CHAPTER - 4

CHAPTER-4

ANALYSIS OF SIMPLE JOINTED PLANAR KINEMATIC CHAINS: PART-2

Introduction

The primary Hamming number technique with some additional constraint is used to reveal the type of freedom possessed by a multi-d.o.f. kinematic chain A computerized methodology is developed based on [35] for the same.

Further, to select the frame and best actuator a matrix called the transmission matrix is introduced [36,37] and the results obtained from this are correlated with Hamming values of the links based on link-adjacency matrix [36,37]. A computerized methodology is formulated for the same. The actuator or input links are likened to self loops in graph theory as they are understood to receive or transmit motion to themselves i.e. drive motors are considered integral part of actuator link.

Literature Review

Methods have already been reported to isolate non-isomorphic kinematic chains out of all possible combinations for a specified number of links and degree of freedom, chains with distinct structures can be expected to possess different characteristics, which are not quite obvious. The designer must have some idea about the expected behavior of the chains at least in comparative sense, so that he can pick up the best chain and select the frame and actuator links in order to obtain best performance. The work done in this direction, if any, is scant. Previous computerized works reported are [39,40].

METHOD

In a multi-degree-of-freedom (multi-d o f.) chain it is necessary to know the type of freedom in order to select the actuator (input) links A multi-d.o f. kinematic chain can possess one of the following three types of freedom: (I) full; (ii) partial; and (iii) fractionated A computer methodology is formulated using Hamming matrices, which are usually used to test isomorphism to reveal type of freedom [35] as follows,

Absence of elements $h_{ij} < (C_i + C_j - 2)$ in the H matrix ensures that only loops with more than four links exist in the chain. In addition, let the number of elements of the value $h_{ij} < (C_i + C_j)$ in the i^{th} row of H matrix be p: then

(a) $P - c_i = 0$ indicates that the i^{th} link participates in 5-bar distinct loops.

(b) $p - c_i = 1$ indicates that the i^{th} link participates in 6-bar distinct loops.

(c) $p - c_i = 2$ indicates that the i^{th} link participates in 7-bar distinct loops,

and so on.

1.Full-freedom: Two-d.o.f. Chains will have full-freedom if all the links participate in 5-bar loops, i.e. for every row of H matrix, $(p - c) > 0$. Three-d.o.f. Chains will have full-freedom if for every row of the H matrix, $(p-c)>1$. However, the possibility of the presence of single-d.o.f. Sub-chain with all 5-bar loops, such as fig.-1, should be checked.

2.Partial freedom: The presence of elements, $h_{ij} < (c_i + c_j - 2)$ indicates partial freedom. Also chains with seeming full freedom may consist of sub-chains with lower d.o.f. It is easy to check for low d o.f Sub-chain as it demands removal of some links from the original chain,

Chapter - 4

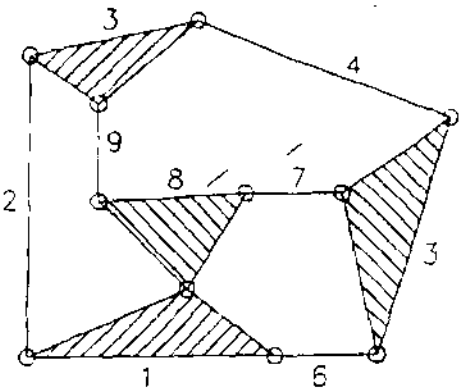

Fig-1

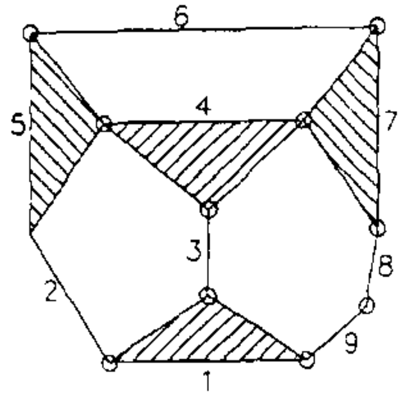

Fig-2

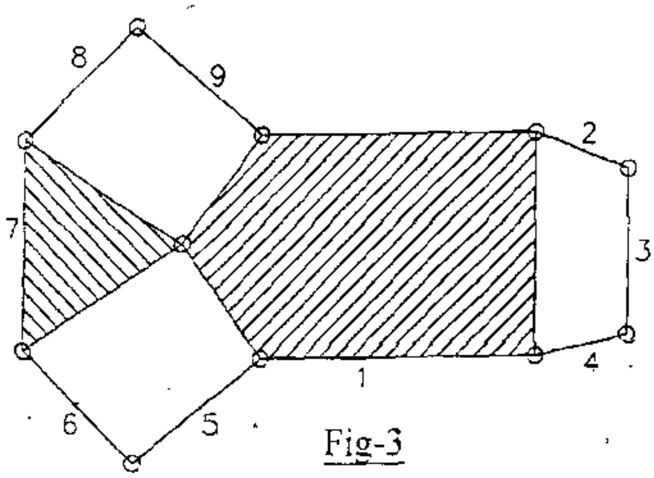

Fig-3

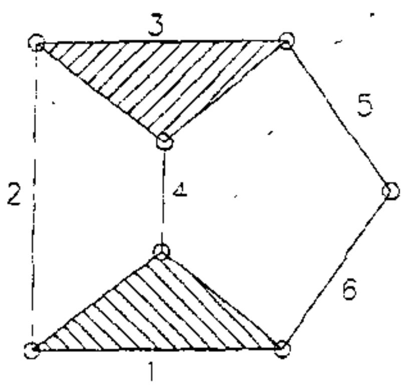

Fig-4

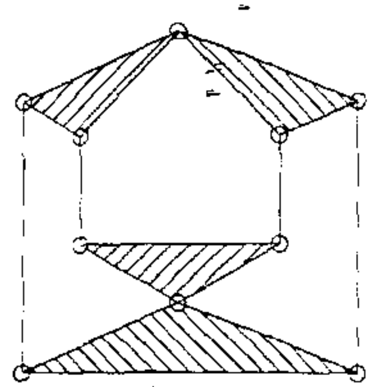

Fig-5

which should satisfy the requirements of a lower d.o.f. chain. For example, if an 8-link and 14-joint 2-d.o.f. chain leaves behind on removal of three links and four joints, as 8-link, and 10-joint chain which conforms to a single d.o.f. then the original chain has partial freedom. But the removal of three links with four joints is possible only if the original chain consists of a string of three binary links. No other combination of links permits it and the presence of three binary links in series (string) can easily be checked from the H matrix.

3.Fractionated freedom

(a).Presence of a link i with connectivity c_i > 1/2 (n – f + 1) ensures fractionated freedom, Where N is the total number of links in a chain and F is the d.o.f. of the chain.

(b).If connectivity of all links is smaller than 1/2 (n – f + 1) fractionated freedom cannot exist.

(c).Presence of a single link with connectivity equal to 1 / 2 (n – f + 1) will result in fractionated freedom if both ends of a string of binary links are connected to the link of highest connectivity.

A computerized methodology is formulated taking these in considerations.

Illustration

For example a nine-link 2-d.o.f. chain (Fig.-1) has full freedom because for every inversion two driving links can be selected arbitrarily ensuring the mobility of the chain, the essential reason being that all the distinct loops are five-bar or 2-d.o.f. loops and there is no sub-chain with single d.o.f. The Hamming matrix of the chain (Fig.-1) is given below:

$$H = \begin{bmatrix} 0 & 5 & 4 & 5 & 4 & 5 & 3 & 6 & 3 \\ 5 & 0 & 5 & 2 & 5 & 2 & 4 & 3 & 2 \\ 4 & 5 & 0 & 5 & 4 & 5 & 5 & 4 & 5 \\ 5 & 2 & 5 & 0 & 5 & 2 & 2 & 5 & 2 \\ 4 & 5 & 4 & 5 & 0 & 5 & 5 & 4 & 5 \\ 5 & 2 & 5 & 2 & 5 & 0 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 & 5 & 2 & 0 & 5 & 2 \\ 6 & 3 & 4 & 5 & 4 & 3 & 5 & 0 & 5 \\ 3 & 2 & 5 & 2 & 5 & 4 & 2 & 5 & 0 \end{bmatrix}$$

Now consider a nine-link 2 d.o.f. Chain with partial freedom (fig - 2) This chain contains a four bar loop consisting of links 4, 5, 6 and 7. It is obvious, for example, when link 4 is fixed links and 7 cannot be taken as independent input links. On the other hand when link 1 is fixed, links 2 and 3 can be taken as driving links. Thus it is clear that there is a restriction on the selection of independent input links. The Hamming matrix for this chain is given below

H =

0	5	5	4	4	5	6	3	5
5	0	2	3	5	2	5	4	2
5	2	0	5	3	4	3	4	2
4	3	5	0	6	1	6	3	5
4	5	3	6	0	5	2	5	5
5	2	4	1	5	0	5	2	4
6	5	3	6	2	5	0	5	3
3	4	4	3	5	2	5	0	4
5	2	2	5	5	4	3	4	0

Lastly, let us consider the chain (Fig.-3) which shows a nine-link with fractionated freedom i.e. the link of the highest connectivity can be cut into two pieces so that two separate chains are created. The d.o.f. of each separated chain is such that their combined d.o.f. is equal to that of the original chain. The H matrix for this chain is given below:

H =

0	7	3	7	7	3	8	3	7
7	0	4	0	2	4	3	4	2
3	4	0	4	4	4	5	4	4
7	0	4	0	2	4	3	4	2
7	2	4	2	0	4	1	4	2
3	4	4	4	4	0	5	2	4
8	3	5	3	1	5	0	5	1
3	4	4	4	4	2	5	0	4
7	2	4	2	2	4	1	4	0

(a) It may, however, be noted that some of higher d.o.f. kinematic chains consisting of distinct loops of full-freedom may contain a sub-chain with a lower degree-of-freedom, such as in Fig.-2 and hence will have partial freedom.

ACTUATOR LOCATION

Selection of the input link or the link to be connected to actuator is important from the point of view of transmission and resolution (control). This can be accomplished again by writing the Hamming matrix but in writing the matrix the effect of the actuator must be introduced in the transmission matrix [36,37]. The driver or input link receiving motion from the actuator can be considered to connect to itself or receive or transfer motion to itself and this can be mathematically represented by replacing the leading diagonal element 0 by 1 of the corresponding link. From the transmission matrix the Hamming values need to be determined all the times and best actuator link (S) can be determined as in [36,37], Out of all possible input links for coupling the actuator, the link with less Hamming values based on transmission matrix [36,37] is the best.

ILLUSTRATION

For example consider the inversion when link 5 is fixed and link 3 is input link in Fig.

$$T = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$H = \begin{bmatrix} 0 & 5 & 2 & 5 & 3 & 4 \\ 5 & 0 & 3 & 0 & 2 & 1 \\ 2 & 3 & 0 & 3 & 3 & 4 \\ 5 & 0 & 3 & 0 & 2 & 1 \\ 3 & 2 & 3 & 2 & 0 & 1 \\ 4 & 1 & 4 & 1 & 1 & 0 \end{bmatrix}$$

Let us now consider the same inversion with link 6 as input link,

$$T = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$H = \begin{bmatrix} 0 & 5 & 1 & 5 & 3 & 3 \\ 5 & 0 & 4 & 0 & 2 & 2 \\ 1 & 4 & 0 & 4 & 2 & 4 \\ 5 & 0 & 4 & 0 & 2 & 2 \\ 3 & 2 & 2 & 2 & 0 & 2 \\ 3 & 2 & 4 & 2 & 2 & 0 \end{bmatrix}$$

It may be noted that the Hamming value of the inversion, Fig.-4 when link 6 is coupled to actuator is higher than the value when link 3 is coupled. The superiority of link 6 as actuator can also be visualized by working out the flow paths For the actuator link to be better the flow paths should be lesser or the flow should be more uniformly distributed In this case actuator 6 is better because of the uniform distributed of flow paths

As a second example consider the nine-link chain, no.1, Fig.-6. The best frame for function generation can be found out to be link 4 or 9. Flow paths are respectively 135 and 128 when (a) link 4 is fixed and 3 and 9 are the actuator links; (b) link 9 is fixed and link 2 and 7 are inputs. Superiority of the latter combination is due to less number of flow paths.

Computer Program

The computer methodology uses matrix notation for mathematical representation of chains. The entire algorithm is based on manipulation of these matrices. Further, number 0 and 1 are particularly suited for storage and manipulation on a computer. The algorithm is developed in C language. The input to the algorithm is kinematic chains of particular category.

Application

The computerized methodology formulated is implemented up to 11-link, 2-d.o.f chains.

RESULTS

The computerized methodology formulated is implemented for complete analysis of simple jointed planar kinematic chains up to 12-links, 2-d.o.f.

Table for Type of Freedom obtained

Category of SJKC	Distinct chain	Total Freedom	Partial Freedom	Fractionated Freedom
7-Link, 2-d.o.f	4	1	2	1
9-Link, 2-d.o.f	40	6	29	5
11-Link, 2-d.o.f	839	98	662	79

CPU time for complete analysis of chain up to 12-link, 1-d.o.f 5min 4 seconds

SJKC: Simple jointed planar kinematic chain

CONCLUSION

The most important aspect of analysis is orderly classification of chains. The primary Hamming number technique with slight additional computation is used to reveal type of freedom possessed by multi-degree-of-freedom chains. It is necessary to know the type of freedom in order to select actuator (input link) which is obtained by slight modification in the original Hamming matrix. The designer must have some idea about expected behavior of chain at least in a comparative sense so that he can pick the best chain, select the frame and actuator links in order to obtain best result.

References

39 N.I.Manolescu, Revue Romaine Sc Tech. Ser Mech.Appl.9, 1263-1313 (1964)

40. T.S.Mruhyanjaya, Mech. Mach. Theory, 14,221-231 (1979)

Appendix 3

As an application of computer methodology developed complete analysis of 9-link 2-d.o.f. is presented below.

TYPE OF FREEDOM AND ACTUATORS

9 LINKS,2-d.o.f simple jointed kinematic chains

1. 2nd HAM=2976 : 1,4 1,5 1,7 1,8 2,3 2,6 2,7 2,9 3,4 5,6 8,9 Total

Best Actuator for 1 Inversion of 1 link is 2

2. 2nd HAM=3492 : 1,4 1,5 1,7 1,8 2,3 2,6 2,7 3,4 3,9 5,6 8,9 Total

Best Actuator for 1 Inversion of 1 link is 2

3. 2nd HAM=3508 : 1,4 1,5 1,7 1,8 2,3 2,6 2,7 3,4 4,9 5,6 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 2

4. 2nd HAM=3492 : 1,4 1,5 1,7 1,8 2,3 2,6 2,7 3,4 5,6 7,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 2

5. 2nd HAM=3808 : 1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 4,9 5,6 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

6. 2nd HAM=3880 : 1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 5,6 5,9 8,9 Total

Best Actuator for 1 Inversion of 1 link is 3

7 2nd HAM=3640 1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 5,6 6,9 8,9 Total

Best Actuator for 1 Inversion of 1 link is 2

8 2nd HAM=3696 . 1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 5,6 7,9 8,9 Total

Best Actuator for 1 Inversion of 1 link is 2

9 2nd HAM=3056 · 1,2 1,5 1,7 1,8 2,3 2,6 2,9 3,4 4,7 5,6 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 2

10. 2nd HAM=3444 : 1,2 1,5 1,7 1,8 2,3 2,6 3,4 4,7 5,6 5,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 2

11. 2nd HAM=3424 · 1,2 1,5 1,7 1,8 2,3 2,6 3,4 4,7 5,6 6,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 6

12. 2nd HAM=3500 : 1,2 1,5 1,7 1,8 2,3 2,6 3,4 4,7 5,6 7,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 2

13. 2nd HAM=3656 : 1,2 1,5 1,7 2,3 2,6 3,4 3,8 4,7 4,9 5,6 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 4

14. 2nd HAM=3792 : 1,2 1,5 1,7 2,3 2,6 3,4 3,8 4,7 5,6 5,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

15. 2nd HAM=3668 : 1,2 1,5 1,7 2,3 2,6 3,4 3,8 4,7 5,6 6,9 8,9 Partial

(1 6) (2) (3) Ternary

Best Actuator for 1 Inversion of 1 link is 6

16. 2nd HAM=3748 : 1,2 1,5 1,7 2,3 2,6 3,4 4,7 4,8 5,6 5,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 4

Best Actuator for 9 Inversion of 9 link is 1

17. 2nd HAM=3696 : 1,2 1,5 1,7 2,3 2,6 3,4 4,7 5.6 5,8 6,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 6

18. 2nd HAM=3552 : 1,2 1,4 1,7 1,8 2,3 2,9 3,4 3,6 5,6 5,7 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

19. 2nd HAM=2832 : 1,2 1,4 1,7 1,8 2,3 3,4 3,6 3,9 5.6 5,7 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

20 2nd HAM=3220 1,2 1,4 1,7 1,8 2,3 3,4 3,6 5,6 5,7 5,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

21 2nd HAM=3624 : 1,2 1,4 1,7 1,8 2,3 3,4 3,6 5,6 5,7 6,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

22. 2nd HAM=3624 : 1,2 1,4 1,7 1,8 2,3 3,4 3,6 5,6 5,7 7,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

23. 2nd HAM=3672 : 1,2 1,4 1,7 2,3 2,8 3,4 3,6 4,9 5,6 5,7 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

Best Actuator for 4 Inversion of 8 link is 3

24. 2nd HAM=3592 : 1,2 1,4 1,7 2,3 2,8 3,4 3,6 5,6 5,7 5,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 5

25. 2nd HAM=3520 : 1,2 1,4 1,7 2,3 3,4 3,6 5,6 5,7 5,8 6,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

26. 2nd HAM=3816 : 1,2 1,4 1,7 2,3 3,4 3,6 5,6 5,7 6,8 7,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

27. 2nd HAM=3448 : 1,2 1,4 1,8 2,3 2,9 3,4 3,5 3,7 5,6 6,7 8,9 Fractional

Best Actuator for 1 Inversion of 1 link is 3

Best Actuator for 2 Inversion of 2 link is 3

28. 2nd HAM=3168 : 1,2 1,4 1,8 2,3 3,4 3,5 3,7 3,9 5,6 6,7 8,9 Fractional

Best Actuator for 1 Inversion of 1 link is 3

29. 2nd HAM=3396 : 1,2 1,4 1,8 2,3 3,4 3,5 3,7 5,6 6,7 6,9 8,9 Partial

Best Actuator for 1 Inversion of 1 link is 3

30. 2nd HAM=3256 : 1,2 1,4 2,3 2,8 3,4 3,5 3,7 3,9 5,6 6,7 8,9 Fractional

Best Actuator for 1 Inversion of 1 link is 3

31. 2nd HAM=3404 : 1,2 1,4 2,3 2,8 3,4 3,5 3,7 4,9 5,6 6,7 8,9 Fractional

Best Actuator for 1 Inversion of 1 link is 3

32. 2nd HAM=3320 : 1,6 1,7 2,3 2,5 3,4 4,5 4,6 4,8 6,9 7,8 7,9 Fractional

Best Actuator for 1 Inversion of 1 link is 4