{2n}\,b_{6n})\,(-b_{2n}) + 2\,(b_{1n}b_{6n} - b_{3n}\,b_{4n})\,(b_{1n})] \times 1$$

(2 B) Values of Function (Watt chain)

$F_n = (b_{3n}b_{5n} - b_{2n} b_{6n})^2 + (b_{1n}b_{6n} - b_{3n} b_{4n})^2 - (b_{1n}b_{5n} - b_{2n} b_{4n})^2 = 0$

Where $b_{1n} = \cos \theta_n - r_1$ and $b_{1,1} = \cos \theta_1 - r_1$,

$b_{2n} = \sin \theta_n$, $b_{2,1} = \sin \theta_1$,

$b_{3n} = r_3 - r_2 \cos \theta_n$, $b_{3,1} = r_3 - r_2 \cos \theta_1$,

$b_{4n} = \cos (\phi_n + \alpha) + r_4 \cos\alpha$, $b_{4,1} = \cos (\phi_1 + \alpha) + r_4 \cos\alpha$,

$b_{5n} = \sin (\phi_n + \alpha) + r_4 \sin \alpha$, $b_{5,1} = \sin (\phi_1 + \alpha) + r_4 \sin\alpha$,

$b_{6n} = r_6 + r_5 \cos \phi_n$, $b_{6,1} = r_6 + r_5 \cos \phi_1$,

(2 C)

ϕ Values (Watt chain)

$x = \cos \phi$

$x^2 (-0.5 \cos 2 \alpha (b_3^2 - b_1^2) + 0.5 \cos 2 \alpha (b_3^2 - b_2^2) + r_5^2 (b_1^2 - b_2^2) - b_3 b_2 \sin\alpha$

$- 2 b_1 b_2 r_5 \cos \alpha + \sin \alpha \cos \alpha (2 b_1 b_2)) + (1-x^2) (0.5 \cos 2 \alpha (b_3^2 - b_1^2)$

$- 0.5 \cos 2 \alpha (b_3^2 - b_2^2) - \sin \alpha \cos \alpha (2 b_1 b_2)) + x (2r_4 \sin^2 \alpha (b_3^2 - b_1^2)$

$+ 2r_4 \cos^2 \alpha (b_3^2 - b_2^2) + 2r_5r_6 (b_1^2 - b_2^2) - 2b_3r_6b_2 \sin \alpha - 2b_3 r_6b \cos \alpha$

$- 2b_3r_5b_2r_4 \sin \alpha - 2b_3 r_5b_1r_4 \cos \alpha + 2r_4 \sin \alpha \cos \alpha (2b_1b_2))$

$+ \sqrt{1-x^2} (2r_4 \sin \alpha \cos \alpha (b_3^2 - b_1^2) - 2r_4 \sin \alpha \cos \alpha(b_3^2 - b_2^2)- 2b_3 r_6b_2 \cos \alpha$

$+ 2b_3r_6b_1 \sin \alpha - (r_4 \sin^2 \alpha - r_4 \cos^2 \alpha) (2b_1b_2)) +x \sqrt{} (1-x2 (\sin 2\alpha$

$(b_3^2 - b_1^2) - \sin 2\alpha (b_3^2 - b_2^2) - 2b_3 r_5b_2 \cos \alpha + 2b_3r_5b_1 \sin \alpha - 2b_1b_2$

$(\cos^2 \alpha - \sin^2 \alpha)) + [0.5 + r_4^2 \sin^2 \alpha] (b_3^2 - b_1^2) + (0.5 + r4^2 \cos^2 \alpha)$

$(b_3^2 - b_2^2) + r_6^2 (b_1^2 - b_2^2) - 2b_3r_6b_2 r_4 \sin \alpha - 2b_3 r_6b_1 \cos \alpha$

$+ (r_4^2 \sin \alpha \cos \alpha 2b_1b_2) = 0$