CHAPTER - 2

CHAPTER-2

STRUCTURAL SYNTHESIS OF PLANAR KINEMATIC CHAINS

Introduction

Machine designers have been synthesizing kinematic chains unconsciously since time immemorial. An earlier stage in the synthesis process is number synthesis wherein one chooses the number of binaries, ternaries etc. so as to build a chain with desired degree of freedom

Unfortunately, however, structural synthesis, wherein the designer determines the exact topological form in which links are combined, has not yielded to such direct approaches. The four-bar, six-bar and eight-bar chains are simple enough to be assembled by visual inspection. Ten-bar chains demand greater skill, patience and draftsmanship.

The reason why designers have been plodding through so many new routes instead of sticking to what ought to have been a straight-as-an-arrow path is easy to visualize. The total set of all possible chains for a given number of links and family descriptions can be extremely long, but yet can be prepared after some effort. The crux of the problem all these years has been the absence of reliable and computationally efficient technique to pick the non-isomorphic chains To put it bluntly, isomorphism tests are cumbersome slow or unreliable.

The present method, entirely different from other methods, envisages generation of n-link, f-degree-of-freedom chains from a basic chain, i.e. distinct (n-2) link, f-d.o.f. chain in two ways [26]. The first of these is joining a string of two binary links to (n-2) link, f-d.o.f chain. It may be noted that all the chains generated in this manner will consist of at least one string of two binary links. To develop the n-link, f-d o.f chains that do not have such strings of binary links, the existence of which cannot be ruled out, the second approach is adopted. In the second approach a (n-2)-link, f-.d.o.f. chain is first converted into (n-1) link, (f+1) d.o.f. chain by incorporating a link at a distinct joint and then reconverting it into n-link, f-d.o.f chains by incorporating a link across two of its links. Each (n-2)-link, f-d.o.f basic chain will lead to a number of n-link,f-d.o.f chains and as a result the number of chains generated by all the basic chains in a particular category will be large. To save the time and effort, care is taken at the beginning to avoid formation of degenerate and isomorphic chains. The Hamming number technique [25], which was originally developed to test isomorphism, is explored further to reveal symmetry, identity among links and joints of a chain without any additional effort. This information is utilized in generating only the distinct chains from each of the basic chains. However, all the distinct chains resulting from all the basic (n-2)-link, f-d.o.f chains, when grouped together, may not be distinct and some of them may be identical. Thus testing for isomorphism which uses secondary and tertiary chain Hamming string becomes essential but this is limited to a very small number of chains there by making present structural synthesis algorithm computationally efficient and stronger.

Literature Review

Machine designers have been synthesizing kinematic chains unconsciously since time immemorial, but it was Releaux who first made a systematic attempt, though his views were limited to type synthesis [1]. But synthesis as an independent area of study gained recognition only after Hartenberg and Denavit published their classic book [1]. In this text and in the considerable work by many other researchers over the last four decades, dimensional synthesis has claimed the lion's share because of the possible solutions are truly infinite. An earlier stage in the synthesis process is number synthesis wherein one chooses the number of binaries, ternaries etc. so as to build a chain with desired degree of freedom. Gruebler [2] proposed a very useful equation and Crossley [3] introduced an algorithm for applying it.

Unfortunately, however, structural synthesis, wherein the designer determines the exact topological form in which links are combined, has not yielded to such direct approaches. The four-bar, six-bar and eight-bar chains are simple enough to be assembled by visual inspection. Ten-bar chains demand greater skill, patience and draftsmanship. Alt [4] attempted to list all constrained distinct ten-bar chains, Crossley [5], too did the same and after comparing the results spotted omissions on both sides. Upon rectifying these errors he arrived, for the first time [6], at the correct count of 230 and got the same number working with Davies [7] by using Frank notation. But along with the notation certain amount of complexity quietly crept in The author's [7] admit, "To arrange .. the links into molecules such that all valencies are satisfied requires some ingenuity". They also warn that "with the method of Franke's census has a tendency to be incomplete with too small a number". Kiper and Schian also applied this notation for generating 12-bar chains, but obtained a count 6855 chains first [8] and 6856 chains later [9] which is still six less than the recent count by Hwang and Hwang [10]. Manolescu [11] had earlier used Assur groups but that method has opposite draw back: isomorphic forms infiltrate into results and so the problem of weeding them out still remains [7]. Manolescu [12] later tried a different approach by transforming Barnov trusseses using "graphisation". Mruthyanjaya [13, 14] started with multiple jointed binary chains and transformed them gradually in stages until all the joints become simple joints. But by far the most popular synthesis has been via graph theory, first suggested by Crossley [15] and owing mainly to Freudenstein's pioneering and sustained work. Dobrajansky and Freudenstein [16] invoked this branch of mathematics for systematic enumeration of spatial mechanisms. Two major references are Freudenstein application [17] of Poyla's theory of enumeration and Woo's paper [18] with figures of all the 230 10-bar chains obtained using the two and three-degree of freedom plane linkages Tuttle et al [20] invoked finite groups yet another branch of mathematics, but suggest that a "a general algorithm will have to use concept from both graph theory and group theory.

Uicker and Raicu [21] proposed the characteristic polynomial, but the computations are rather tedious. Moreover, the method yields identical polynomials for some distinct-ten bar and larger chains, a deficiency pointed out by Sohn and Freudenstein [19] and also by Mruthyunjaya and Balasubramanian [14]. The degree matrix proposed in latter paper was claimed to be more reliable, but computations more involved due to presence of numerals larger than 1 in the adjacency matrix. In addition, the method fails for two pairs of 12-bar 1-d.o.f. chains as pointed by Hwang and Hwang [10] Dube and Rao [22] proposed a distance matrix instead of adjacency matrix for computing the polynomial. The mincode method proposed by Ambekar and Agrawal [23] also suffers from this handicap To put it bluntly, isomorphism tests are cumbersome slow or unreliable.

Recently Rao and Raju [25] proposed Secondary Hamming number as a simple and fast test for isomorphism This test discriminates among all the 230 1-d.o.f. 10-bar chains as well as 97 3-d.o f chains subsequently, this test was applied successfully on the two pairs of 1-d o f. 12-bar chains that disobeyed the degree code method, as pointed out in [10].

Conclusion from literature review

The scan on literature on the methods of synthesis points out following objectives,

- Must generate all chains of a given category;
- Ratio of total number of chains generated to the number of distinct chains must be low;
- Should aid in elimination of degenerate chains and structure;
- Should facilitate synthesis of chain with different type of joint;
- Sketching of chains must be simple;
- Should be compatible for computerization.

Definitions and Terminology's

Distinct link pair: Any two links considered together forms a link pair. Some of the pairs may be identical to other link pairs in every respect and such link pairs are not considered distinct.

Distinct joint: Joints having unique joint strings.

Avoiding adjacent link pair: Link pairs, formed by forbidden links and links participating in 4-bar loop.

Dyad: Two binary links with a simple joint.

Simple jointed kinematic chain: Closed, connected and constrained assemblage of links with turning joints.

Structure: Chain with zero degree of freedom

Degenerate Kinematic chain: A kinematic chain with positive degree of freedom having a sub-chain with zero or less degrees of freedom

Parent chain: Simple jointed kinematic chain, which is the starting point for the generation of next generation of chain with more number of links.

Isomorphism:Two chains (graphs) are isomorphic if there is one-to–one correspondence between links (vertices) of one chain (graph) and those of other such that two links (vertices) of a chain (graph) are connected by a joint (edge) if and only if the corresponding links (vertices) of the other chain (graph) are connected by a joint (edge).

Method

The methodology is to generate n-link, f- d.o.f chains from a basic chain, i.e., (n-2)-link, f-d.o.f chains in three steps. Each (n-2)-link, f-d.o.f basic chain will usually lead to a number of n-link, f-d.o.f basic chains and as a result number of chains generated by all the basic chains in a particular category will be quite large. The methodology avoids formation of degenerate chains and isomorphic chains right at the beginning. The Hamming number technique [25], which was originally, developed to test

isomorphism, is explored further to reveal symmetry, identity among links and joints of a chain without any additional effort. The information is utilized in generating only the distinct chains from each of the basic chains, which is the strength of the methodology.

STEP-I

A string of two binary links (dyad) is added to two eligible links of the basic chain such that number of joint increases by three and number of links increases by two. This method of generation is termed as distinct link pair method and abbreviated as DLP. **Distinct link pair (DLP)**

(n-2)-link,f-d o f .. .. n-link ,f-d o f

basic chain **synthesized chain**

STEP-II

It consists of two stages

Stage-I The basic chain (n-2)-link, f-d.o.f is converted into (n-1)-link, f+1-d.o.f chain by incorporating a binary link at the eligible joints of the basic chains. This methodology of generation is termed as distinct joint method and abbreviated as DJ. Here intermediate chains (n-1)-link,f+1-d.o.f are generated.

Distinct joint (DJ)

(n-2)link,f-d.of.............. .... (n-1)link,f+1-d.o.f

Stage-II

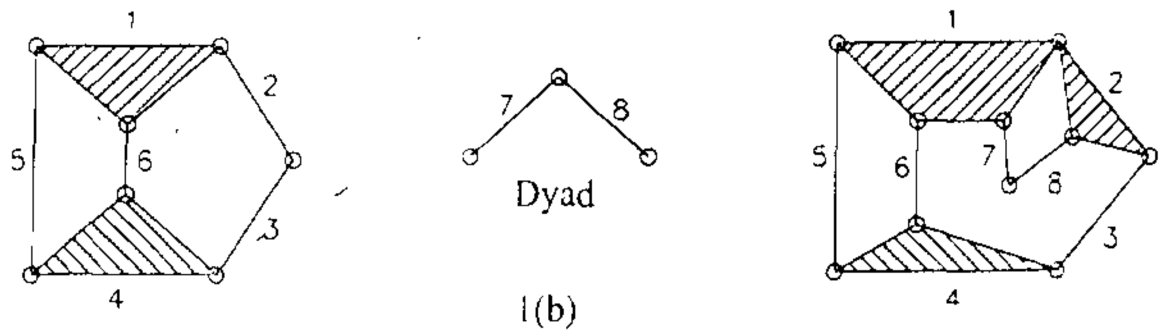
The intermediate chains (n-1)-link, f+1-d.o.f are converted into n-link, f-d.o.f chains by introducing a binary link across eligible links called avoiding adjacent link pair such that number of links increases by one and joints increases by two. The generation by this methodology is termed as avoiding adjacent link pair method and abbreviated as AALP.

Avoiding adjacent link pair (AALP)

(n-1)-link,f+1-d.o.f.. ....n-link,f-d.o.f

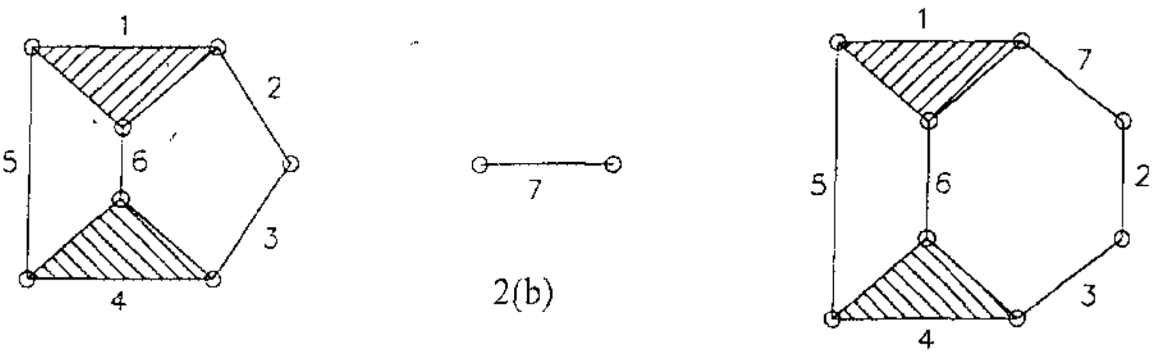
STEP-III

All the chains generated through step-I and step-II form the basic chain, when grouped together lead to few identical chains. To eliminate Isomorphic chains Primary Hamming number; Secondary Hamming number technique is used.



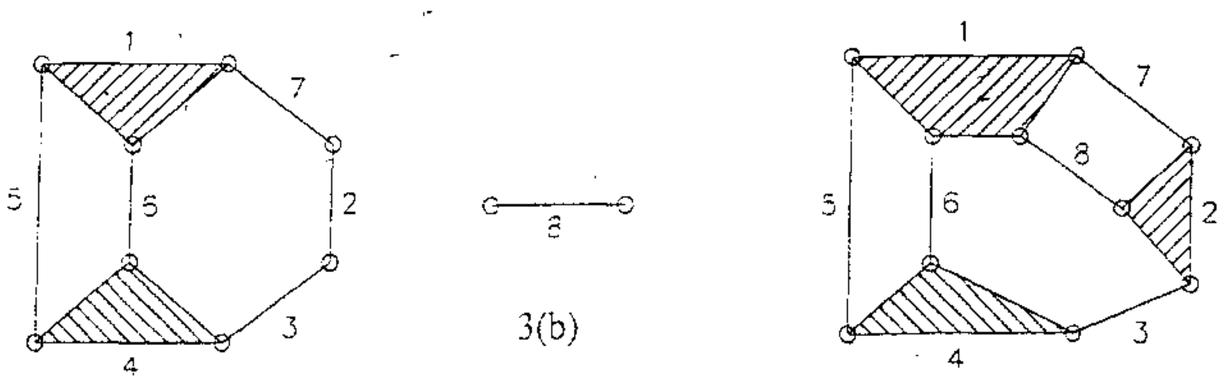
1(a) 1(c)

Fig-1 (DLP)



2(a) 2(c)

Fig-2 (DJ)



3(a) 3(c)

Fig-3 (AALP)

Illustration

Consider the generation of 8-link, 1-d.o.f chains from one of its basic chain 6-link, 1-d.o.f shown in Fig.1:

Step-I is to identify distinct link pair of the chain in Fig-1 so that chain may be converted into 8-link, 1-d o.f using DLP. The connectivity matrix, Hamming matrix and Frequency matrix is obtained as in [25]. Possible link pairs of chain Fig.1 are 12,13,14,15,16,23,24,25,26,34,35,36,45,46 and 56. To identify a distinct link pair a table is formulated below,

Table for computing distinct link pairs

Link Pair	Link pair – Hamming string (1)	Hamming No. H_{ij} (2)	Link directly connected/ separated (3)	Eligible	Not eligible
12	20 – 301101 – 16 – 111201	5	C	E	-
13	20 – 301101 – 16 – 111201	3	S	E	-
14	20 – 301101 – 20 – 301101	2	S	E	-
15	20 – 301101 – 14 – 200202	5	C	E	-
16	20 – 301101 – 14 – 200202	5	C	-	NE
23	16 – 111201 – 16 – 111201	4	C	E	-
24	20 – 301101 – 16 – 111201	3	S	-	NE
25	16 – 111201 – 14 – 200202	2	S	E	NE

26	16 – 111201 – 14 – 200202	2	S	-	NE
34	20 – 301101 – 16 – 111201	5	C	-	NE
35	16 – 111201 – 14 – 200202	2	S	-	NE
36	16 – 111201 – 14 – 200202	2	S	-	NE
45	20 – 301101 – 14 – 200202	5	C	-	NE
46	20 – 301101 – 14 – 200202	5	C	-	NE
56	20 – 301101 – 14 – 200202	0	S	E	

First column of the table contains all possible link pairs. Second column contains Link pair Hamming strings, where in rows of frequency matrix of corresponding participating links are arranged in descending order of their link Hamming numbers. For e.g., link pair 12 of Fig.1 has participating links 1 and 2. Frequency matrix details of link 1 is 20-301101 and link 2 is 16-111201, they are arranged in descending order of link Hamming number as 20-301101-16-111201. In column 3 the Hamming value from the Hamming matrix corresponding to link pair is entered. For e.g., for link pair 12 the value is 5, which is the value of h_{12} in the Hamming matrix of Fig.1. Column 4 contains details whether the two links forming the pair are in direct contact or not. For e.g., in link pair 12, link 1 is in direct contact with link 2 hence C is entered or otherwise S is to be entered meaning the two links are separated. In the same fashion table is completed for all possible link pair as explained for link pair 12. Link pairs having unique values in column 2,3 and 4 are termed as distinct link pair (DLP). For Fig.1 eligible link pairs are 12,13,14,15,23,25 and 56. Possible link pairs are 15 in number but method has identified only 07 distinct link pairs which will be used to generate 8-link, 1-d.o.f chains. Hence 08 chains are eliminated. Let us consider generation of 8-link, 1-d.o.f chain from Fig.1 using 12 as a link pair.

FIG.1 (a,b,c)

STEP-II has two stages. In stage I distinct joints of the basic chain (Fig.1) are identified so that it can be converted into intermediate chain 7-link, 2-d.o.f. Joints in the basic chain (Fig.1) are 12,15,16,23,34,45 and 46. To obtain distinct joints a table is formulated below,

Table for computing distinct joint

Joints	**Link pair - Hamming string (2)**	**Hamming No. (3)**	**Eligible**	**Not eligible**
12	20 – 301101 - 16 – 111201	5	E	-
15	20 – 301101 - 14 – 200202	5	E	-
16	20 – 301101 - 14 – 200202	5	-	NE
23	16 – 111201 - 16 – 111202	3	E	-
34	20 – 301101 - 16 – 111201	5	-	NE
45	20 – 301101 - 14 – 200202	5	-	NE
46	20 – 301101 - 14 – 200202	5	-	NE

Column 1 in the table contains all the joints present in the basic chain (Fig.1). Column 2 has details of joint strings, where the link Hamming string of link participating in the joint is listed in descending order. For e.g., joint 12 has link participating 1 and 2 hence their link Hamming value with Frequency Hamming details of links forming that joint is entered. Column 3 has value of Hamming element, from the Hamming matrix of the chain corresponding to the participating link, in the joint. For e.g., joint 12 has Hamming value equal to 5 as element $h_{12}=5$ in the Hamming matrix. Table is completed for all other rows as has been done for the first row. All the rows are compared one to one from every other row the joint corresponding to the unique rows is termed as distinct joints. The possible joints are 12,15,16,23,34,45 and 46. The distinct joints from the methodology are 12,15 and 23. Lets us consider the generation of a 7-link, 2-d.o.f chain from Fig.2 (a,b,c).

In stage-II the intermediate chain 7-Link, 2-d.o.f are converted into 8-link, 1-d.o.f by adding binary link across the avoiding adjacent link pair of the intermediate chains. Formulation of avoiding adjacent link pair is similar to the link pair with an additional consideration that two links should not be directly connected and corresponding Hamming element value should not be zero as is shown in table below for AALP (56). The same is tabulated below.

Table for computing Avoiding adjacent link pair

AALP	AALP Hamming -string	Hamming number	Eligible	Non-Eligible
12	25-401101-22-042001	3	E	-
13	25-401101-20-210301	5	E	-
14	25-401101-25-401101	2	E	-
15	25-401101-18-210202	5	E	-
16	25-401101-18-210202	-	-	NE
24	25-401101-22-042001	3	-	-
25	22-042001-18-210202	4	E	-
26	22-042001-18-210202	4	-	NE
35	20-210301-18-210202	2	E	-
36	20-210301-18210202	2	-	NE
37	20-210301-20-210301	2	E	-
47	25-401101-20-210301	5	-	NE
56	18-210202-18-210202	0	-	NE
57	20-210301-18-210202	2	-	NE
67	20-210301-18-210202	2	-	NE

Possible avoiding adjacent link pairs is 12,13,14,15,24,25,26,35,36,37,47,56,57 and 67. Distinct avoiding adjacent link pair is 12,13,14,15,25,35 and 37. Let us consider generation of 8-link, 1-d.o.f chain from 7-link, 2-d.o.f chain using avoiding adjacent link pair (AALP) 12.

Number of such chains will be generated from basic chains, when grouped together some of them may be identical. To eliminate isomorphic chain Primary Hamming number technique and Secondary Hamming number technique are used as an isomorphic index [25]

Applications

The computerized methodology is implemented to generate single and multi-degree of freedom chains up to 13-links, 2-d o f

Results

Kiper and Schian [8,9] reported that there are 6856 kinematic chains with 12-links, 1-d.o.f Hwang and Hwang [10] have reported 6862 distinct chains with 12-links, 1-d.o.f. Authors have synthesized 6856 chains. Further more author have synthesized 29008 chains with up to 13-link, 2-d.o f against 29704 reported by Hwang and Hwang. Isomorphism test eliminates the counter examples reported in [48]. Number of Inversions and type of freedom agree with the results reported earlier Some new results are reported.

TABLE: Number of kinematic chains generated by the methodology in the specified category

Number of links	**Degree of freedom**	**Number of planar kinematic chains**	**CPU time**
6	1	2	.5 sec
7	2	4	.5 sec
8	1	16	1.2 sec
9	2	40	3.2 sec
10	1	230	1min 10sec
11	2	839	13min
12	1	6856	02 hrs 29 min
13	2	29008	06 hrs 02 min

Conclusion

The genesis of the chains is clearly defined, as synthesis algorithm is progressive All the time the algorithm is processing real chains so chances of missing few distinct chains are scarce The distinct chains are generated from basic chain and as isomorphic, structure and degenerate chains are eliminated in the beginning, ratio of distinct chain to total number of generated chain is high. *Further, the algorithm generates higher chain* through distinct link pair, distinct joint and forbidden link pair therefore checking of isomorphism is restricted to comparatively lower number of chains which further strengthen the algorithm. Data generated during the synthesis procedure leads to kinematic analysis. Intermediate chains are the by-products (higher d.o.f. chains). With slight modification the algorithm can be applied to computer aided structural synthesis of kinematic chains with multiple joints. Methodology leads to an unified computer methodology for the synthesis and analysis of simple *jointed kinematic chains of any number of links and* d.o.f. Sketching of simple jointed kinematic chain is progressive and hence easy. The processing time indicates that the algorithm is computationally efficient and compatible for digital computers

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Appendix

SYNTHESIS DETAILS OF SINGLE DEGREE OF FREEDOM CHAINS

****

6 Link,1 d.o.f. to 8 Link,1 d.o.f. using Distinct Link Pairs

****

##########################<< 1 >>################################

1st Ham=100 2nd Ham=888

Original :1,2 1,4 1,5 2,3 2,6 3,4 5,6

Pairs : 1,2 1,3 1,4 2,3 2,4 3,4 3,6 4,6 5,6

1. LINK PAIR 1 2 Ham=2288

2. LINK PAIR 1 3 Ham=2508

3. LINK PAIR 1 4 Ham=2464

4. LINK PAIR 3 4 Ham=2576

5. LINK PAIR 3 6 Ham=2456

6. LINK PAIR 4 6 Ham=2656

##########################<< 2 >>################################

1st Ham=100 2nd Ham=808

Original :1,2 1,4 1,5 2,3 3,4 3,6 5,6

Pairs : 1,2 1,3 1,5 1,6 2,3 2,4 2,5 3,5 3,6 5,6

7. LINK PAIR 1 3 Ham=2096

8. LINK PAIR 1 5 Ham=2488

9. LINK PAIR 1 6 Ham=2448

10. LINK PAIR 2 4 Ham=2448

11 LINK PAIR 2 5 Ham=2612

12 LINK PAIR 5 6 Ham=2600

****

7 Link,2 d.o.f. to 8 Link,1 d.o.f. using Avoiding adjacent link

****

###########################<< 1 >>##################################

1st HAM=148 2nd HAM=1468

ORIGINAL . 1,4 1,5 1,7 2,3 2,6 2,7 3,4 5,6

FLPs . 1,2 1,3 3,5 3,6 3,7 4,5 4,6

13. FLP 3 5 Ham=2672

14. FLP 3 6 Ham=2424

###########################<< 2 >>##################################

1st HAM=148 2nd HAM=1460

ORIGINAL : 1,2 1,5 1,7 2,3 2,6 3,4 4,7 5,6

FLPs : 1,3 1,4 3,5 3,6 3,7 4,5 4,6

###########################<< 3 >>##################################

1st HAM=148 2nd HAM=1284

ORIGINAL : 1,2 1,4 1,7 2,3 3,4 3,6 5,6 5,7

FLPs : 1,5 1,6 2,5 2,6 6,7

15. FLP 1 5 Ham=2072

16. FLP 6 7 Ham=2608

###########################<< 4 >>##################################

1st HAM=144 2nd HAM=1160

ORIGINAL : 1,2 1,4 2,3 3,4 3,5 3,7 5,6 6,7

FLPs : 1,6

SYNTHESIS OF MULTI DEGREE OF FREEDOM PLANAR KINEMATIC CHAINS

****

7 Link,2 d.o.f. to 9 Link,2 d.o.f. using Distinct Link Pairs

****

###########################Chain 1 #################################

1st Ham=148 2nd Ham=1468

Original :1,4 1,5 1,7 2,3 2,6 2,7 3,4 5,6

Pairs : 1,2 1,3 1,4 1,7 3,4 3,5 3,6 3,7

1. LINK PAIR 1 2 Ham=2976

2. LINK PAIR 1 3 Ham=3492

3. LINK PAIR 1 4 Ham=3508

4. LINK PAIR 1 7 Ham=3492

5. LINK PAIR 3 4 Ham=3808

6. LINK PAIR 3 5 Ham=3880

7. LINK PAIR 3 6 Ham=3640

8. LINK PAIR 3 7 Ham=3696

###########################Chain 2 #################################

1st Ham=148 2nd Ham=1460

Original :1,2 1,5 1,7 2,3 2,6 3,4 4,7 5,6

Pairs : 1,2 1,3 1,4 1,5 1,6 1,7 3,4 3,5 3,6 3,7 4,5 5,6

9. LINK PAIR 1 2 Ham=3056

10. LINK PAIR 1 5 Ham=3444

11. LINK PAIR 1 6 Ham=3424

12. LINK PAIR 1 7 Ham=3500

13. LINK PAIR 3 4 Ham=3656

14. LINK PAIR 3 5 Ham=3792

15. LINK PAIR 3 6 Ham=3668

16. LINK PAIR 4 5 Ham=3748

17. LINK PAIR 5 6 Ham=3696

###########################Chain 3 #################################

1st Ham=148 2nd Ham=1284

Original 1,2 1,4 1,7 2,3 3,4 3,6 5,6 5,7

Pairs 1,2 1,3 1,5 1,6 1,7 2,4 2,5 2,6 5,6 6,7

18 LINK PAIR 1 2 Ham=3552

19 LINK PAIR 1 3 Ham=2832

20. LINK PAIR 1 5 Ham=3220

21. LINK PAIR 1 6 Ham=3624

22. LINK PAIR 1 7 Ham=3624

23. LINK PAIR 2 4 Ham=3672

24. LINK PAIR 2 5 Ham=3592

25. LINK PAIR 5 6 Ham=3520

26. LINK PAIR 6 7 Ham=3816

##########################Chain 4 ###############################

1st Ham=144 2nd Ham=1160

Original :1,2 1,4 2,3 3,4 3,5 3,7 5,6 6,7

Pairs : 1,2 1,3 1,5 1,6 2,3 2,4 2,5

27. LINK PAIR 1 2 Ham=3448

28. LINK PAIR 1 3 Ham=3168

29. LINK PAIR 1 6 Ham=3396

30. LINK PAIR 2 3 Ham=3256

31. LINK PAIR 2 4 Ham=3404

****

8 Link,1 d.o.f. to 9 *Link*,2 d.o.f. using *Distinct Joints*

****

######################## CHAIN 1 ########################

1st HAM=208 2nd HAM=2288

ORIGINAL : 1,2 1,4 1,5 1,7 2,3 2,6 2,8 3,4 5,6 7,8

JOINTs . 1,2 1,4 3,4

######################## CHAIN 2 ########################

1st HAM=212 2nd HAM=2508

ORIGINAL : 1,2 1,4 1,5 1,7 2,3 2,6 3,4 3,8 5,6 7,8

JOINTs : 1,2 1,4 1,5 1,7 2,3 2,6 3,4 3,8 5,6 7,8

######################## CHAIN 3 ##########################

1st HAM=212 2nd HAM=2464

ORIGINAL : 1,2 1,4 1,5 1,7 2,3 2,6 3,4 4,8 5,6 7,8

JOINTs : 1,2 1,5 2,3 2,6 5,6

######################## CHAIN 4 ##########################

1st HAM=216 2nd HAM=2576

ORIGINAL : 1,2 1,4 1,5 2,3 2,6 3,4 3,7 4,8 5,6 7,8

JOINTs : 1,2 1,5 5,6

######################## CHAIN 5 ##########################

1st HAM=216 2nd HAM=2456

ORIGINAL : 1,2 1,4 1,5 2,3 2,6 3,4 3,7 5,6 6,8 7,8

JOINTs : 1,2 1,4 2,3 3,4 3,7 7,8

32. JOINT 3 7 HAM=3544

######################## CHAIN 6 ##########################

1st HAM=216 2nd HAM=2656

ORIGINAL : 1,2 1,4 1,5 2,3 2,6 3,4 4,7 5,6 6,8 7,8

JOINTs : 1,2 1,4 1,5 3,4 4,7 7,8

33. JOINT 4 7 HAM=3752

######################## CHAIN 7 ##########################

1st HAM=208 2nd HAM=2096

ORIGINAL : 1,2 1,4 1,5 1,7 2,3 3,4 3,6 3,8 5,6 7,8

JOINTs : 1,2 1,5 5,6

######################## CHAIN 8 ##########################

1st HAM=212 2nd HAM=2488

ORIGINAL : 1,2 1,4 1,5 1,7 2,3 3,4 3,6 5,6 5,8 7,8

JOINTs 1,2 1,5 1,7 2,3 3,6 5,6 5,8 7,8

34 JOINT 1 7 HAM=3424

######################## CHAIN 9 ##########################

1st HAM=212 2nd HAM=2448

ORIGINAL : 1,2 1,4 1,5 1,7 2,3 3,4 3,6 5,6 6,8 7,8

JOINTs : 1,2 1,5 1,7 2,3 3,6 5,6 6,8 7,8

35. JOINT 1 7 HAM=3468

######################## CHAIN 10 ##########################

1st HAM=216 2nd HAM=2448

ORIGINAL : 1,2 1,4 1,5 2,3 2,7 3,4 3,6 4,8 5,6 7,8

JOINTs : 1,2 1,5 5,6

######################## CHAIN 11 ##########################

1st HAM=216 2nd HAM=2612

ORIGINAL : 1,2 1,4 1,5 2,3 2,7 3,4 3,6 5,6 5,8 7,8

JOINTs : 1,2 1,4 1,5 2,3 2,7 3,4 3,6 5,6 5,8 7,8

36. JOINT 2 7 HAM=3784

######################## CHAIN 12 ##########################

1st HAM=216 2nd HAM=2600

ORIGINAL : 1,2 1,4 1,5 2,3 3,4 3,6 5,6 5,7 6,8 7,8

JOINTs : 1,2 1,5.5,6 5,7 7,8

37. JOINT 5 7 HAM=3664

######################## CHAIN 13 ##########################

1st HAM=216 2nd HAM=2672

ORIGINAL : 1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 5,6 5,8

JOINTs : 1,4 1,5

38. JOINT 1 5 HAM=3368

######################## CHAIN 14 ##########################

1st HAM=216 2nd HAM=2424

ORIGINAL 1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 5,6 6,8

JOINTs 1,4 1,7 2,3 2,7 3,4 3,8

######################## CHAIN 15 ##########################

1st HAM=212 2nd HAM=2072

ORIGINAL : 1,2 1,4 1,7 1,8 2,3 3,4 3,6 5,6 5,7 5,8

JOINTs . 1,2 2,3 3,6

######################## CHAIN 16 ##########################

1st HAM=216 2nd HAM=2608

ORIGINAL : 1,2 1,4 1,7 2,3 3,4 3,6 5,6 5,7 6,8 7,8

JOINTs : 1,2 1,7

39. JOINT 1 7 HAM=3480

****

7 Link,2 d.o.f. to 9 Link,2 d.o.f. using Distinct Joints & Avoiding adjacent link pair

****

################################Chain 1 ###########################

1st Ham =148 2nd Ham=1468 :

ORIGINAL : 1,4 1,5 1,7 2,3 2,6 2,7 3,4 5,6

JOINTS : 1,4 1,7 3,4

################################Chain 2 ###########################

1st Ham =148 2nd Ham=1460 :

ORIGINAL : 1,2 1,5 1,7 2,3 2,6 3,4 4,7 5,6

JOINTS : 1,2 1,5 1,7 3,4 5,6

################################Chain 3 ###########################

1st Ham =148 2nd Ham=1284 :

ORIGINAL : 1,2 1,4 1,7 2,3 3,4 3,6 5,6 5,7

JOINTS : 1,2 1,7 5,6

################################Chain 4 ###########################

1st Ham =144 2nd Ham=1160 :

ORIGINAL 1,2 1,4 2,3 3,4 3,5 3,7 5,6 6,7

40 JOINT 1 2 FLP 1 2 Ham=3320

Representation of 8-Link, 1- d.o.f. Chains

Primary Hamming	Secondary Hamming	Connectivity	Representation Code
1. 208	2288	1,2 1,4 1,5 1,7 2,3 2,6 2,8 3,4 5,6 7,8	4 6(1) DLP(12)
2. 212	2508	1,2 1,4 1,5 1,7 2,3 2,6 3,4 3,8 5,6 7,8	4 6(1) DLP(13)
3 212	2464	1,2 1,4 1,5 1,7 2,3 2,6 3,4 4,8 5,6 7,8	4 6(1) DLP(14)
4. 216	2576	1,2 1,4 1,5 2,3 2,6 3,4 3,7 4,8 5,6 7,8	4 6(1) DLP(34)
5. 216	2456	1,2 1,4 1,5 2,3 2,6 3,4 3,7 5,6 6,8 7,8	4 6(1) DLP(36)
6. 216	2656	1,2 1,4 1,5 2,3 2,6 3,4 4,7 5,6 6,8 7,8	4 6(1) DLP(46)
7. 208	2096	1,2 1,4 1,5 1,7 2,3 3,4 3,6 3,8 5,6 7,8	4 6(2) DLP(13)
8. 212	2488	1,2 1,4 1,5 1,7 2,3 3,4 3,6 5,6 5,8 7,8	4 6(2) DLP(15)
9. 212	2448	1,2 1,4 1,5 1,7 2,3 3,4 3,6 5,6 6,8 7,8	4 6(2) DLP(16)
10. 216	2448	1,2 1,4 1,5 2,3 2,7 3,4 3,6 4,8 5,6 7,8	4 6(2) DLP(24)
11. 216	2612	1,2 1,4 1,5 2,3 2,7 3,4 3,6 5,6 5,8 7,8	4 6(2) DLP(25)
12. 216	2600	1,2 1,4 1,5 2,3 3,4 3,6 5,6 5,7 6,8 7,8	4 6(2) DLP(56)
13. 216	2672	1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 5,6 5,8	4 6(1) DJ(12) FLP(35)
14. 216	2424	1,4 1,5 1,7 2,3 2,6 2,7 3,4 3,8 5,6 6,8	4 6(1) DJ(12) FLP(36)
15. 212	2072	1,2 1,4 1,7 1,8 2,3 3,4 3,6 5,6 5,7 5,8	4 6(1) DJ(15) FLP(15)
16. 216	2608	1,2 1,4 1,7 2,3 3,4 3,6 5,6 5,7 6,8 7,8	4 6(1) DJ(15) FLP(67)

No. Of Chains = 16